Solutions

Chapter 1

Concept Application Exercises

Exercise 1.1

1. $\vec{F} = q[\vec{v} \times \vec{B}] = 2\times10^{-6}[(2\hat{i} - 3\hat{j} + 4\hat{k}) \times (3\hat{j} - 2\hat{k})]$

$= 2\times10^{-6}[-6\hat{i} + 4\hat{j} + 6\hat{k}]$ N

By Newton's law of motion,

$$\vec{a} = \frac{\vec{F}}{m} = \frac{2\times10^{-6}}{5\times10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k})$$

$$= 0.8(-3\hat{i} + 2\hat{j} + 3\hat{k}) \text{ m s}^{-2}$$

2. $\vec{F} \perp \vec{B} \quad \therefore \quad \vec{a} \perp \vec{B} \quad \therefore \quad \vec{a} \cdot \vec{B} = 0$

$\therefore \quad (2\hat{i} + x\hat{j}) \cdot (-3\hat{i} + 2\hat{j} - 4\hat{k}) = 0 \quad \Rightarrow -6 + 2x = 0 \quad \Rightarrow x = 3.$

3. (a) Let velocity of the particle is $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

then $\vec{F} = q\vec{v} \times \vec{B}$

$$\Rightarrow \quad -3.36\times10^{-7}\,\hat{i} + 7.42\times10^{-7}\,\hat{j}$$

$$= -5.6\times10^{-9}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \times (-1.25)\,\hat{k}$$

$$\Rightarrow \quad -3.36\,\hat{i} + 7.42\,\hat{j} = 7\times10^{-2}(-v_x\,\hat{j} + y_y\,\hat{i})$$

$$\Rightarrow \quad v_x = -\frac{7.42\times10^2}{7} = -\,106 \text{ m s}^{-1}$$

$$\text{and} \quad v_y = \frac{-3.36\times10^2}{7} = -\,48 \text{ m s}^{-1}$$

(b) Because $\vec{F} \perp \vec{v}$, so $\vec{v} \cdot \vec{F} = 0$. Let us check it

$$\vec{v} \cdot \vec{F} = (-106\,\hat{i} - 48\,\hat{j} + v_z\hat{k}) \cdot (-3.36\times10^{-7}\,\hat{i} + 7.42\times10^{-7}\,\hat{j})$$

$$= 106\times3.36\times10^{-7} - 48\times7.42\times10^{-7} = 0$$

4. (a) Let $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, then $\vec{F} = q\vec{v} \times \vec{B}$

$$\Rightarrow \quad 8.40\times10^{-2}\,\hat{i} - 5.60\times10^{-2}\,\hat{k}$$

$$= 7\times10^{-6}(-4\times10^{3})\,\hat{j} \times (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k})$$

$$\Rightarrow \quad 84\,\hat{i} - 56\,\hat{k} = 28(-B_x\hat{k} + B_z\hat{i})$$

$$\Rightarrow \quad B_x = \frac{-56}{28} = -2 \text{ T}, \; B_z = -\frac{84}{28} = -3 \text{ T}$$

(b) Because $\vec{F} \perp \vec{B}$, so $\vec{B} \cdot \vec{F} = 0$.

Let us check it

$$\vec{B} \cdot \vec{F} = (-2\,\hat{i} + B_y\,\hat{j} - 3\,\hat{k}).(8.40\times10^{-2}\,\hat{i} - 5.70\times10^{-2}\,\hat{k}) = 0$$

5. In first case, force is along $-\hat{k}$, so $\vec{B}$ should be in xy plane or z component of B should be zero. In second case, force is along x axis, so B cannot be along x-axis. Finally we find that B is along negative y direction.

Let $\quad \vec{B} = -B_0\hat{j}$ and applying $\vec{F} = q\vec{v} \times \vec{B}$

$$-(1.28\times10^{-13}\hat{k}) = (1.6\times10^{-19}) \times [(2\hat{i} + 3\hat{j}) \times (-B_0\hat{j})] \times 10^{6}$$

$$1.28 = 1.6\times2\times B_0 \text{ or } B_0 = 0.4 \text{ T}$$

$$\vec{B} = -(0.4\,\hat{j}) \text{ T}$$

6. (a) $\vec{F} = q\vec{v} \times \vec{B} = -qv[B_x(\hat{j} \times \hat{i}) + B_z(\hat{j} \times \hat{k})]$

$$= qvB_x\hat{k} - qvB_z\hat{i}$$

(b) $B_x > 0$, $B_z < 0$, sign of B_y does not matter.

(c) $\vec{F} = |q|VB_x\hat{i} - |q|VB_x\hat{k}$

7. (a) $$\vec{F} = q\vec{v} \times \vec{B} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & v \\ B_x & B_y & B_z \end{vmatrix}$$

$$= -qvB_y\hat{i} + qvB_x\hat{j}$$

But $\quad \vec{F} = 3F_0\hat{i} + 4F_0\hat{j}$,

So $\quad 3F_0 = -qvB_y$ and $4F_0 = qvB_x$

$$B_y = -\frac{3F_0}{qv}, \; B_x = \frac{4F_0}{qv}, \; B_z \text{ is arbitraty.}$$

(b) $$B = \frac{6F_0}{qv} = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad \Rightarrow \quad B_z = \pm\frac{11F_0}{qv}$$

8. The force associated with the magnetic field must point in the $\hat{j}$ direction in order to cancel the force of gravity in the $-\hat{j}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{k}$ direction (since $\hat{i} \times (-\hat{k}) = \hat{j}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by equation (with $\phi = 90°$). Therefore, with $m = 1.0\times10^{-2}$ kg, $v = 2.0\times10^{4}$ m/s, and $q = 5.0\times10^{-5}$ C, we find

$$\vec{B} = B_z\hat{k} = -\left(\frac{mg}{qv}\right)\hat{k} = -\left(\frac{1.0\times10^{-2}\times10}{5.0\times10^{-5}\times2.0\times10^{4}}\right)\hat{k} = (-0.10 \text{ T})\hat{k}$$

9. The force on the electron is

$$\vec{F_B} = q\vec{v} \times \vec{B} = q(v_x\hat{i} + v_y\hat{j}) \times (B_x\hat{i} + B_y\hat{j}) = q(v_xB_y - v_yB_x)\hat{k}$$

$$\Rightarrow \quad \vec{F} = q(v_x(3B_x) - v_yB_x)\hat{k}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\hat{k}$ where $F_z = 6.4\times10^{-19}$ N, then we are led to the condition

$$q(3v_x - v_y)B_x = F_z \Rightarrow B_x = \frac{F_z}{q(3v_x - v_y)}$$

Substituting $v_x = 2.0$ m/s, $v_y = 4.0$ m/s, and $q = -1.6\times10^{-19}$ C, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)} = \frac{6.4\times10^{-19}}{(-1.6\times10^{-19})[3(2.0) - 4.0]} = -2.0 \text{ T}$$

10. The magnetic force has a magnitude of $F = |q|\,vB\sin\theta$. The field B and the directional angle θ are the same for each particle. Particle 1, however, travels faster than particle 2. By itself, a faster speed v would lead to a greater force magnitude F. But the force on each particle is the same. Therefore, particle 1 must have a smaller charge to counteract the effect of its greater speed.

We have $\underbrace{F = |q_1|v_1B\sin\theta}_{\text{Particle 1}}$ and $\underbrace{F = |q_2|v_2B\sin\theta}_{\text{Particle 2}}$

Dividing the equation for particle 1 by the equation for particle 2 and remembering that $v_1 = 3v_2$ gives

$$\frac{F}{F} = \frac{|q_1|v_1B\sin\theta}{|q_2|v_2B\sin\theta} \quad \text{or} \quad 1 = \frac{|q_1|v_1}{|q_2|v_2} \quad \text{or} \quad \frac{|q_1|}{|q_2|} = \frac{v_2}{v_1} = \frac{v_2}{3v_2} = \frac{1}{3}$$

11. The positive plate has a charge q and is moving downward with a speed v at right angles to a magnetic field of magnitude B. The magnitude F of the magnetic force exerted on the positive plate is $F = |q|vB \sin 90°$. The charge on the positive plate is related to the magnitude E of the electric field that exists between the plates by $|q| = \varepsilon_0 AE$, where A is the area of the positive plate. Substituting this expression for $|q|$ into $F = |q|\,vB \sin 90°$ gives the answer in terms of known quantities.

$$F = (\varepsilon_0 AE)vB$$
$$= [8.85\times10^{12}](5.0\times10^{-4})(200)\times(20)(5.0) = 8.85\times10^{13}\text{ N}$$

An application of right-hand rule shows that the magnetic force is perpendicular to the plane of the page and directed out of the page, toward the reader.

12. (a) According to right hand rule, if you extend your right hand so that your fingers point along the direction of the magnetic field B and your thumb points in the direction of the velocity v of a positive charge, your palm will face in the direction of the force F on the positive charge.

For the electron in question, the fingers of the right hand should be oriented downward (direction of B) with the thumb pointing to the east (direction of v). The palm of the right-hand points due north (the direction of F on a positive charge). Since the electron is negatively charged, it will be deflected due south.

(b) The acceleration of an electron is given by Newton's second law, where the net force is the magnetic force. Thus,

$$a = \frac{F}{m} = \frac{|q_0|vB\sin\theta}{m}$$

Since the kinetic energy is $\text{KE} = \frac{1}{2}mv^2$, the speed of the electron is $v = \sqrt{2(\text{KE})/m}$. Thus, the acceleration of the electron is

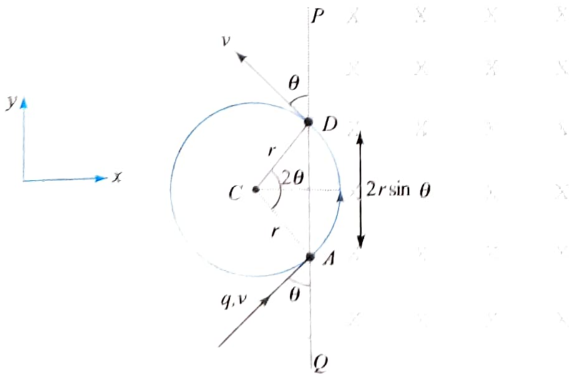
$$a = \frac{|q_0|vB\sin\theta}{m} = \frac{|q_0|\sqrt{\frac{2(\text{KE})}{m}}B\sin\theta}{m}$$

$$= \frac{(1.60\times10^{-19}\text{ C})\sqrt{\frac{2(4.5\times10^{-15}\text{ J})}{9.11\times10^{-31}\text{ kg}}}\times(2.25\times10^{-5}\text{ T})\sin 90°}{9.0\times10^{-31}\text{ kg}}$$

$$= 4.0\times10^{14}\text{ m/s}^2$$

Exercise 1.2

1. The particle will move in the field as shown in figure.



Angle subtended by the arc at the centre $= 2\theta$

(a) Time spent by the charge in magnetic field

$$\omega t = 2\theta \Rightarrow \frac{qB}{m}t = 2\theta \Rightarrow t = \frac{m2\theta}{qB}$$

(b) Distance travelled by the charge in magnetic field

$$= r(2\theta) = \frac{mv}{qB}\times 2\theta$$

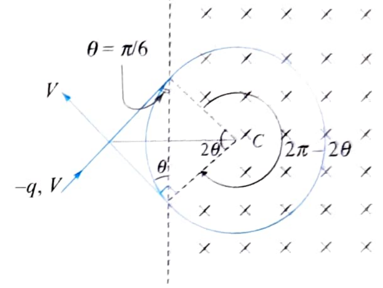
(c) Impulse = change in momentum of the charge

$$= (-mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}) - (mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j})$$
$$= -2mv\sin\theta\,\hat{i}$$

2. (a) $2\pi - 2\theta = 2\pi - 2\times\frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} = \omega t = \frac{qB}{m}t$

$$\Rightarrow \quad t = \frac{5\pi m}{3qB}$$

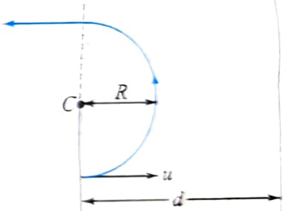
(b) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$



(c) Impulse = charge in linear momentum

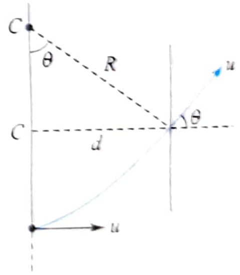
$$= m(-v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j}) - m(v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j})$$
$$= -2mv\sin\theta\,\hat{i} = -2mv\sin\frac{\pi}{6}\,\hat{i} = -mv\,\hat{j}$$

3. (a) $d > \frac{mu}{qB}$ means $d > R$, the particle will complete semicircle in the field.



$\therefore$ time spent $t = \frac{T}{2} = \frac{\pi m}{qB}$

(b) $d < \frac{mu}{qB} \Rightarrow q < R$, the particle will not be able to complete semicircle and it will come out from other side of the field.



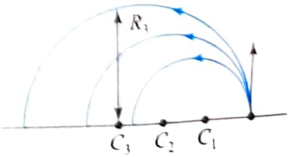
$$\sin\theta = \frac{d}{R}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{d}{R}\right)$$

time spent $\omega t = \theta$

$$\Rightarrow \quad t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$$

4. The path of the particle will be circular. Larger the velocity, larger will be the radius.

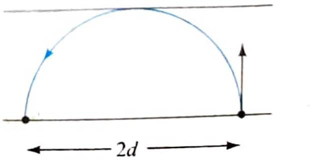


For particle not to strike, $R < d$

$$\therefore \quad \frac{mv}{qB} < d \Rightarrow v < \frac{qBd}{m}$$

For limiting case, $v = \frac{qBd}{m}$, $R = d$

$\therefore$ Coordinate where the particle strikes = $(-2d, 0, 0)$



5. (a) $p = mv = m\left(\dfrac{RqB}{m}\right) = RqB$

$= (5.0 \times 10^{-3}\text{ m})\,(6.0 \times 10^{-19}\text{ C})\,(2.0\text{ T})$

$= 6.0 \times 10^{-21}\text{ kg m s}^{-1}$

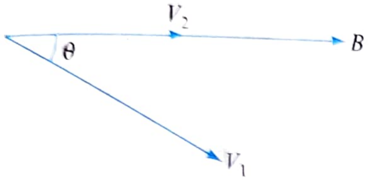
(b) $L = Rp = (5.0 \times 10^{-3}\text{ m}) \times 6.0 \times 10^{-21}$

$= 3.0 \times 10^{-23}\text{ kg m}^2\text{ s}^{-1}$

6. $qV = \dfrac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{\dfrac{2qV}{m}}$

To meet them again and again:

$$v_2 = v_1\cos\theta = \sqrt{\frac{2qV}{m}}\cos\theta$$



Time interval: $T = \dfrac{2\pi m}{qB}$

Distance after which they meet = 1 pitch = v_2T

$$= \sqrt{\frac{2qV}{m}}\cos\theta \times \frac{2\pi m}{qB} = \pi\sqrt{\frac{8Vm}{qB^2}}\cos\theta$$

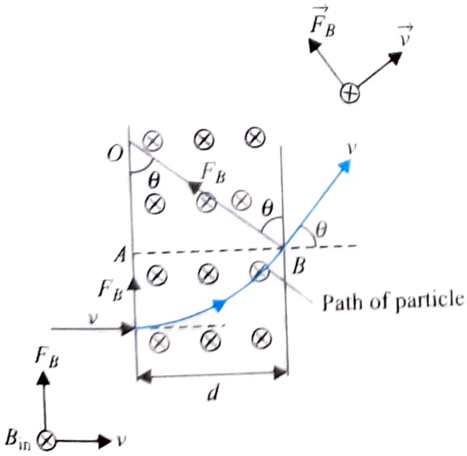
7. $qV = \dfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2qV}{m}}$

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2mV}{qB^2}} \Rightarrow m \propto r^2$$

$$\frac{m_{\text{heavy}}}{m_{\text{light}}} = \left(\frac{l+d}{l}\right)^2$$

8. The radius of circulation is $R = \dfrac{mv}{Bq} = \dfrac{p}{Bq} = \dfrac{\sqrt{2mK}}{Bq}$.

As given, angle of deviation is θ; it is the angle between directions of final velocity and initial velocity.



In ΔOAB, $\sin\theta = \dfrac{d}{R} = \dfrac{dBq}{mv} = \dfrac{dBq}{p} = \dfrac{dBq}{\sqrt{2mK}}$

(a) $p_\alpha = p_p$, $\dfrac{\sin\theta_p}{\sin\theta_\alpha} = \dfrac{q_p}{q_\alpha} = \dfrac{1}{2}$

(b) $K_\alpha = K_p$; hence $\dfrac{\sin\theta_p}{\sin\theta_\alpha} = \dfrac{q_p}{q_\alpha} \times \sqrt{\dfrac{m_\alpha}{m_p}}$

$$= \frac{1}{2} \times \sqrt{\frac{4}{1}} = 1$$

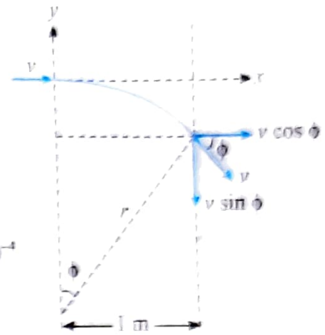
(c) When a charged particle is accelerated through a potential difference V, gain of kinetic energy is qV.

$K_p = q_pV$, $K_\alpha = q_\alpha V$

Now, $\sin\theta = \dfrac{dBq}{\sqrt{2mqV}} = \dfrac{dB\sqrt{q}}{\sqrt{2mV}}$

Hence, $\dfrac{\sin\theta_p}{\sin\theta_\alpha} = \sqrt{\dfrac{q_p}{q_\alpha}} \times \sqrt{\dfrac{m_\alpha}{m_p}} = \sqrt{\dfrac{1}{2}} \times \sqrt{\dfrac{4}{1}} = \sqrt{2}$

9. (a) Radius of circular path $r = \dfrac{mv}{eB}$



From figure, $\sin\phi = \dfrac{1}{r} = \dfrac{eB}{mv}$

$\Rightarrow \quad mv\sin\phi = eB = p_y$

Hence y-component of linear momentum of proton

$$p_y = eB = 1.6 \times 10^{-19} \times 5 \times 10^{-4}$$

$$= 8.0 \times 10^{-23}\text{ kg m/s}$$

(b) Kinetic energy of the proton

$$K = 50\text{ MeV} = 50 \times 1.6 \times 10^{-19}\text{ J}$$

Hence $\dfrac{1}{2}mv^2 = 50 \times 1.6 \times 10^{-19}\text{ J}$

$$v = \sqrt{\frac{2 \times 50 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27}}} = 10^8\text{ m/s}$$

Here $\sin\phi = \dfrac{eB}{mv}$

$$\sin\phi = \frac{1.6 \times 10^{-19} \times 5 \times 10^{-4}}{1.6 \times 10^{-27} \times 10^8} = \frac{1}{2}$$

$\Rightarrow \quad \phi = 30°$

10. $d = n\,(v\cos\phi)\,T = (v\cos\phi)\dfrac{2\pi mn}{qB}$, where $n = 1, 2, 3, \ldots\ldots$

where $e\Delta V = \dfrac{1}{2}mv^2$

- $v = \sqrt{\dfrac{2\Delta Ve}{m}}$

Putting the value, we get $B = \dfrac{n}{d}\,\dfrac{2\pi m}{q}\left(\sqrt{\dfrac{2\Delta Ve}{m}}\right)\cos\phi$

For min B, $n = 1$

11. (a) The particle will move in circular paths, as velocity vector is perpendicular to magnetic field. Time period of both the particles is same $\left(T = \dfrac{2\pi m}{Bq}\right)$

So, for collision not to take place,

$$r_1 + r_2 < d$$

$$\frac{mv}{Bq} + \frac{2mv}{2Bq} < d \quad \text{or} \quad v < \frac{Bqd}{2m}$$

Therefore, maximum speed should be $\dfrac{Bqd}{2m}$

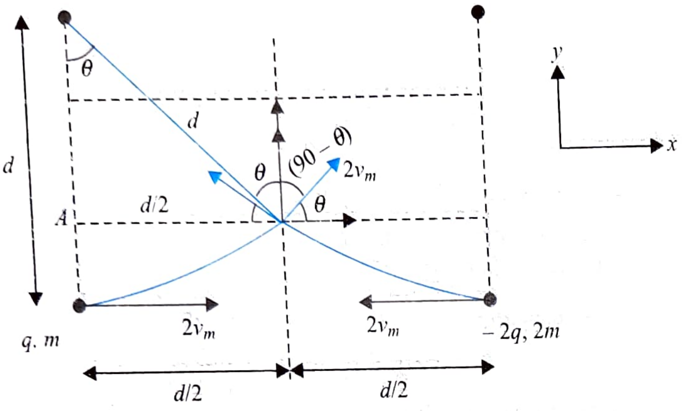
i.e., $v_m < \dfrac{Bqd}{2m}$

(b) From symmetry, it can be concluded that collision occurs at $d/2$ if

$$v = 2v_m = \frac{qBd}{m}$$

$$r = \frac{mv}{qB} = d, \sin\theta = \frac{d/2}{d} = \frac{1}{2}; \theta = \frac{\pi}{6}$$

$$t = T\left(\frac{\theta}{2\pi}\right) = \frac{2\pi m}{qB}\left(\frac{\pi/6}{2\pi}\right) = \frac{\pi m}{6qB}$$



(c) After collision, charge on the combined particle $= -q$,
Mass $= 3m$

The combined particle will have velocity in y direction just after collision. Using conservation of linear momentum

$$(mv\cos\theta\, i + mv\sin\theta\, j) + (-2mv\cos\theta\, i + 2mv\sin\theta\, j) = 3m\vec{v}$$

$$3m\vec{v} = -mv\cos\theta\, i + 3mv\sin\theta\, j$$

$$\vec{v} = \left(-\frac{v}{2\sqrt{3}}\hat{i} + \frac{v}{2}\hat{j}\right)$$

$$|\vec{v}| = v\sqrt{\left(\frac{1}{4\times 3} + \frac{1}{4}\right)} = v\sqrt{\frac{1}{4}\left(\frac{4}{3}\right)} = \frac{v}{\sqrt{3}}$$

$$|v| = \frac{1}{\sqrt{3}} \times 2 \times \frac{qBd}{2m} = \frac{qBd}{\sqrt{3}m}$$

$$\therefore \quad r = 3m \times \frac{qBd}{\sqrt{3}m \times qB} = \sqrt{3}\, d$$

12. Just before striking the plate, the electric force on the charged particle is $F_E = qE = \dfrac{qV}{d}$, in the upward direction. Since the kinetic energy of the electron is $K = \dfrac{1}{2}mv^2 = qV$, $v = \sqrt{\dfrac{2qV}{m}}$. On the other hand, the magnetic force is

$$F_B = qvB = qB\sqrt{\frac{2qV}{m}}$$

in the downward direction. To prevent the electron from striking the plate, we require $F_B = F_E$, or

$$qB\sqrt{\frac{2qV}{m}} > \frac{qV}{d} \Rightarrow B > \frac{V}{d}\sqrt{\frac{m}{2qV}} = \sqrt{\frac{mV}{2qd^2}}$$

Exercise 1.3

1. Magnetic force on wire $abcdef$ in uniform magnetic field is $\vec{F}_m = I(\vec{L} \times \vec{B})$, $\vec{L}$ is displacement between free ends of the conductor from initial to final point. $\vec{L} = (l)\hat{i}$ and

$$\vec{B} = (B)\hat{j};\ F_m = I(\vec{L} \times \vec{B}) = BIL(\hat{i} \times \hat{j}) = BIl(\hat{k})$$

2. Net force on the loop $= 3(\vec{F}_{AD})$

Force on wire ACD = Force on AD = Force on AED

$$\Rightarrow \quad F_{\text{net}} = 3(i)(AD)(B) = (3)(3.0)(2\sqrt{2})(2.0)\,\text{N}$$

$= 36\sqrt{2}$ N. Direction of this force is towards EC.

3. The net force from A to B is $d\vec{F} = I(d\vec{L} \times \vec{B})$

$$\int_A^B d\vec{F} = \int_A^P I[d\vec{L}_1 \times \vec{B}] + \int_P^Q I[d\vec{L}_2 \times \vec{B}] + \int_Q^R I[d\vec{L}_3 \times \vec{B}]$$

$$+ \int_R^T I[d\vec{L}_4 \times \vec{B}] + \int_T^B I[d\vec{L}_5 \times \vec{B}]$$

The entire path can be broken down into elemental vectors joined to each other in sequence. We know, from polygon law of addition of vectors, that vector joining the tail of the first vector to the head of the last vector is the resultant.

$$\vec{F} = I(\vec{L} \times \vec{B}),$$

where $|\vec{L}| = a + \sqrt{c^2 - b^2} + 2r + d$

$$F_{\text{net}} = IB(a + \sqrt{c^2 - b^2} + 2r + d)$$

and its direction is upward on the plane of paper.

4. We have for mid-air suspension,

$$mg = ilB$$

$$\Rightarrow \quad B = \frac{mg}{il} = \frac{0.2 \times 10.0}{2 \times 1.0} = 1.0\ \text{T}$$

5. $\vec{F} = I\vec{l} \times \vec{B}$

$$F = IlB\sin\theta$$

(a) When the current is flowing from east to west,

$$\theta = 90°$$

Hence $F = IlB = (1\text{A})(1\text{m})(3 \times 10^{-5}\ \text{T}) = 3 \times 10^{-5}$ N

The direction of the force is downwards. This direction may be obtained by either Fleming's left hand rule or the directional property of cross product of vectors.

(b) When the current is flowing from south to north,

$$\theta = 0° \quad \Rightarrow \quad F = 0$$

Hence, there is no force per unit length on the conductor.

6. In order to use equation $\vec{F} = I\vec{l} \times \vec{B}$, we would need to find the angle between $\vec{l}$ and $\vec{B}$. We avoid this task by using unit vector notation. From the problem figure, we see that

$$\vec{l} = a\hat{i} - a\hat{j} + a\hat{k}$$

Thus, the force is $\vec{F} = I\vec{l} \times \vec{B} = IaB(\hat{i} - \hat{j} + \hat{k}) \times (\hat{j})$

$$= IaB(-\hat{i} + \hat{k})$$

The force lies in the xz plane and has a magnitude

$$F = \sqrt{2}\, IaB = 10\ \text{N}$$

7. (a) The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{l} \times \vec{B} \quad \text{or} \quad F = ilB$$

It acts towards right in the figure provided. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to $\mu_0 mg$. Thus,

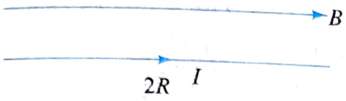
$$\mu_0 mg = ilB \text{ or } \mu_0 = \frac{ilB}{mg}$$

(b) If the friction coefficient is $\mu = \dfrac{\mu_0}{2} = \dfrac{ilB}{2mg}$, the wire will slide towards right. The frictional force by the rails is

$$f = \mu mg = \frac{ilB}{2} \text{ towards left}$$

The resultant force is $ilB - \frac{ilB}{2} = \frac{ilB}{2}$ towards right. The acceleration will be $a = \frac{ilB}{2m}$. The wire will slide towards right with this acceleration.

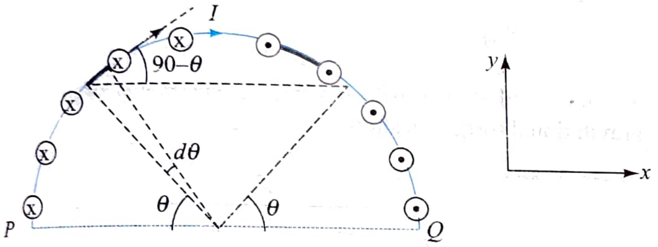
8. The wire is equivalent to a straight wire as shown in the figure shown below.



As $F = ilB \sin\theta$

$\because \quad \theta = 0$, therefore, $F_{\text{res}} = 0$

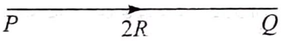
Forces on individual parts are marked in figure by ⊗ and ⊙. By symmetry, there will be pair of forces forming couples.

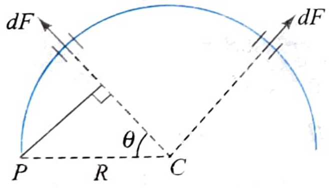


$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta)2R\cos\theta$$

$$= \frac{i\pi R^2}{2}B \quad \text{or} \quad \vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j})$$

9. $\vec{F}_{\text{net}} = I\,2R\,B$. The wire is equivalent to





Force on each element is radially outward, $\tau_C = 0$

Torque about point $P = \tau_p = \int_0^{\pi}[I(Rd\theta)B\sin 90°]R\sin\theta$

$$= 2\,IBR^2$$

10. Magnetic force is along the positive x-axis. If motion is to occur along the incline, $\sum F = 0$: $IL\,|\,B_y\,|\cos 37° = mg\sin 37°$ from which

$$I = \frac{mg\tan 37°}{L\,|\,B_y\,|} = \frac{(0.050)(10)(0.75)}{(0.50)(0.02)} = 37.5 \text{ A}$$

11. Initially, the rod will be in equlibrium if

$$2T_0 = mg \text{ with } T_0 = kx_0 \qquad \text{...(i)}$$

When current is passed through the rod, it experiences a force $F = Bil$ vertically upward. Now for equilibrium, in this situation

$$2T + Bil = mg \text{ with } T = kx \qquad \text{...(ii)}$$

From equations (i) and (ii), we get

$$\frac{T}{T_0} = \frac{mg - Bil}{mg} \quad \text{or} \quad \frac{x}{x_0} = 1 - \frac{Bil}{mg}$$

Solve to get $B = \frac{mg(x_0 - x)}{ilx_0} = \frac{(1\times 10^{-3})(10)(3\times 10^{-2})}{(20)(25\times 10^{-2})(4\times 10^{-2})}$

$$= 1.5 \times 10^{-3} \text{ T}$$

12. By impulse of magnetic force, if the wire attains a velocity v in upward direction then we use

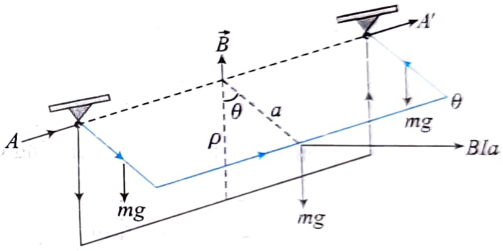
$$BIl\delta t = mv \Rightarrow v = \frac{Bql}{m}$$

Maximum height attained by wire MN is given as

$$h = \frac{v^2}{2q} = \frac{B^2q^2l^2}{2m^2g}$$

13. Let the side of square wire frame is a. The horizontal wire of the frame will experience a rightward force BIl and due to this force it experiences a torque about the axis AA' and the frame will tilt. Torque on frame about AA' is given as

$$\tau = Bla \times a\cos\theta = Bla^2\cos\theta$$



This is the deflecting torque and at equilibrium. This torque is balanced by the torque on the frame due the weight mg of the three sides of the square acting at centre of the wires as shown in figure. so at equilibrium, balancing of torques gives

$$Bla^2\cos\theta = \frac{Mg}{3}\times a\sin\theta + 2\left(\frac{Mg}{3}\right)\times\left(\frac{a}{2}\right)\sin\theta$$

$$\Rightarrow \quad Bla^2\cos\theta = \frac{2}{3}Mga\sin\theta$$

$$\Rightarrow \quad B = \frac{2Mg}{3la}\tan\theta$$

Mass of square wire frame can be written in terms of its density, length and cross sectional area as

$$M = 3aS\rho$$

$$\Rightarrow \quad B = \frac{2(3aS\rho)g}{3la}\tan\theta \Rightarrow B = \frac{2aS\rho g}{la}\tan\theta$$

Exercise 1.4

1. (a) $\phi = 90°$: $\tau = NIAB\sin(90°) = NIAB$

Direction: $\hat{k}\times\hat{j} = -\hat{i}$, $U = -MB\cos\phi = 0$

(b) $\phi = 0°$: $\tau = NIAB\sin(0) = 0$, no direction,

$M = NIA$, $U = -MB\cos\phi = -NIAB$

(c) $\phi = 90°$: $\tau = NIAB\sin(90°) = NIAB$

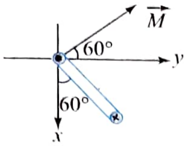
Direction: $-\hat{k}\times\hat{j} = \hat{i}$, $U = -MB\cos\phi = 0$

(d) $\phi = 180°$: $\tau = NIAB\sin(180°) = 0$, no direction,

$U = -MB\cos\phi = NIAB$

2. Magnitude of dipole moment $|\vec{M}| = IA = Il^2$

Direction of magnetic moment is found using right hand rule.



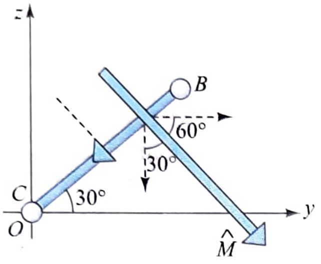
$$\vec{M} = \left(\cos 30°(-\hat{i}) + \sin 30°(\hat{j})\right)M$$

$$= \frac{Il^2}{2}(-\sqrt{3}\,\hat{i} + \hat{j})$$

3. Magnetic moment of the loop

$$|\vec{M}| = iA = 4.0 \times 20 \times 10 \times 10^{-4} = 8 \times 10^{-2}\ \text{A m}^2$$

For direction:



Unit vector in the direction of

$$\widehat{M} = \cos 60°\,\hat{j} - \cos 30\,\hat{k} = \frac{\hat{j}}{2} - \frac{\sqrt{3}}{2}\hat{k}$$

$$\therefore \quad \vec{M} = |\vec{M}|\,\widehat{M} = 4 \times 10^{-2}(\hat{j} - \sqrt{3}\,\hat{k})\ \text{Am}^2$$

4. The magnetic dipole moment of the current carrying coil is given by $\vec{M} = NIA\hat{n}$

$$= 100 \times 0.5 \times 0.08 \times 0.04\,\hat{i} = 0.16\,\hat{i}\ \text{A m}^2$$

The torque acting on the coil is

$$\vec{\tau} = \vec{M} \times \vec{B} = M(\hat{i} \times \hat{j})B = 0.16 \times \frac{0.05}{\sqrt{2}}\hat{k}$$

$$= 4\sqrt{2} \times 10^{-3}\ (\text{Nm})\,\hat{k}$$

5. The magnetic moment of the loop is in the positive z-direction (right hand thumb rule).

(a) The magnetic moment of the loop is given by

$$\vec{M} = NIA\hat{k} = (10)(5.0)(0.40)^2\hat{k} = 8.0\ \text{Am}^2\,\hat{k}$$

(b) The torque on the current loop is given by

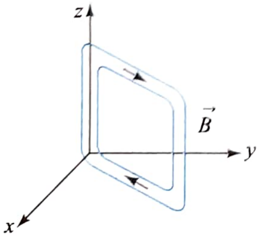
$$\vec{\tau} = \vec{M} \times \vec{B} = (8.0\hat{k}) \times (0.3\hat{i} + 0.4\hat{k}) = 2.4\ \text{Nm}\,\hat{j}$$

(c) The potential energy is the negative dot product of $\vec{M}$ and $\vec{B}$:

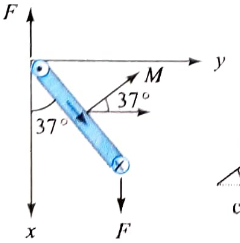
$$U = -\vec{M}\cdot\vec{B} = -(5.76\hat{k})\cdot(0.3\hat{i} + 0.4\hat{k}) = -2.30\ \text{J}$$

6. (a) From Fig. (b), we see that the unit vector normal to loop

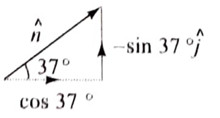
$$\hat{n} = -\sin 37°\hat{i} + \cos 37°\hat{j} = -0.6\hat{i} + 0.8\hat{j}$$



(a)



(b)



(c)

The magnetic moment is

$$\vec{M} = NIA\hat{n} = (5)(2)(0.2)^2(-0.6\hat{i} + 0.8\hat{j})$$

$$= -0.24\hat{i} + 0.32\hat{j}\ \text{A m}^2$$

(b) The torque, $\hat{\tau} = \vec{M} \times \vec{B} = (-0.24\hat{i} + 0.32\hat{j}) \times (0.5\hat{j})$

$$= -0.12\hat{k}\ \text{N m}$$

(c) The potential energy of the loop is $U = -MB\cos\theta$ where $M = NIA = 0.4\ \text{Am}^2$ and the position of minimum energy is $\theta = 0$. Thus, the external work, $W_{ext} = +\Delta U$, needed to rotate it to the given orientation, is given by

$$W_{ext} = U_f - U_i = (-MB\cos 37°) - (-MB\cos 0°)$$

$$= (0.4)(0.5)(1 - 0.8) = 0.04\ \text{J}$$

The external work is positive since the dipole moment is rotated away from alignment with the field.

7. The normal to the loop, OP, makes an angle $\theta = 37°$ with the $+x$ direction, the field direction. Hence,

$$\tau = NIAB\sin\theta = (1)(10\ \text{A})(\pi \times 25 \times 10^{-4}\ \text{m}^2)(0.04\ \text{T})\sin 37°$$

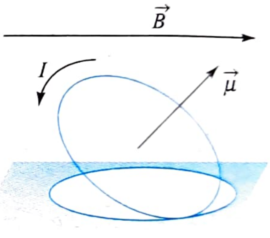
$$= 6\pi \times 10^{-4}\ \text{Nm}$$

The right hand rule shows that the loop will rotate about the y-axis, so as to decrease the angle labelled 37°.

8. $\theta = \dfrac{NBAI}{k}, \theta' = \dfrac{10NB(A/16)I}{k}$

$$\Rightarrow \quad \theta' = \frac{5}{8}\theta$$

9. The loop will start to lift off when the magnetic torque equals the gravitational torque as shown in figure



The magnetic torque acting on the loop, $\tau_m = MB = I\pi R^2 B$.

The gravitational torque exerted on the loop, $\tau_g = mgR$.

So $I\pi R^2 B = mgR \quad \Rightarrow \quad I = \dfrac{mg}{\pi RB}$

10. $\tau_{max} = MB = IAB = I\pi r^2 B = 5 \times \pi \times (0.05)^2 \times 3 \times 10^{-3}$

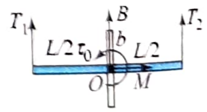
$$= 375\,\pi \times 10^{-7}\ \text{Nm}$$

$$U_{max} = +MB,\ U_{min} = -MB$$

11. Let mass of the hanging system is m, then

$2T_0 = mg \rightarrow$ in the absence of magnetic field

(a) when magnatic field is applied:



$$\tau_0 = MB = \pi b^2 IB \quad ..(i)$$

$$T_1 + T_2 = mg \quad ...(ii)$$

Taking torque about O:

$$\tau_0 + T_2\frac{L}{2} = T_1\frac{L}{2} \quad ...(iii)$$

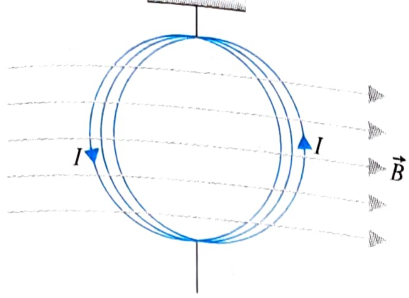
Solve equations (i), (ii) and (iii) to get

$$T_1 = \frac{mg}{2} + \frac{\pi b^2 BI}{L} \quad \text{and} \quad T_2 = \frac{mg}{2} - \frac{\pi b^2 BI}{L}$$

(b) In this case, magnetic field will produce no torque, so tension will remain same, $T_0 = mg/2$.

12. Magnetic force $F_m = I\left(\oint d\vec{l} \times \vec{B}\right)$

For coil or close loop $\oint d\vec{l} = 0$ so $\vec{F}_m = 0$



The torque $\vec{\tau}$ on a coil of any shape having N turns and current I in a magnetic field B is given by $\tau = NIAB\sin\theta$

$$\tau = 100\times 10\times \pi\times (5\times 10^{-2})^2\times 2.0\times \sin 90° = 5\pi\ \text{N}$$

The direction of $\vec{\tau}$ is vertically upwards. To prevent the coil from turning, an equal and opposite torque must be applied.

13. We assume equal and opposite currents in wires PQ and RS. Then we split the given loop in three independent square loops and find magnetic moment vector of the loop which is only due to the square coil $PQRS$ as top and bottom square coils magnetic moments are equal and opposite, so will cancel each other. Thus magnetic moment of the given loop is given as

$$\vec{M} = Ia^2\hat{j} = 2\times (2)^2\hat{j} = 8\hat{j}$$

The magnetic induction vector in space is given as

$$\vec{B} = 2\hat{j}$$

The torque on the given loop is given as

$$\vec{\tau} = \vec{M}\times\vec{B} = 0$$

14. The average current in the loop is $i = \frac{q}{T} = \frac{q}{2\pi r/v} = \frac{qv}{2\pi r}$ and its magnetic dipole moment is

$$\vec{M} = iA = \left(\frac{qv}{2\pi r}\right)(\pi r^2) = \frac{1}{2}qvr$$

With $\vec{\tau} = \vec{M}\times\vec{B}$, we find the maximum torque exerted on the loop by a uniform magnetic field to be

$$\tau_{\max} = MB = \frac{1}{2}qvrB$$

Exercises

Single Correct Answer Type

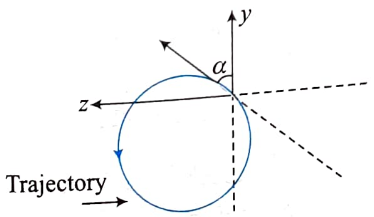
1. **(1)** It is clear from Right Palm Rule that the charges are positively charged.

As $\quad r = \frac{mv}{qB}$

The radius of path is not the only function of either m or q. For (2), (3) and (4) we cannot make clear statement, but statement (1) is certainly true.

2. **(1)** $r = \frac{mv}{qB} \quad\Rightarrow\quad r \propto m$ (for v same for both sense)

3. **(3)** We can see from trajectory that both y- and z-coordinates can become negative.



This trajectory lies in y–z plane. So, x-coordinate is always zero.

4. **(2)** For the particle to move along anticlockwise path, force should be along $\hat{j}$. Velocity is along $\hat{i}$.

Now, $\vec{F}_m = -e(\vec{v}\times\vec{B})$

In terms of unit vectors only,

$$\hat{j} = -(\hat{i}\times ?) \quad \text{or} \quad \hat{j} = ?\times\hat{i}$$

Clearly, $?$ is $\hat{k}$.

5. **(3)** $F = Bqv$

But $\quad \frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$

$\therefore \quad F = Bq\sqrt{\frac{2eV}{m}}$

$\Rightarrow \quad F \propto \sqrt{V}$ and

$F' \propto \sqrt{2\text{V}}$

$\frac{F'}{F} = \sqrt{2} \quad \text{or} \quad F' = \sqrt{2}F$

6. **(2)** The charged particle has circular path in the case when only magnetic field is present.

7. **(4)** Depending on the direction of magnetic field, tension may increase or decrease.

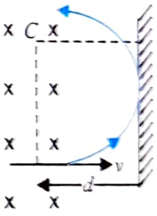
8. **(1)** Time interval in which $\vec{v}$ returns to its initial value is same as time period of the particle, hence the required time $= \frac{2\pi m}{eB}$

9. **(1)** $\vec{F} \propto \left(\vec{v}\times\vec{B}\right) = \hat{k}\left[aD - dA\right]$

10. **(1)** The particle moves in a circular path with radius d if it is to just miss the wall.

$\Rightarrow \quad mv = Bqr\ ,\ r = d$

or $\quad B = \frac{v}{(q/m)d} = \frac{v}{sd}$



11. **(2)** For a particle moving in any combination of electric and magnetic fields, work is done only by the electric field.

Energy of the particle = work done by the electric field = electric field × displacement in the direction of the electric field.

12. **(3)** $T = \frac{2\pi m}{Bq} \quad \text{or} \quad T \propto \frac{m}{q}$

$$\frac{T_\alpha}{T_p} = \frac{4m}{2q}\times\frac{q}{m} = 2$$

or $\quad T_\alpha = 2\left[\frac{25}{5}\right]\mu\text{s} = 10\ \mu\text{s}$

13. **(3)** $\frac{1}{2}mv^2 = qV \quad \text{or} \quad v = \sqrt{\frac{2qV}{m}}$

Centripetal force, $\frac{mv^2}{R} = qvB$

$\therefore \quad v = \left(\frac{qB}{m}\right)R$

Hence, $\sqrt{\frac{2qV}{m}} = \left(\frac{qB}{m}\right)R$

or $$R = \left(\frac{2mV}{q}\right)^{1/2} \times \frac{1}{B}$$

Here, V, q and B are constant. Hence, $m \propto R^2$

So, $$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

14. (2) $Bqv = \frac{mv^2}{r}$ or $Bqr = mv$

For electron as well as proton B is the same, r is the same and numerically charge q is same; therefore mv is constant.

$$m_e v_e = m_p v_p \text{ or } v_p = \left(\frac{m_e}{m_p}\right) v_e$$

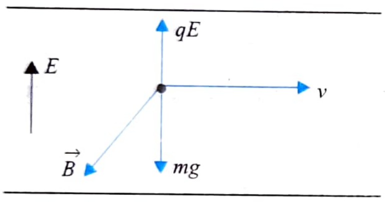
or $$v_p = \left(\frac{0.90 \times 10^{-30}}{1.8 \times 10^{-27}}\right)(3.0 \times 10^6) = 1.5 \times 10^3 \text{ m/s}$$

15. (2) Since the charged particle passes straight without deflection, therefore

$$Bqv = mg$$

or $$v = \frac{mg}{Bq} = \frac{10^{-3} \times 10}{1 \times 10^{-5}} = 10^3 \text{ ms}^{-1}$$

16. (1) Net force on the particle should be zero.

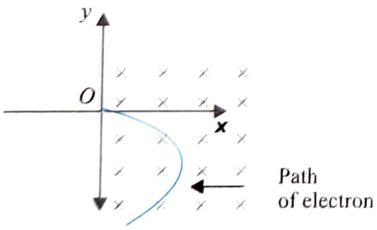


$$qE = 10^{-6} \times 200 = 2 \times 10^{-4} \text{ N}$$
$$mg = 2 \times 10^{-5} \times 9.8 = 1.96 \times 10^{-4} \text{ N}$$

Since $qE > mg$, so magnetic force qvB should act downward to balance the forces

$$qE = mg + qvB \Rightarrow 2 \times 10^{-4} = 1.96 \times 10^{-4} + 10^{-6} v \times 2$$
$$\Rightarrow v = 2 \text{ m/s}$$

17. (4) y will be less than zero. Speed will remain same. The trajectory will be as shown in figure.



18. (3) Knowledge based.

19. (2) Knowledge based.

20. (2) Due to linear scale, $i \propto \theta$.

21. (2) As the loop is placed in horizontal plane, so area vector is along vertical direction. From $\vec{\tau} = I(\vec{A} \times \vec{B})$, as $\vec{A}$ is in vertical direction, $\vec{\tau}$ would be the plane of loop only. So, option (1) is wrong because for rotation of loop about its own axis $\vec{\tau}$ must be along vertical direction. (2) is correct because we can produce torque in the plane of the loop and due to this the loop can tip over.

22. (1) See the direction of torque about centre.

23. (4) Net force on a current carrying loop in a uniform magnetic field is zero. So, magnetic force cannot balance its weight.

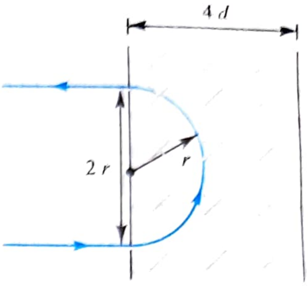
24. (2) As we know that

$$\vec{F} = \oint I d\vec{l} \times \vec{B} \quad \text{and} \quad \vec{\tau} = \vec{M} \times \vec{B}$$

25. (2) $T = 2\pi m/qB$, $\left(\frac{m}{q}\right)_{\text{He}} > \left(\frac{m}{q}\right)_{\text{p}} > \left(\frac{m}{q}\right)_{e}$

$$\therefore \quad t_{\text{He}} > t_{\text{p}} > t_e$$

26. (2) $r = \frac{mv}{qB} = \frac{v}{B\alpha} = \frac{(2\alpha d)(B)}{(B\alpha)} = 2d$



i.e., the electron will move out after travelling on a semicircular path of radius $r = 2d$.

27. (2) $r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB} = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{m}{q}} \times \frac{\sqrt{2V}}{B}$

$$\Rightarrow \frac{r_2}{r_1} = \sqrt{\frac{4}{1}} \times \sqrt{\frac{1}{2}} = \sqrt{2} \Rightarrow r_2 = \sqrt{2} r_1 = 5\sqrt{2} \text{ cm}$$

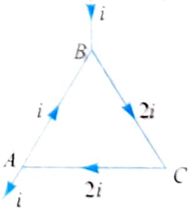
28. (2) $r = \frac{mv\sin\theta}{qB}$, Pitch: $p = v\cos\theta T = v\cos\theta \frac{2\pi m}{qB}$

Now, $p = r$, solve to get $\theta = \tan^{-1} 2\pi$

29. (4) $R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2(\text{KE})}{m}} = \frac{\sqrt{2m}}{qB}\sqrt{\text{KE}}$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\sqrt{\text{KE}}}{\sqrt{\text{KE}/2}} \Rightarrow R_2 = \frac{R_1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ cm}$$

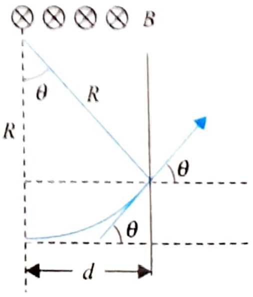
30. (1) The equivalent figure can be redrawn as shown in figure.



Force on $AB = ilB$, Force on $BCA = 2ilB$ in opposite direction to that on BA.

Hence, net force $= 2ilB - ilB = ilB$

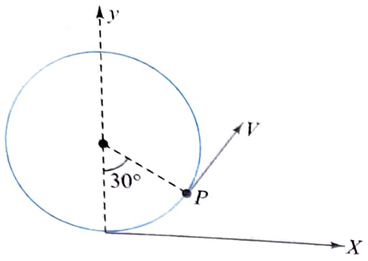
31. (4) $\sin\theta = (d/R)$



$$d = R\sin\theta = \frac{mv}{qB}\sin\theta$$

$$\Rightarrow \quad \frac{q}{m} = \frac{v\sin\theta}{Bd}$$

32. (2) Time period, $T = \dfrac{2\pi m}{qB} = \dfrac{2\pi}{\pi \times 2} = 1\text{ s}$

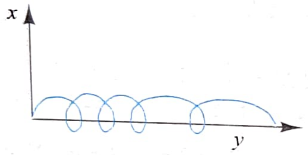


Thus, particle will be at point P after $t = \dfrac{1}{12}$ s

$$\vec{v} = 10[\cos 30\hat{i} + \sin 30\hat{j}]$$

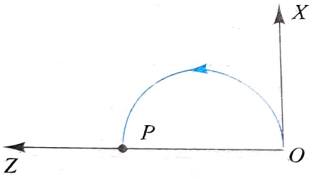
$$\vec{v} = 10\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right] = 5\left[\sqrt{3}\hat{i} + \hat{j}\right]\text{ ms}^{-1}$$

33. (4) The particle will move in a non-uniform helical path with increasing pitch as shown in figure.



Its time period will be: $T = \dfrac{2\pi m}{qB} = 2\pi$ s

Changing the view, the particle seems to move in a circular path in (X–Z) plane as shown in figure.



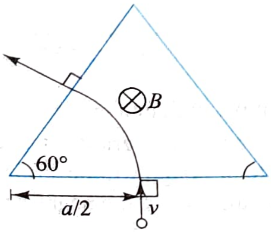
After π seconds, the particle will be at point 'P' (after completing half circle), hence X-coordinate of P will be 0.

$$y(\pi) = 0(\pi) + \frac{1}{2}\frac{Eq}{m}(\pi)^2 = \frac{\pi^2}{2},$$

$$z = 2r = 2mv/qB = 2\text{ m}$$

Hence, the coordinates of the particle are $\left(0, \dfrac{\pi^2}{2}, 2\right)$

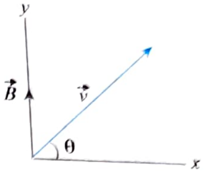
34. (2) The charged particle moves in a circle of radius $a/2$.



$$qvB = \frac{mv^2}{a/2} \quad \text{or} \quad B = \frac{2mv}{qa}$$

35. (2) Each and every pair of loop elements located symmetrically w.r.t. central line experience zero net force, so total magnetic force experienced by the loop is zero.

36. (3) Since the proton is entering the magnetic field at some angle other than 90°, its path is helix.



Corresponding velocity of the proton along X-axis,

$$v_x = v\cos 60° = 2\times 10^6 \times \frac{1}{2} = 10^6\text{ ms}^{-1}$$

Due to velocity component v_x, the radius of the helix is described and is given by the relation

$$r = \frac{mv_x}{qB} = \frac{1.6\times 10^{-27}\times 10^6}{1.6\times 10^{-19}\times 0.10} = 0.1\text{ m}$$

$$\text{Now, } T = \frac{2\pi r}{v_x} = \frac{2\pi \times 0.1}{10^6} = 2\pi \times 10^{-7}\text{ s}$$

37. (1) $K = \dfrac{1}{2}mv^2 = eV \quad \text{or } v = \sqrt{\dfrac{2eV}{m}}$

$$\text{Also, } F = evB = e\left[\sqrt{\frac{2eV}{m}}\right]\times B$$

$$\text{Therefore, } \frac{F}{2F} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V}{V'}}$$

$$\therefore \quad \frac{V'}{V} = \frac{4}{1}$$

38. (4) $\vec{E}$ is parallel to $\vec{B}$ and $\vec{v}$ is perpendicular to both. Therefore, path of the particle is a helix with increasing pitch. Speed of particle at any time t is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \qquad \text{...(1)}$$

Here, $v_y^2 + v_z^2 = v_0^2$

and $v = 2v_0 \Rightarrow v_x = \sqrt{3}v_0$

$$v_x = a_x t \Rightarrow \sqrt{3}v_0 = \frac{qE}{m}t \Rightarrow t = \frac{\sqrt{3}mv_0}{qE}$$

39. (3) Using, impulse = change in linear momentum, we have

$$\int F\,dt = mv \quad \text{or} \quad \int (iBl)\,dt = mv$$

$$\text{or} \quad Blq = mv \qquad \left(\text{as} \int i\,dt = q\right)$$

$$\therefore \quad q = \frac{mv}{Bl}$$

40. (1) $F_{CAD} = F_{CD} = F_{CAD}$

$\therefore$ Net force on the frame $= 3F_{CD}$

$= (3)(2)(1)(4) = 24$ N $\quad (F = ilB)$

41. (3) $p = \dfrac{2\pi m}{Bq}(v\cos 45°) = \dfrac{2\pi m}{Bq}(v\sin 45°)$

$$\therefore \quad \frac{mv\sin 45^\circ}{Bq} = \frac{p}{2\pi} = \text{radius of helix}$$

42. **(4)** Torque τ acting on a current carrying coil of area A placed in a magnetic field of induction B is given by

$$\tau = NIBA \sin\theta$$

where I = current in the coil, θ = angle which the normal to the plane of the coil makes with the lines of induction B.

Here, $N = 1, \quad B = 1.5 \times 10^{-2}$ T

$A = 0.05 \times 0.08 = 40 \times 10^{-4}$ m^2

$I = 10.0$ amp, $\theta = 90^\circ = \pi/2$

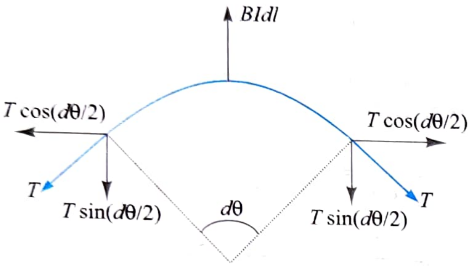
$\tau = (1.5 \times 10^{-2})(10.0) \times (1)(40 \times 10^{-4})\sin(\pi/2)$

$= 6 \times 10^{-4}$ Nm

43. **(3)** $BI(dl) = 2T\sin(d\theta/2)$

$\Rightarrow \quad BI(rd\theta) = 2T(d\theta/2) \qquad (\theta \text{ is small, } \sin\theta = \theta)$

$\Rightarrow \quad T = BIr = BIl/2\pi$



44. **(1)** Torque due to magnetic field $\tau_{\text{mag}} = MB_0 = I\pi R^2 B_0$ ...(i)

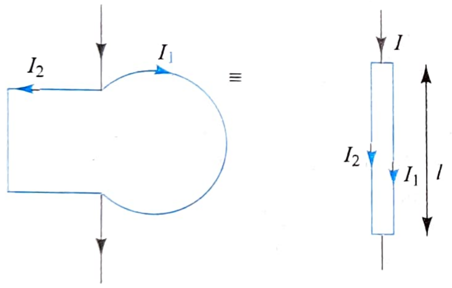
Torque due to weight about the point where string is connected

$\tau_{\text{weight}} = mgR$ ...(ii)

If ring remains horizontal, then $\tau_{\text{mag}} = \tau_{\text{weight}}$

$$I\pi R^2 B_0 = mgR \quad \Rightarrow \quad I = \frac{mg}{\pi R B_0}$$

45. **(2)** $F = BI_1 l + BI_2 l$

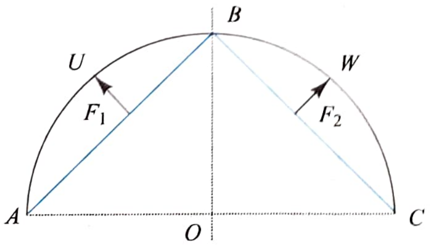


$\Rightarrow \quad F = BI(I_1 + I_2) = BIl$

46. **(1)** Radius of circular path of the charged particle $r = \dfrac{mV}{qB}$

Here $R < r$, hence the particle will press the outer wall of the pipe hence the force applied by the pipe on the particle should be towards the centre of the pipe.

47. **(3)** The part AUB of the wire can be replaced by straight wire AB and BWC can be replaced by BC.



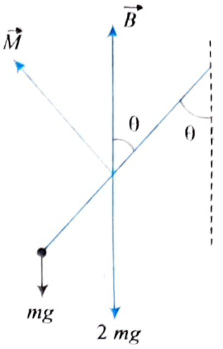
Force experienced by AB is, $F_1 = IB_0\left(\sqrt{2}R\right)$

Force experienced by BC is, $F_2 = I \times 2B_0 \times \sqrt{2}R$

Resultant magnetic force acting on the wire is,

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{10}\, IB_0 R$$

48. **(1)** Let a is the side of square.



Torque of current $= MB\sin(90 - \theta)$

$= MB\cos\theta = i\,a^2 B\cos\theta$

Mass of each side $= m = \rho A a$

Torque of gravity $= 2mg\,\dfrac{a}{2}\sin\theta + mg\,a\sin\theta$

$= 2\,mg\,a\sin\theta = 2\rho Aga^2\sin\theta$

Now, both torques should be same

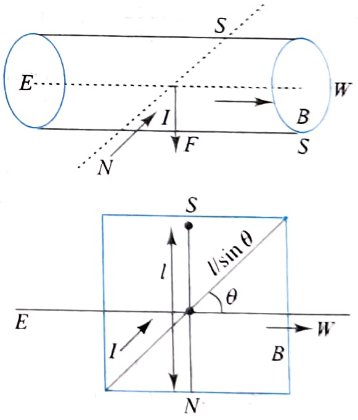
i.e., $\quad ia^2 B\cos\theta = 2\rho Aga^2\sin\theta$

$$\Rightarrow \quad \cot\theta = \frac{2\rho Ag}{iB}$$

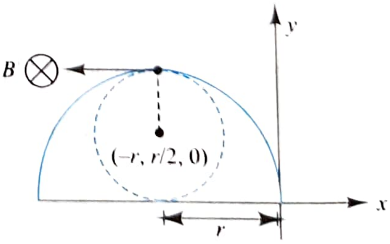
49. **(3)** Initially: $1.2\,N = 1(\vec{l} \times \vec{B})$ downward

In the given condition:

$$F = I\frac{l}{\sin\theta}B\sin\theta = IlB = 1.2\text{ N downward}$$



50. **(2)** Two particles will meet at P. After they meet they will stick together. Momentum mv will remain same. But charge is doubled, so radius is halved.



Finally, both move in dotted circle.

51. **(3)** After two and a half time periods, its x-coordinate will be $2.5P_0$ (so x-coordinate of image will be $17.5P_0$). It will be at distance $2R_0$ (equal to diameter) on the negative z-axis and y-coordinate will be zero.

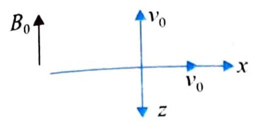
52. (1) Wire *abc* can be replaced by a straight wire *ac* for the computation of force.

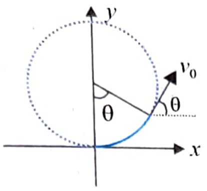
Length of $\overrightarrow{ac}$ can be written as,

$$\vec{l} = \vec{r}_C - \vec{r}_A = \left[(\hat{i}+\hat{j})-(\hat{i}+\hat{k})\right]50\times10^{-2} = 0.50[\hat{j}-\hat{k}]$$

Required force, $\vec{F} = I\left(\vec{l}\times\vec{B}\right) = 0.6\hat{i}$

53. (1)

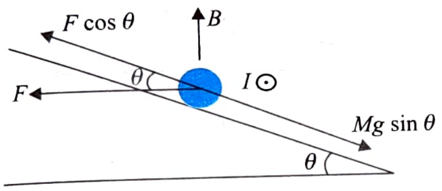




$$\vec{v} = v_0\cos\theta\hat{i} + v_0\sin\theta\hat{j} - v_0\hat{k}$$

where $\theta = \omega t = \dfrac{qB_0}{m}t = \alpha B_0 t$

54. (3) $F\cos\theta = Mg\sin\theta$



$$IBL\cos\theta = Mg\sin\theta \;\Rightarrow\; I = \frac{Mg\tan\theta}{LB}$$

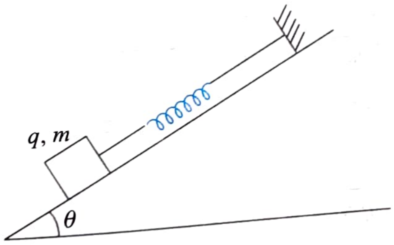
55. (1) Due to torque of magnetic field, ring will rotate about vertical diameter. $\tau = I\alpha \;\Rightarrow\; MB = I\alpha$

$$\Rightarrow \quad i\pi r^2 B = \frac{1}{2}mr^2\alpha$$

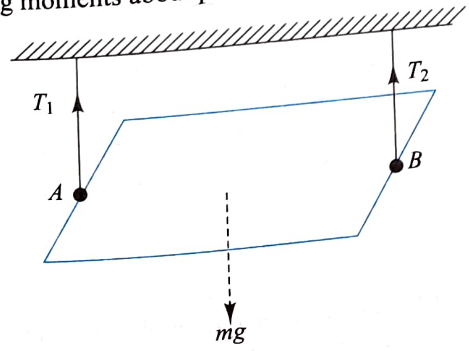
$$\Rightarrow \quad \alpha = \frac{2iB\pi}{m} = \frac{2\times4\times10\pi}{2} = 40\pi\ \text{rad s}^{-2}$$

56. (1) The ratio M/L is always $\dfrac{q}{2m}$

57. (1) There will be no effect of magnetic force on time period because the magnetic force will be perpendicular to the inclined plane.



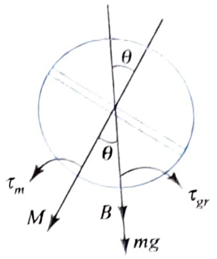
58. (3) Taking moments about point B to be zero.



$$T_1.l + iblB = mg\frac{l}{2};\quad T_1 = \frac{mg-2ibB}{2}$$

59. (1) effective length from A to C is $2\sqrt{2}a$

60. (1) The gravitational torque must be counter balanced by the magnetic torque about O, for equilibrium of the sphere. The gravitational torque $\tau_m = \pi i r^2 B\sin\theta$



$$\therefore \quad \pi i r^2 B\sin\theta = mgr\sin\theta$$

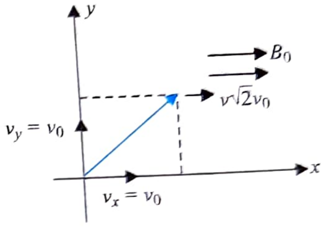
$$\Rightarrow \quad B = \frac{mg}{\pi i r}$$

61. (3) The particles will not collide if

$$d > 2(r_1 + r_2)$$

or $\quad d > 2\left(\dfrac{mv_1}{Bq} + \dfrac{mv_2}{Bq}\right)$ or $d > \dfrac{2m}{Bq}(v_1 + v_2)$

62. (4) $\alpha = \dfrac{q}{m}$, path of the particle will be a helix of time period,



$$T = \frac{2\pi m}{B_0 q} = \frac{2\pi}{B_0\alpha}$$

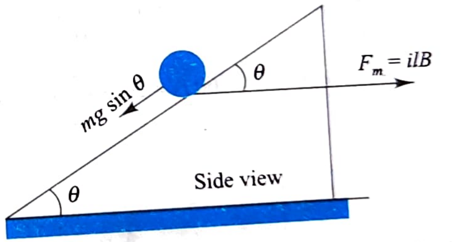
The given time $t = \dfrac{\pi}{B_0\alpha} = \dfrac{T}{2}$

∴ Coordinates of particle at time $t = T/2$ would be $(v_x\, T/2, 0, -2r)$

Here, $r = \dfrac{mv_0}{B_0 q} = \dfrac{v_0}{B_0\alpha}$

∴ The coordinates are $\left(\dfrac{v_0\pi}{B_0\alpha}, 0, \dfrac{-2v_0}{B_0\alpha}\right)$.

63. (2) Magnetic force acts in the direction shown in figure



Rod will move downward with constant velocity if net force on it is zero.

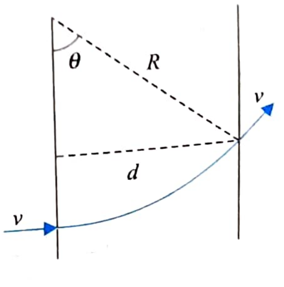
or $\quad F_m\cos\theta = mg\sin\theta$

or $\quad ilB\cos\theta = mg\sin\theta$

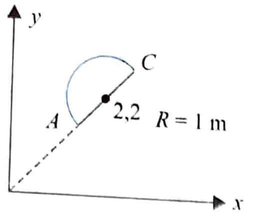
$$\therefore \quad B = \left(\frac{mg}{il}\right)\tan\theta$$

64. (3) $R = \dfrac{mv}{qB}$, $\sin\theta = \dfrac{d}{R} = \dfrac{dqB}{mv}$

$$t = \frac{2\pi m}{qB}\frac{\theta}{2\pi} = \frac{m}{qB}\theta = \frac{m}{qB}\sin^{-1}\left[\frac{dqB}{mv}\right]$$

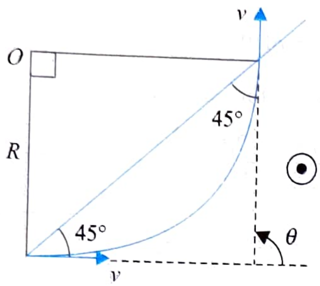


65. **(2)** $\vec{l} = \vec{AC} = 2R(\cos 45°\hat{i} + \sin 45°\hat{j}) = \sqrt{2}(\hat{i} + \hat{j})$



$$\vec{F} = I\vec{l} \times \vec{B} = 1 \times \sqrt{2}(\hat{i} + \hat{j}) \times (3\hat{i} + 4\hat{j} + \hat{k})$$
$$= \sqrt{2}[\hat{i} - \hat{j} + \hat{k}]$$

66. **(2)** Path of electron is shown in figure. Deviation of electron will be $\theta = 90°$.



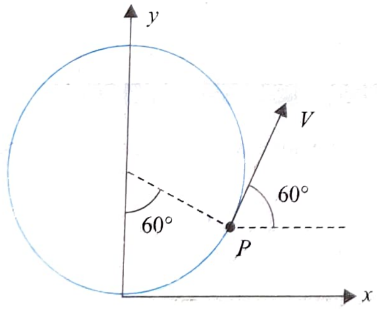
67. **(1)** Change in kinetic energy is only due to work done by electric field. Hence work done by electric field is change in KE:

$$qEa = \frac{1}{2}m\left[(2v)^2 - v^2\right] \Rightarrow E = \frac{3mv^2}{2qa}$$

68. **(1)** Time period $T = 2\pi\dfrac{m}{qB} = 1\,\text{s}$

Thus, the particle will be at P after $t = \dfrac{1}{6}$ s

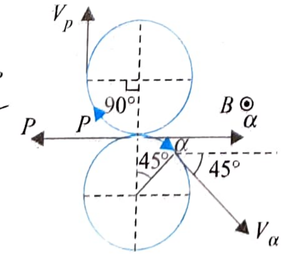
$$\therefore \quad \vec{V} = 10(\cos 60\hat{i} + \sin 60\hat{j}) = 5(\hat{i} + \sqrt{3}\hat{j})\ \text{m/s}$$



69. **(3)** $\omega = qB/m$,

$$\omega_p = eB/m, \omega_\alpha = \frac{2eB}{4m} = 2\omega_P$$

So proton will cover double angle in same time as that of α-particle. Clearly from the above figure angle between v_α and v_p is 135°



70. **(2)** $\vec{M} = Ia^2[-\hat{i} + \hat{j}]$, $\vec{\tau} = \vec{M} \times \vec{B}$,

Solve to get $\tau = 0$

71. **(4)** Since the particle is moving undeflected, so magnetic force = electric force

$$\Rightarrow \quad qvB\sin\theta = qE \quad \Rightarrow \quad v = \frac{E}{B\sin\theta}$$

Pitch, $d = v\cos\theta\, T = v\cos\theta\dfrac{2\pi m}{qB}$

$$= \frac{E}{B\sin\theta}\cos\theta\frac{2\pi m}{qB} = \frac{2\pi mE}{qB^2\tan\theta}$$

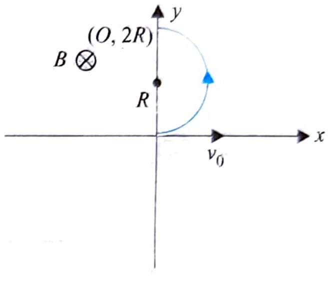
72. **(1)** Magnetic field is uniform, apply $\vec{F} = I\vec{L} \times \vec{B}$

where $\vec{L} = 0.25\hat{i} + \hat{j}$ and $\vec{B} = 4(\cos 60\,\hat{i} + \sin 60\,\hat{j})$

73. **(2)** $\vec{B}_A = \dfrac{\mu_0 R}{4\pi R}\hat{k} + \dfrac{\mu_0 I}{2R}\hat{i} - \dfrac{\mu_0 I}{4\pi R}\hat{j}$

$$B_A = \frac{\mu_0 I}{4\pi R}\sqrt{(2\pi - 1)^2 + (1)^2} = \frac{\mu_0 I}{4\pi R}\sqrt{2(2\pi^2 - 2\pi + 1)}$$

74. **(2)** y coordinate $= 2R = \dfrac{2mv_0}{qB}$

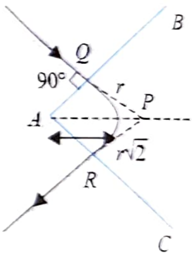


75. **(4)** Since, $QP = r$, $AP = r\sqrt{2}$ and $\angle AQP = 90°$

$AQ = r$ and $\angle QAP = 45°$

The same logic concludes $AR = r$ and $\angle PAR = 45°$

So, $\angle QAR = 90°$



$$r = \frac{mv}{Bq}$$

Distance travelled $= \dfrac{\pi r}{2}$

Time $= \dfrac{\pi}{2} \cdot \dfrac{mv}{Bq} \cdot \dfrac{1}{v} = \dfrac{m\pi}{2Bq}$

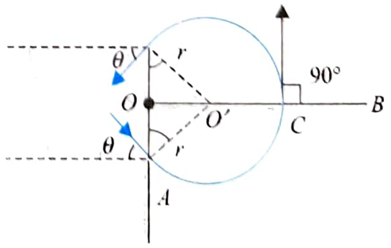
76. **(3)** Work done $= q\,E.\,2a$ = change in KE

$$q \cdot \frac{mv^2}{qa} \cdot 2a = \frac{1}{2}mv_Q^2 - \frac{1}{2}mv^2$$

$$4v^2 = v_Q^2 - v^2 \quad \Rightarrow \quad v_Q = v\sqrt{5}$$

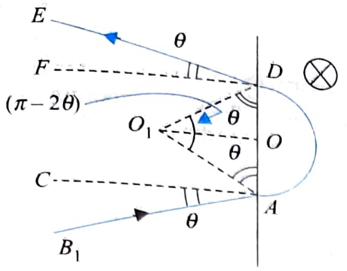
77. **(4)** $r = \dfrac{mv}{qB} = \dfrac{2K}{qvB}$

Thus, $OA = r\cos\theta$ (given)

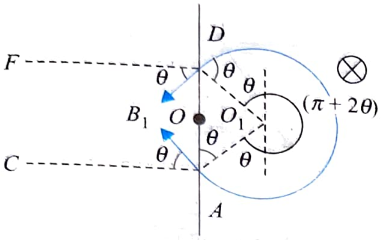


On line OB, centre O' of part of circle (in which particle moves) lies. The tangent at C with OB makes an angle of 90°. Particle leaves circular path at C. Thus, required angle is 90°.

78. (3) The diagrams are self-explanatory.



When θ is negative.



When θ is positive.

79. (1) O will be the midpoint of incident and point of emergence. Hence, $OA = OB$

80. (1) The geometry of the figures of the solution of problem (78) suggests $\phi = \theta$.

81. (1) See diagrams of the solution of problem (78)

When θ is + ve, angle at centre $= \pi + 2\theta$

When θ is –ve, angle at centre $= \pi - 2\theta$

When $\theta = 0$, incidence is normal, a semicircle is completed and

$$T = 2\pi \frac{m}{BQ} \text{ (time period)}$$

When θ is positive, time spent in magnetic field

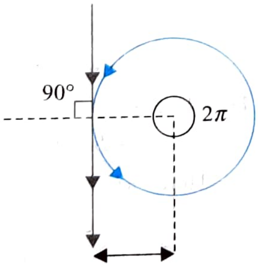
$$= \frac{T}{2\pi} \times (\pi + 2\theta)$$

When θ is negative, time spent in magnetic field

$$= \frac{T}{2\pi} \times (\pi - 2\theta)$$

$$\text{Hence, ratio} = \frac{(\pi + 2\theta)}{(\pi - 2\theta)} = \frac{(\pi/2 + \theta)}{(\pi/2 - \theta)}$$

82. (2) An observation will show that when θ is positive and actually 90°, angle formed will be 2π at the centre and centre of the circle formed by the charged particle will be at a distance of radius of the circle from AD.



It will be also true for θ to be negative. (θ measured anticlockwise from normal AC).

$$\text{Hence range is } \pm r = \pm \frac{mv}{QB}.$$

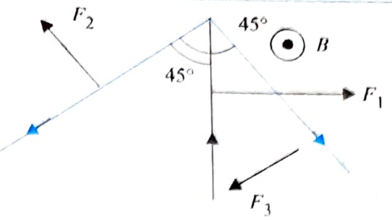
83. (2) The forces will act as shown in figure

Magnitude of F_2 and F_3 will be same.

$$F_2 = F_3 = ILB = 0.5 \times 5 \times 1$$
$$= 2.5 \text{ N} (I_2 = I_3 = 0.5 \text{ A})$$

Resultant of F_2 and F_3

$$= \sqrt{F_2^2 + F_3^2} = 2.5\sqrt{2} \cong 3.5 \text{ N}$$



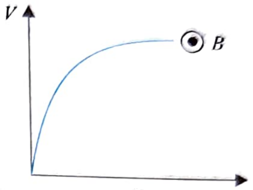
Along negative x-axis

$F_1 = 1 \times 4 \times 1 = 4$ N along positive x-axis

Net resultant force $= 4 - 3.5 = 0.5$ N along positive x-axis.

84. (3) See solution of the previous problem.

85. (2) The force is along x-direction, the system enters the magnetic field with some velocity along y-direction.



So, the system will move in circular path.

86. (3) Since, the system will move in circular path, the velocity will be constant. Hence, the acceleration will be zero.

87. (2) Angle θ for one segment, $\theta = \frac{2\pi}{8} = \frac{\pi}{4}$

$$M_0 = IA = IA = I\left[r^2 \frac{\theta}{2} - r_1^2 \frac{\theta}{2}\right]$$

$$= I\left[(2a)^2 \frac{\theta}{2} - a^2 \frac{\theta}{2}\right] = I \cdot \frac{3a^2\theta}{2}$$

$$\text{For 4 segments, } M = 4I \cdot 3a^2 \frac{\theta}{2} = 6I a^2 \theta$$

Magnetic moment due to circle $= I \pi a^2$

Total magnetic moment $= 6I a^2 \theta + I \pi a^2$

$$= 6I a^2 \times \frac{\pi}{4} + I \cdot \pi a^2 = \frac{5}{2} I \pi a^2$$

88. (1) Force on TP

$$= -F_{\text{loop}} = I aB \sin(90° + 60°) \frac{IaB}{2} \odot$$

$$F_{\text{loop}} = \frac{IaB}{2} \otimes$$

Alternatively

$$F_{PQ} = IaB \sin(90° + 30°) \otimes = IaB \frac{\sqrt{3}}{2} \otimes$$

$$F_{QP} = I(2a)B \sin 30° \otimes = IaB \otimes$$

$$F_{RS} = I(a)B \sin 60° \otimes = IaB \frac{\sqrt{3}}{2} \odot$$

$$F_{ST} = IaB \sin(90° + 60) \odot = \frac{IaB}{2} \odot$$

$$\text{So, } F_{\text{net}} = \frac{IaB}{2} \otimes$$

Multiple Correct Answers Type

1. (1), (2)

If electric field is parallel to magnetic field, the charged particle moving parallel to $\vec{E}$ experiences a force in the direction of $\vec{E}$; due to $\vec{B}$ there will not be any force.

Hence, no deflection.

The particle may go undeflected in the case when forces due to electric field and magnetic field balance each other.

2. (1), (2)

If a charged particle goes unaccelerated in a region containing electric and magnetic fields,

$$|q\vec{E}| = |q\vec{v}\times\vec{B}| \;\Rightarrow\; |\vec{E}| = |\vec{v}\times\vec{B}| \;\Rightarrow\; E = vB\sin\theta$$

3. (1), (2)

The pairs $\vec{F},\vec{v}$ and $\vec{F},\vec{B}$ are always at right angle to each other, because $\vec{F}$ is always perpendicular to the plane containing $\vec{B}$ and $\vec{v}$. Vectors $\vec{B}$ and $\vec{v}$ may have any angle between them.

4. (1), (2), (3), (4)

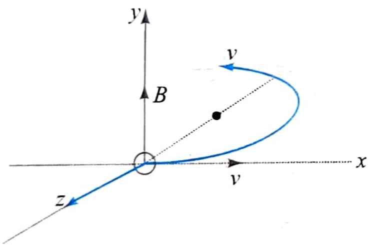
Options (1) and (2) are theoretical facts. As in case of moving charged particle in magnetic field $\vec{F}_{\text{mag}} \perp \vec{v}$, hence power associated will be zero (option (3) is correct).

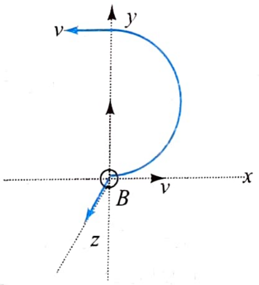
If both the electric and magnetic fields exist: If $\vec{B} \parallel \vec{E}$, the path of the charged particle will be helical.

If $\vec{B}$ is not parallel to $\vec{E}$, the radius of the charged particle will not be constant. Hence, the path will not be circular (option (4) is correct).

5. (1), (2)

Use right hand rule or $\vec{F} = q\vec{v}\times\vec{B}$ for explanation.



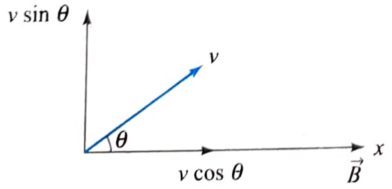


If we apply magnetic field along y-direction, the circular path in x–z plane, the particle can move parallel to magnetic direction.

If we apply the magnetic field in z direction, the plane of circular path will be x–y plane.

6. (1), (4)

When the particle moves in a helical path, the plane of the helical path will be slightly inclined with x-axis. Hence, options (1) and (4) are correct.



7. (1), (2), (3)

$$\text{Pitch} = \left(\frac{2\pi m}{QB}\right)v\cos\theta \text{ and radius } r = \frac{mv\sin\theta}{QB}$$

Maximum distance of the particle from the x-axis = $2r$

$$\therefore \quad \left(\frac{2\pi m}{QB}\right)v\cos\theta = 2\frac{mv}{QB}\sin\theta$$

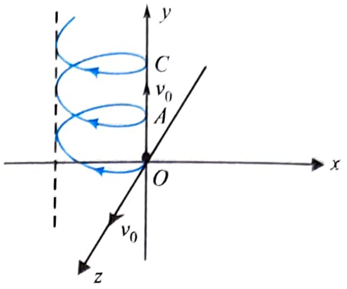
or $\quad \tan\theta = \pi$

Hence, the options (1), (2) and (3) are not correct.

8. (2), (4)

The path of the particle will be helix as shown in figure. Clearly x-coordinate is always negative. z-coordinate can be negative and positive both. x and z coordinate will be zero at the same time at points A, C etc.

$$y = v_0 t \;\Rightarrow\; y \propto t$$



9. (1), (2), (3)

As $\vec{F}\perp\vec{v}$, so $\vec{F}\cdot\vec{v} = 0 \;\Rightarrow\; m\vec{a}\cdot\vec{v} = 0$

$$\Rightarrow \quad m(2\hat{i} + x\hat{j})\cdot(3\hat{i} + 4\hat{j}) = 0$$

$$\Rightarrow \quad 6 + 4x = 0 \;\Rightarrow\; x = -1.5$$

Let the magnetic field is $\vec{B} = a\hat{i} + b\hat{j} + c\hat{k}$, then from $\vec{F} = q\vec{v}\times\vec{B}$

$$\Rightarrow \quad m(2\hat{i} + x\hat{j}) = q(3\hat{i} + 4\hat{j})\times(a\hat{i} + b\hat{j} + c\hat{k})$$

$$\Rightarrow \quad 2m\hat{i} - 1.5m\hat{j} = q(3b\hat{k} - 3c\hat{j} - 4a\hat{k} + 4c\hat{i})$$

$$4c = 2m \;\Rightarrow\; c \neq 0$$

$$3b - 4a = 0 \;\Rightarrow\; b = \frac{4}{3}a$$

a and b both may or may not be zero.

10. (1), (2), (3)

If a charge particle experiences no electromagnetic force, then electric field must be zero (as $\vec{F} = q\vec{E}$). The magnetic field may or may not be zero as a charge particle at rest or moving in the direction of magnetic field experiences no force.

1. When both $\vec{E}$ and $\vec{B}$ are 0, it is undeflected.
2. If $\vec{E}$ is non-zero, it will move in the direction of E and remains undeflected.
3. $\vec{E}$ is non-zero, force will be in direction of E. B is non-zero and V is parallel to $\vec{B}$.
4. $\vec{F} = q\vec{v}\times\vec{B}$ will become zero due to $\vec{B}$ and proton will move in a direction of $\vec{E}$ only.

11. (2), (3), (4)

$\vec{\tau} = \vec{M}\times\vec{B}$ and $U = -\vec{M}\cdot\vec{B}$

Here, $\vec{M}$ and $\vec{B}$ are anti-parallel.

$\therefore \quad \vec{\tau} = \vec{0}$ and $U = +MB$ (maximum)

$\therefore \quad$ (2), (3) and (4).

12. (1), (2)

Let d = distance of the target T from the point of projection. P will strike T if d is an integral multiple of the pitch.

$$\text{Pitch} = \left(\frac{2\pi m}{QB}\right)v\cos\theta$$

$$= k\left(\frac{v}{B}\right)$$

where k = constant

Initially, pitch: $d = k\left(\frac{v_0}{B_0}\right)$

For the given options if pitch $d' = \frac{d}{2}, \frac{d}{3}, \frac{d}{4}, \ldots$, etc., then charge will hit the target.

13. **(1), (3)**

In the absence of a magnetic field, the particle will experience gravitational force mg. As a result, the particle will not continue moving in the horizontal direction but will describe a parabolic path. So, a magnetic field must be present and its direction must be perpendicular to the direction of the velocity. The magnetic force experienced by the particle is given by $\vec{F} = q(\vec{v} \times \vec{B})$.

The magnitude of the force is $F = qvB \sin \theta$. If the particle is to move in the horizontal direction, this force must balance the force of gravity, i.e., $mg = qvB \sin \theta$

The minimum value of B corresponds to $\sin \theta = 1$ or $\theta = 90°$. Thus, $mg = qvB$

or $$B = \frac{mg}{qv} = \frac{0.5 \times 10^{-3} \times 9.8}{2.5 \times 10^{-8} \times 6 \times 10^4} = 3.27 \text{ T}$$

Hence, the correct options are (1) and (3).

14. **(1), (2), (3)**

To find the Ampere's force on a conductor of any shape, replace the conductor by an imaginary straight conductor joining the two ends of the given conductor. So, if B is in x-direction, then the imaginary straight conductor will be along the field and the force acting on it will be zero. If B is in y direction, then the force will be λBI acting along the z direction. Similarly, if B is in the z direction, then the force will be λBI, acting along the negative y direction.

15. **(1), (4)**

The magnitude of the magnetic field depends only on the distance from the x-axis. Points A and C are at distances of 1 unit each from the x-axis. Points B and D are at distances of $\sqrt{2}$ unit each from the x-axis. Magnetic field at point D,

$$B = \frac{\mu_0 I}{2\sqrt{2}\pi}$$

It is obvious that field B is inclined at an angle of 45° with the x–y plane.

16. **(1), (3)**

1. Velocity of the particle can be constant if the electric force balances the magnetic force, i.e., $q\vec{E} = q\vec{v} \times \vec{B}$.
2. If $E = 0$, the particle can trace circular path if $v \perp B$.
3. If $E = 0$, kinetic energy remains constant, because magnetic force is not doing any work.

17. **(1), (2), (4)**

$$|\vec{v}| = \sqrt{8^2 + 6^2} = 10 \text{ m s}^{-1}$$

Radius of circular path $r = \frac{mv}{qB} = \frac{10}{2} = 5$ m

As the magnetic field is in z-direction, hence the path of the charged particle will be circular of radius 5 m in x-y plane; hence options (1) and (2) may be possible.

Time period of circular path $T = \frac{2\pi r}{v} = \frac{2\pi \times 5}{10} = \pi$ seconds

18. **(2), (3)**

$\vec{V} \perp \vec{B}$

Therefore, path of the particle is a circle. In magnetic field, speed of the particle remains constant. Therefore, distance moved by the particle in time $t = \frac{\pi}{B_0 \alpha}$ is $v_0 t$ or $\frac{\pi v_0}{B_0 \alpha}$.

Magnitude of velocity is always v_0, hence option (4) is incorrect.

19. **(1), (2), (3), (4)**

Option (2) is obvious

$$r = \frac{mv_0}{qB}$$

$$PQ = 2r \sin \alpha = 2\frac{mv_0}{qB} \sin \alpha$$

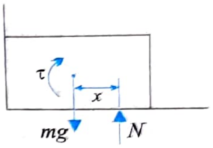
$$\alpha = \beta$$

Time taken $= T = \frac{2\pi r}{v}$

$$T = \frac{2\pi m}{qB}$$

For t time, $t = \frac{T}{2\pi}(2\pi - 2\alpha) = \frac{2m}{qB}(\pi - \alpha)$

20. **(2), (4)**



Torque of magnetic field: $\tau = MB_0 = Ia^2B_0$

For rotational equilibrium: $\tau = mgx$

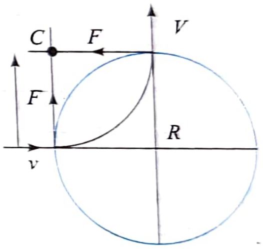
$\Rightarrow \quad Ia^2B_0 = mgx$

$\Rightarrow \quad x = \frac{Ia^2B_0}{mg}$

For the block not to topple:

$$x < \frac{a}{2} \Rightarrow I < \frac{mg}{2aB_0}$$

21. **(1), (2), (3)**



The particle will move along an arc which is part of a circle of radius:

$$r = \frac{mv}{Bq}$$

From figure, we can see that $r = R$

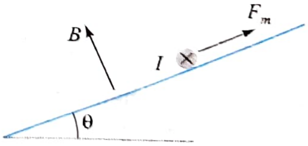
$$\therefore \quad R = \frac{mv}{Bq}$$

$$T = \frac{\pi r/2}{v} = \frac{\pi r}{2v} = \frac{\pi R}{2v} \qquad \left(\because r = R = \frac{mv}{Bq}\right)$$

$$\therefore \quad T = \frac{\pi m}{2Bq}$$

22. **(1), (2)**

When a charge is passed through a wire, a current flows in the wire (for a very small time). As a result of this, the wire experiences a magnetic force for a very small duration, i.e, during the time for which charge has been passed through the wire, due to which it acquires some velocity; then onwards, it moves under gravity. For the wire to move up the incline, the positive charge has to pass through the wire in a direction coinciding with into the plane of paper.



$$F_m = IBl = \frac{dq}{dt} \times B\ell = ma$$

[For this small duration dt, we can neglect gravitational force because I would be very large due to small passage time of charge.]

$$\Rightarrow \quad m\frac{dv}{dt} = \frac{dq}{dt} \times Bl \quad \Rightarrow \quad m\, dv = dq \times B\ell$$

$$\Rightarrow \quad mv = q \times Bl$$

$$\Rightarrow \quad v = \frac{q \times Bl}{m} \quad \Rightarrow \quad q = \frac{mv}{Bl}$$

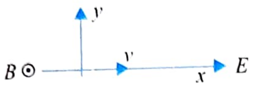
where q is the charge passed through the wire and v is the velocity acquired by the wire just after charge had been passed through it.

From kinematics, $v^2 = 2g \sin\theta\, s$

$$q = \frac{m\sqrt{2gs\sin\theta}}{Bl}$$

23. (3), (4)

(1) If $v = 0$, $E = 0$ then no force will act on particle, hence particle remains at rest



(2) here $v = 0$, $E \neq 0$, due to E the particle will gain velocity in x direction. Due to this velocity it will experience force due to magnetic filed along negative y axis. As a result of it, the particle will move in curved path that will not be a circle

(3) If $v = 0$, $E \neq 0$, then particle will execute cycloidal motion. See theory for this.

(4) If $E = 0$, then path will be circular.

24. (1), (2), (3), (4)

Net force on charge should be zero in region 1,

$$qE = qvB \quad \Rightarrow \quad v = E/B$$

In region 1, net force is zero on particle, hence on work is done on particle in region 1.

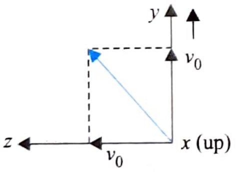
In region 2, only magnetic force is there, hence no work is done in this region also.

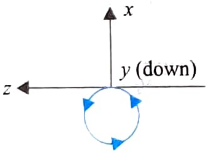
In region 2: $r = \dfrac{mv}{qB_0} = \dfrac{E/B}{(q/m)B_0} = \dfrac{E}{sBB_0}$

If $l_2 > r$ or $l_2 > \dfrac{E}{sBB_0}$, then particle will complete half revolution in region 2 and come out of it with same velocity in opposite direction.

25. (2), (4)

Velocity of proton makes an angle of 45° with the direction of magnetic field.





Therefore, path of the proton is a helix. The plane of the circle of this helix is the plane formed by negative x and z-axis. Therefore, x-coordinate can never be positive and z co-ordinate can be positive as well as negative. Further x and z co-ordinates will become zero simultaneously after every pitch and y-co-ordinate of proton at any time t is $y = v_0 t$ i.e. $y \propto t$

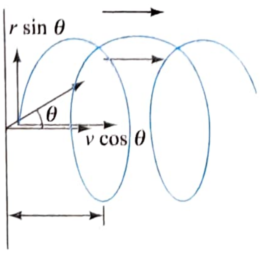
Linked Comprehension Type

For Problems 1–3

1. (2) **2. (1)** **3. (3)**

The first particle will have a helical path and the second particle will move rectilinearly along the field. For the two particles to meet again and again, $v_{\parallel}T = v'T$

where v' is the speed of the second particle.



$\therefore \quad v' = v_{\parallel} = v\cos\theta$

$$\frac{1}{2}mv^2 = qV \quad \Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$$\therefore \quad v' = \sqrt{\frac{2qV}{m}}\cos\theta$$

$$T = \frac{2\pi m}{qB}$$

Distance travelled = pitch

$$\text{Distance} = \sqrt{\frac{2qV}{m}}\cos\theta \times \frac{2\pi m}{qB}$$

$$\text{Distance} = \sqrt{\frac{2Vm}{q}}\frac{2\pi}{B}\cos\theta$$

For Problems 4–5

4. (3) **5. (1)**

For the first case: $\vec{F} = q\vec{v} \times \vec{B}$

$$\Rightarrow \quad -5\sqrt{2} \times 10^{-3}\,\hat{k}$$

$$= 10^{-5} \times \frac{10^6}{\sqrt{2}}\left(\hat{i} + \hat{j}\right) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$= \left(\frac{10}{\sqrt{2}}\right)\left[B_z\hat{i} - B_z\hat{j} + \left(B_y - B_x\right)\hat{k}\right]$$

$$\Rightarrow \quad B_z = 0, B_y - B_x = -10^{-3}\text{ T} \qquad \text{...(i)}$$

Similarly, for the second case:

$$F_2\hat{j} = (10^{-5})(10^6\,\hat{k}) \times \left[\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\right]$$

$$F_2\hat{j} = 10\left(B_x\hat{j} - B_y\hat{i}\right)$$

$$F_2 = 10B_x,\ B_y = 0 \qquad \text{...(ii)}$$

Using eqs. (i) and (ii), we get $B_x = 10^{-3}$ T

Thus, $\vec{B} = (10^{-3}\text{ T})\,\hat{i}$

Also, $F_2 = 10B_x = 10^{-2}$ N.

For Problems 6–7

6. (1) **7. (4)**

Particle's acceleration is in y direction,

$$\frac{dv_y}{dt} = \frac{qE}{m} = \text{constant}$$

The motion of the particle is equivalent to circular motion in xz plane with uniform acceleration in y-direction.

Hence, $v_0^2 = v_x^2 + v_z^2 = \text{constant}$

The magnetic force cannot change the magnitude of v_0.

The y-component of velocity,

$$v_y = \frac{qE}{m}t$$

The y-coordinate at time t, $y = \dfrac{1}{2}\dfrac{qE}{m}t^2$

The time period of circular motion in xz plane,

$$T = \frac{2\pi m}{Bq}$$

Let the particle cross y-axis after n rotations, then

$$t = nT = \frac{2\pi mn}{qB}$$

Thus, $y_n = \frac{qE}{2m} \times \left(\frac{2\pi mn}{qB}\right)^2 = \frac{2\pi^2 mn^2 E}{qB^2}$

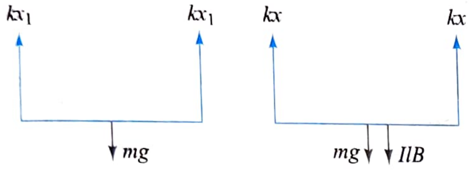
As $\quad v_y = a_y t = \left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{qB}\right) = \frac{2\pi nE}{B}$

Thus, $\tan\alpha = \frac{v_0}{v_y} = \frac{v_0 B}{2\pi nE} \Rightarrow \alpha = \tan^{-1}\left(\frac{v_0 B}{2\pi nE}\right)$

For Problems 8–9

8. (2) **9. (3)**

$I = \frac{\varepsilon}{R} = \frac{24}{12} = 2\text{A}$



Magnetic force $= IlB\sin\frac{\pi}{2} = 2\times 5\times 10^{-2}\times B = \frac{B}{10}$

$$2kx_1 = mg;\ 2k = \frac{mg}{x_1} \quad \text{and}$$

$$2k\,(x_1 + x_2) = mg + IlB$$

$$2kx_1 + 2kx_2 = mg + IlB$$

$$\frac{mg}{x_1} x_2 = IlB$$

$$10\times 10^{-3}\times 10\times \frac{0.3}{0.5} = \frac{B}{10} \Rightarrow B = 600\times 10^{-3} = 0.6\ \text{T}$$

For Problems 10–11

10. (1) **11. (1)**

(a) $F_1 = mg$

When bar is just ready to levitate,

$$IlB = mg,\ I = \frac{mg}{lB} = \frac{(0.750\ \text{kg})(9.80\ \text{m/s}^2)}{(0.500\ \text{m})(0.450\ \text{T})} = 32.67\ \text{A}$$

$$\varepsilon = IR = (32.67\ \text{A})\ (25.0\ \Omega) = 817\ \text{V}$$

(b) $R = 2.0\ \Omega,\ I = \varepsilon/R = (817\ \text{V})/(2.0\ \Omega) = 408.5\ \text{A}$

$$F_1 = IlB = 92\ \text{N}$$

$$a = (F_1 - mg)/a = 113\ \text{ms}^{-2}$$

For Problems 12–13

12. (1) **13. (4)**

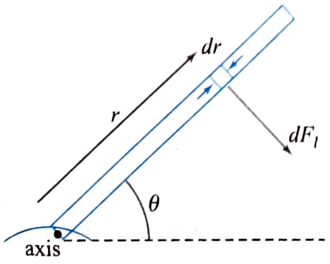
$F = ILB$, to the right.

$$v^2 = 2ad \Rightarrow d = \frac{v^2}{2a} = \frac{v^2 m}{2ILB}$$

For Problems 14–15

14. (1) **15. (1)**

By examining a small piece of the wire (figure), we find:



Divide the rod into infinitesimal sections of length dr. The magnetic force on this section is $dF_1 = IB\,dr$ and is perpendicular to the rod. The torque dt due to force on this section is $d\tau = rd\,F_1 = IBr\,dr$

The total torque is

$$\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442\ \text{N m}^{-1},$$

clockwise. This is the same torque calculated from a force diagram in which the total magnetic force $F_l = ILB$ acts at the center of the rod.

F_1 produces a clockwise torque, so the spring force must produce a counterclockwise torque. The spring force must be to the left, the spring is stretched.

Find x, the amount the spring is stretched:

axis at hinge, counterclockwise torques positive

$$(kx)l\sin 53° - \frac{1}{2}Il^2B = 0$$

$$x = \frac{IlB}{2k\sin 53.0°} = \frac{(6.50\ \text{A})(0.200\ \text{m})(0.340\ \text{T})}{2(4.80\ \text{Nm}^{-1})\sin 53.0°}$$

$$= 0.05765\ \text{m}$$

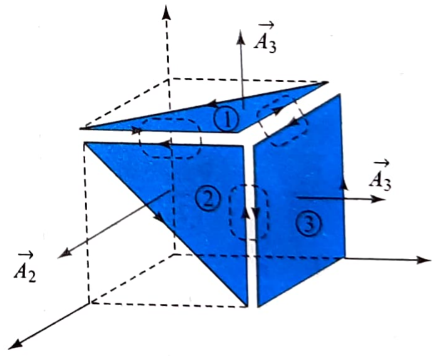
For Problems 16–18

16. (1) **17. (2)** **18. (3)**

The net force on a current carrying loop of any arbitrary shape in a uniform magnetic field is zero.

$$\vec{F}_{\text{net}} = 0$$

The given loop can be considered to be a superposition of three loops as shown in figure. The area vector of the three loops (1), (2) and (3) are



$$\vec{A}_1 = \left(\frac{1}{2}\times 10\times 10\times 10^{-4}\right)\hat{j}\ \text{m}^2$$

$$\vec{A}_2 = \left(\frac{1}{2}\times 10\times 10\times 10^{-4}\right)\hat{k}\ \text{m}^2$$

$$\vec{A}_3 = \left(10\times 10\times 10^{-4}\right)\hat{i}\ \text{m}^2$$

Magnetic moment vector,

$$\vec{\mu} = i\vec{A} = 10(0.01\hat{i} + 0.005\hat{j} + 0.005\hat{k})\ \text{Am}^2$$

$$= (0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k})\ \text{Am}^2$$

Torque,

$$\vec{\tau} = (0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k})$$

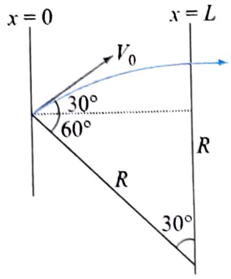
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.1 & 0.05 & 0.05 \\ 2 & -3 & 1 \end{vmatrix} = 0.2\hat{i} - 0.4\hat{k}\ \text{Nm}$$

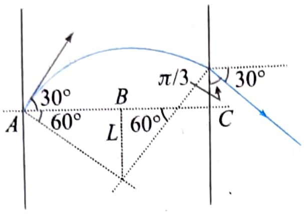
For Problems 19–20

19. (2) **20. (4)**

$\sin 30^\circ = L/R$

$$L = \frac{mV_0}{2qB} = \frac{50 \times 10^{-3} \times 100}{2 \times 1 \times 25} = \frac{1}{10}\ \text{m} = 10\ \text{cm}$$

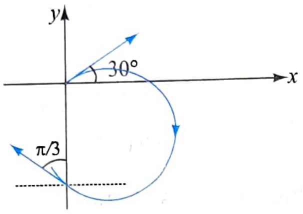




$L = AB + BC$

$= r\cos 60^\circ + r\cos 60^\circ = r$

$$L = \frac{mV_0}{qB} = 20\ \text{cm}$$



⇒ This is minimum L.

For other case (figure):

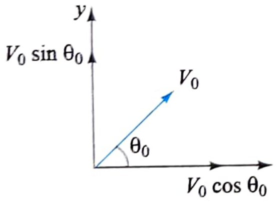
For this case L can extend upto ∞

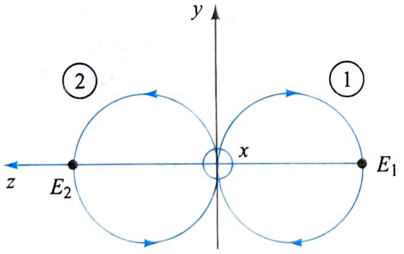
For Problems 21–22

21. (1), (2) **22. (2)**

As the magnetic field is along the x-axis, the magnetic force will be along (–) z-axis at $t = 0$. So, the particle will move in helical path along (1). At $t = T_0$, the direction of field changes, so force becomes along + z-direction, and now the particle will move in helical path along (2). It will be moving along x-axis, so that resulting path will be helical.

At $t = \frac{T_0}{2}$, particle will be at E_1.





x-coordinate $= \frac{P_0}{2}$ (half of pitch)

y-coordinate $= 0$ (from figure)

and z-coordinate $= -2R_0$ (from figure)

Hence, (1) is correct.

Similarly at $t = \frac{3T_0}{2}$ particle will be at E_2.

∴ The coordinates are $\left(\frac{3P_0}{2}, 0, 2R_0\right)$

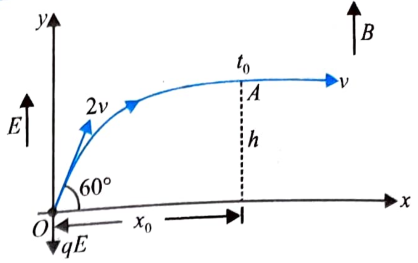
Hence, (2) is correct.

From figure, we can see that distance between two extremes: E_1 and E_2 is $4R_0$.

For Problems 23–25

23. (1) **24. (2)** **25. (3)**

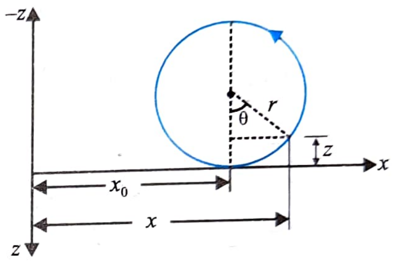
First particle will travel along parabolic path OA. Let time from O to A is t. $a_y = \frac{-qE}{m}$



$$x = \frac{\sqrt{3}\,mv^2}{qE} = (2v\cos 60^\circ)\,t_0 \quad \Rightarrow \quad t_0 = \frac{\sqrt{3}\,mv}{qE}$$

$$v_y = u_y + a_y t_0 = 2v\sin 60^\circ - \frac{qE}{m}\,\frac{\sqrt{3}\,mv}{qE} = 0$$

Hence at point A, velocity will be purely along x-axis and it will be $2v\cos 60^\circ = v$. Now magnetic field is switched on along y-axis. Now its path will be helical as shown below with increasing pitch towards negative y-axis.



$$r = \frac{mv}{qB}$$

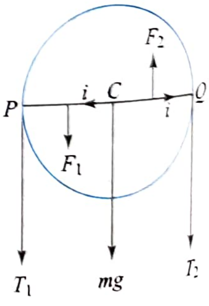
$$x = x_0 + r\sin\theta = (2v\cos 60^\circ)t_0 + \frac{mv}{qB}\sin\omega t$$

$$= v\sqrt{3}\,\frac{mv}{qE} + \frac{mv}{qB}\sin\left(\frac{qB}{m}t\right)$$

z-coordinate: $z = -(r - r\cos\theta) = -\frac{mv}{qB}\left[1 - \cos\left(\frac{qB}{m}t\right)\right]$

For Problems 26–28

26. (1) **27. (2)** **28. (2)**



PC and CQ are in Parallel. So, equivalent resistance of the circuit is $R/2$. In static condition, current through the battery is

$I = 2V/R$.

The current through CP and CQ is

$i = I/2 = V/R$

Hence, $F_1 = F_2 = Bir$

Consider the FBD of ring and the blocks

FBD of ring: Consider torque about 'C',

$$T_1 r + F_1 r/2 + F_2 r/2 = T_2 r \quad \text{...(i)}$$

FBD of blocks: $T_1 = mg$ and $T_2 = 2mg$

So, $mgr + Bir^2/2 + Bir^2/2 = 2mgr$

or $i = \frac{mg}{Br}$ or $\frac{V}{R} = \frac{mg}{Br}$ or $V = \frac{mgR}{Br}$

Net torque of tension

$$= (T_2 - T_1)r = mgr = \frac{BVr^2}{R}$$

For Problems 29–31

29. (1) Angle remains constant, because magnitudes of components of velocities along and perpendicular to magnetic field remain constant.

30. (2) $f = \frac{qB}{2\pi m} = \frac{10^3}{19} \times \frac{\sqrt{3800}}{2\pi} = \frac{10^4}{\pi\sqrt{38}}$

31. (4) Pitch $= TV\cos\theta$

$$= \frac{1}{f} V \frac{\vec{V}\vec{B}}{BV} = \frac{1}{f}\frac{\vec{V}\vec{B}}{B} = \frac{\pi\sqrt{38} \times 400}{10^4\sqrt{3800}} = \frac{4\pi}{10^3}\,\text{m} = \frac{\pi}{250}\,\text{m}$$

For Problems 32–34

32. (4) At speed $v_0 = F_e = F_m$

$$qE = qv_0B \Rightarrow v_0 = \frac{E}{B}$$

Hence (4) is correct option.

33. (3) Greater the distance d, larger would be the deviation produced in the path. Hence the particle will be unable to enter the detector for greater distance d, thus narrowing the range of speeds detected.

34. (1) For $v = 0,\ a = \frac{qE}{m}$

For $v = v_0,\ a = \frac{qE - qvB}{m}$

or $a = \frac{-qB}{m}v + \frac{qE}{m}$ $(y = -mx + c)$

For $v = v_0, a = 0$

For $v > v_0\ a = \frac{qBv}{m} - \frac{qE}{m}$ $(y = mx - c)$

Hence graph (1) is correct

For Problems 35–37

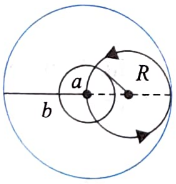
35. (1) In the absence of magnetic field, the electron moves in a straight hit to the anode due to the presence of electrostatic force.

Hence choice (1) is correct.

36. (2) When $V = 0$, the electron moves in homogeneous static field B. The magnetic field acts orthogonal to the velocity and hence electron moves in a circle.

Hence choice (2) is correct.

37. (3) From the figure, we see that in critical core the radius R of the circle satisfies



$$\sqrt{a^2 + R^2} = b - R$$

or $a^2 + R^2 = b^2 + R^2 - 2bR$

or $R = \left(\frac{b^2 - a^2}{2b}\right)$ ...(i)

The radius R of the circular path is determined by equating centripetal force and Lorentz force

$$eB_cv_0 = \frac{mv_0^2}{R} \Rightarrow B_c = \frac{mv_0}{eR}$$

Using equation (i)

$$B_C = \frac{2mbv_0}{e(b^2 - a^2)}$$

Hence choice (3) is correct.

For Problems 38–40

38. (2) Velocity at A $v = \sqrt{2as}$

$$= \sqrt{2 \times \left(\frac{qE}{m}\right)s} = \sqrt{\frac{2 \times 1 \times 10 \times 1.8}{1}} = 6\text{m/s}$$

In magnetic field, speed does not change. Hence particle will collide with speed 6 m/s

39. (4) In magnetic field path of the particle is circle. Radius of circular path is

$$r = \frac{mv}{qB} = \frac{(1)(6)}{(1)(5)} = 1.2\text{ m}$$

$$d = 2.4\text{ m} - 1.8\text{ m} = 0.6\text{ m}$$

Since $d < r\sin\theta = \frac{d}{r} = \frac{0.6}{1.2} = \frac{1}{2}$

$\Rightarrow$ $\theta = 30°$

$AE = AD - DE = r - r\cos\theta$

$$= r(1 - \cos\theta) = 1.2\left(1 - \frac{\sqrt{3}}{2}\right) = 0.6(2 - \sqrt{3})$$

$$FC = BF\tan\theta = \frac{0.6}{\sqrt{3}}$$

$\therefore$ y-co-ordinate $= AE + FC$

$$= 0.6\left(2 - \sqrt{3}\right) + \frac{0.6}{\sqrt{3}}$$

$$0.6\left[2 - \sqrt{3} + \frac{1}{\sqrt{3}}\right] = \frac{1.2\left(\sqrt{3} - 1\right)}{\sqrt{3}}\text{ m}$$

Hence choice (4) is correct and other choices are wrong.

40. (1) Total time $t = t_{OA} + t_{AB} + t_{BC}$

$$t_{OA} = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2sm}{qE}} = \sqrt{\frac{2 \times 1.8 \times 1}{(1)(10)}}$$

$$= 0.6\text{sec} = 3/5\text{sec}$$

$$t_{AB} = \left(\frac{30°}{360°}\right)T = \frac{1}{12} \times \frac{2\pi m}{qB}$$

$$\frac{(2\pi)(1)}{(12)(1)(5)} = \frac{\pi}{30}\text{sec}$$

$$t_{BC} = \frac{BC}{v} = \frac{0.6\sec\theta}{v} = \frac{0.6\left(\frac{2}{\sqrt{3}}\right)}{6} = \frac{1}{5\sqrt{3}}\text{sec}$$

Hence $\frac{3}{5} + \frac{\pi}{30} + \frac{1}{5\sqrt{3}} = \frac{1}{5}\left(3 + \frac{\pi}{6} + \frac{1}{\sqrt{3}}\right)$ sec

Hence choice (1) is correct.

For Problems 41–43

41. (2) Since electrons are deflected downward, hence electric field must be upward. The magnetic force in order to cancel the electric force must point upward. From right hand rule, magnetic field must point out of page. Hence choice (2) is correct.

42. (2) Since $d_1 = \frac{eEL^2}{2mv^2}$ or $\frac{e}{m} = \frac{2d_1v^2}{EL^2}$

Here $d_1 = 2 \times 10^{-3}$ m

$v = 3 \times 10^7$ m/s $L = 0.1$m

$E = 1800$ V/m

$$\frac{e}{m} = \frac{2 \times (2 \times 10^{-3}) \times (3 \times 10^7)^2}{1800 \times (0.1)^2} = 2 \times 10^{11}\text{ C/kg}$$

Hence choice (2) is correct.

43. (1) In the passage, we have

$$d_1 = \frac{eEL^2}{2mv^2}\text{ and }\frac{e}{m} = \frac{2d_1E}{B^2L^2}$$

By increasing speed of electron to two times, d_1 will become 1/4th. For this magnetic field should be halved, so that we can correctly measure e/m. Hence choice (1) is correct.

Matrix Match Type

1. i. → a., c., d.; ii. → a., c., d.; iii. → a., b.; iv. → a., b.

i. Kinetic energy of the particle can remain constant, if both the fields are present. This is possible if the force due to both fields cancel each other.

Kinetic energy of the particle can also remain constant if only magnetic field is present, because magnetic field does not do any work.

Obviously KE will remain constant if no field is present.

ii. This is possible if either both the fields are present or no field is present. This is also possible if only magnetic field is present and the particle is at rest or moving in the direction of field.

iii. This is possible if electric field is present, magnetic field may or may not be present.

iv. This is possible if only electric field is present and velocity and electric field are along the parallel lines. Magnetic field may also be present if it is parallel to velocity.

2. i.→ a., b., d.; ii. → a., b., c., d.; iii. → a., b., d.; iv. → b.

i. Velocity of the particle may be constant, if forces of electric and magnetic fields balance each other. Then, path of particle will be straight line. Also, path of particle may be helical if magnetic and electric fields are in same direction. But path of particle cannot be circular. Path can be circular if only magnetic field is present, or if some other force is present which can cancel the effect of electric field.

ii. Here, all the possibilities are possible depending upon the combinations of the three fields.

iii. This situation is similar to part (i).

iv. In a uniform electric field, path can be only straight line or parabolic.

3. i. → d.; ii. → a.; iii. → b.; iv. → c.

We know that $r = mv/qB$. So, larger the mass, larger is the radius. Since radius of path '4' is largest, so path '4' should correspond to ion B.

Hence, ii. → a.

Now path '2' should correspond to ion C, because charge on both B and C is negative, so their paths should be on the same side.

Hence, iii. → b.

Now, radii of paths '2' and '3' are same. If path '2' corresponds to C, then path '3' should correspond to ion A, because masses of A and C are same.

Hence, i. → d.

Now, remaining path '1' should correspond to D.

Hence, iv. → c.

4. i. → a., c.; ii. → a., b.; iii → d.; iv. → a.

i. Since $\vec{E} = 0$ and $\vec{B} \neq 0$, so path will be straight line if velocity is parallel to $\vec{B}$. Or path will be circular if $\vec{V} \perp \vec{B}$. Or path will be helical (with uniform pitch) if $\vec{V}$ is at some other angle to $\vec{B}$.

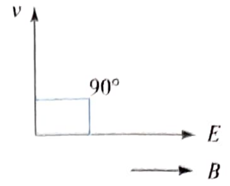
Hence, i. → a, c

ii. Since $\vec{E} \neq 0$ and $\vec{B} = 0$, so path will be straight line if $\vec{v} \parallel \vec{E}$ or parabola otherwise.

Hence, ii. → a, b

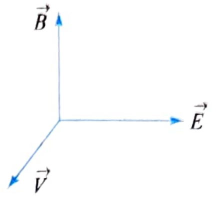
iii. $\vec{E} \neq 0, \vec{B} \neq 0, \vec{E} \parallel \vec{B}$

Helical path with non-uniform pitch



Hence, iii. → d.

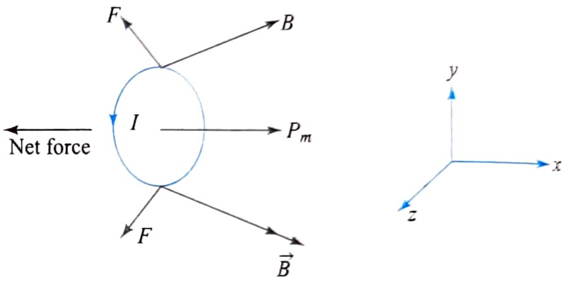
iv. Straight line path if $\vec{V} \times \vec{B} = \vec{E}$



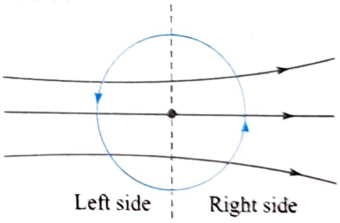
Hence, iv. → a.

5. i. → b.; ii. → a., c.; iii. → a., c.; iv. → a., d.

i.

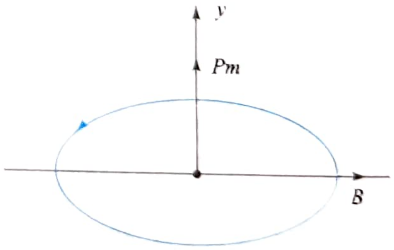


ii. Force on left side is along z-axis and on right side is along $-z$-axis. But former is greater because of higher magnetic field on left side. Hence, net force is along z-direction or along P_m.



iii. Now net force is reversed, but P_m is also reversed.

iv.



Force on left side is more than on right side. And net force will be along y-axis or along P_m.

6. i. → c., d.; ii. → c., d.; iii. → b., c.; iv. → a., c.

i. Because the magnetic field is parallel to x-axis, the force on wire parallel to x-axis is zero. The force on each wire parallel to y-axis is $B_0 \frac{i}{2} l$. Hence, net force on the loop is $B_0 il$.

Similar is the situation in case (ii)

Hence, i. → c, d ii. → c, d

iii. Since direction of current from entry point in the loop to exit point in the loop is along the diagonal of the loop, the direction of external uniform magnetic field is also along the same diagonal. Hence, net force on the loop is zero.

iv. The direction of current from entry point in the loop to exit point in the loop is along the diagonal (of length $\sqrt{2}\,l$) of the loop. The direction of external uniform magnetic field is also perpendicular to the same diagonal. Hence, magnitude of net force on the loop is $B_0 i\left(\sqrt{2}l\right)$. Since force on each wire on the loop passes through centre of the loop, net torque about centre of the loop is zero.

7. **i. → a., b., c., d.; ii. → a., b., d.; iii. → d.; iv. → c.**

i. If both are zero, then no force will act on the particle and then particle will move with uniform velocity.

If both are non-zero, then it is possible that both the forces acting on the particle cancel each other and particle move in straight line.

If one of them is not zero, then particle can move without change in its direction if it moves along the direction of non-zero field.

ii. The velocity will definitely change if only E is non-zero, otherwise velocity may remain same.

iii. This is possible only if B is non-zero and E is zero.

iv. This is possible only if E is non-zero and B is zero.

8. **i. → b., d.; ii. → a., b., c.; iii. → b., d.; iv. → b., c.**

i. If a charged particle at rest experiences a force, then this should be electrostatic force. So electric field should be present, magnetic field may or may not present.

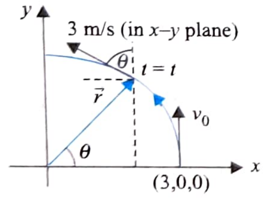
ii. This is possible if (i) both fields are zero (ii) both fields are not zero and their forces balance (iii) $E = 0$, $B \neq 0$ and velocity is parallel to magnetic field.

But if only E is there, then velocity cannot remain same.

iii. speed can vary only if electric field is present. Particle will go undeviated with varying speed if v, E and B all are in same direction. B may also be zero.

iv. For helical path, B should be present, E may or may not be.

9. **i. → c., d.; ii. → c., d.; iii. → a.; iv. → b.**



$$\theta = \omega t = 1 \times t = t$$

$$\omega = \frac{qB_0}{m} = 1 \text{ rad/s}, \quad R = \frac{mv_\perp}{B_0 q} = 1 \times 3 = 3 \text{ m}$$

$$T = \frac{2\pi}{\omega} = 2\pi \text{ sec}, \text{ pitch } = 4 \times 2\pi = 8\pi \text{ m}$$

$$\vec{r} = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + 4t\,\hat{k}$$

$$\vec{v} = -3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 4\hat{k}$$

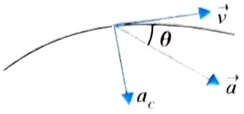
$$\vec{a} = -3\cos t\,\hat{i} - 3\sin t\,\hat{j}$$

$$\vec{v} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3\sin t & +3\cos t & +4 \\ -3\cos t & -3\sin t & 0 \end{vmatrix}$$

$$= (+12\sin t)\hat{i} - (+12\cos t)\hat{j} + 9\hat{k}$$

$$= +12\sin t\,\hat{i} - 12\cos t\,\hat{j} + 9\hat{k}$$

$$\Rightarrow |\vec{v} \times \vec{a}| = 15$$



$$a_C = \frac{v^2}{r} \quad \Rightarrow \quad a \sin\theta = v^2/r$$

$$\Rightarrow \quad a\frac{|\vec{v} \times \vec{a}|}{va} = \frac{v^2}{r} \quad \Rightarrow \quad r = \frac{v^3}{|\vec{v} \times \vec{a}|}$$

$$\Rightarrow \quad \text{radius of curvature} = \frac{v^3}{|\vec{v} \times \vec{a}|} = \frac{5^3}{15} = \frac{25}{3} \text{ m}$$

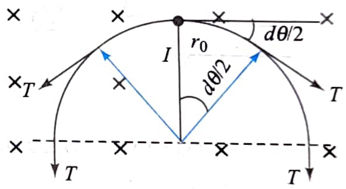
Numerical Value Type

1. **(3)** $IBl = \mu mg$

$$\Rightarrow \quad \frac{6}{20} \times 0.8 \times \frac{5}{100} = \mu \frac{10}{1000}(10)$$

$$\Rightarrow \quad \mu = 0.12 = 3/25$$

2. **(1)** For the equilibrium of a small part of semicircular arc subtending an angle of $d\theta$ at the centre,



$$\text{or,} \quad 2T\sin\left(\frac{d\theta}{2}\right) = BI\,r_0\,d\theta$$

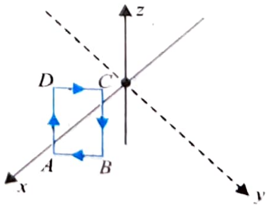
$$B = \frac{T}{Ir_0} = \frac{1.5}{(10)(0.15)} = 1 \text{ T}$$

3. **(2)** $B = 10^4$ G $= 1$ T

For equilibrium, magnetic force should be equal to weight of wire.

$IBl = mg \quad \Rightarrow \quad I \times 1 \times 0.10 = 20 \times 10^{-3} \times 10 \quad \Rightarrow \quad I = 2\text{A}$

4. **(2)** $\vec{m} = I_2\vec{S}$

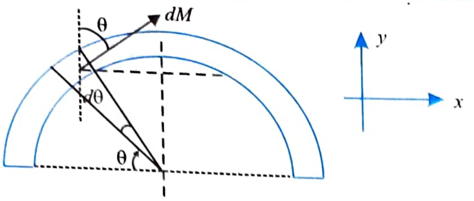


$$\vec{S} = \overrightarrow{BA} \times \overrightarrow{AD}, \ \overrightarrow{BA} = 2d\hat{i} - 2a\hat{j}, \ \overrightarrow{AD} = 2b\hat{k}$$

$$\vec{S} = 2(d\hat{i} - a\hat{j}) \times 2b\hat{k}$$

$$\vec{M} = -4bI(d\hat{j} + a\hat{i}) \quad \Rightarrow \quad |\vec{M}| = 4bI\sqrt{d^2 + a^2} = 2 \text{ J/T}$$

5. **(5)** The magnetic moment of circular current is given by AI, I being the circulating current and A is the area of cross-section; the direction is perpendicular to the plane of current. Now, for an element of toroid of length rdq, its magnetic moment is along the direction of arrow as shown, of magnitude (perpendicular to its cross-section) = current × area × number of turns in the length rdq.



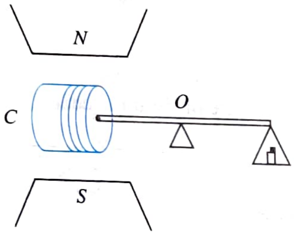
$$dM = I\,\frac{\pi d^2}{4}\left(\frac{N}{\pi r}\,r\,d\theta\right) = \frac{N}{4}\,d^2 I\,d\theta$$

Resolving dM into components along x and y, we get $dM\sin\theta$ and $dM\cos\theta$; components along y from neighbouring elements cancel out to zero, and components along x are added. So

$$M = \int dM\sin\theta = \int_0^{\pi}\frac{N}{4}\cdot d^2 I\sin\theta\,d\theta = \frac{Nd^2 I}{2}$$

Putting the given values we get $M = 5$ Am^2.

6. (4)



Magnetic torque = $NISB\sin(\vec{S},\vec{B})$ $NISB\sin 90°$

Gravitational torque = $(\Delta m \times g)l$

For equilibrium $NISB = \Delta mgl \;\Rightarrow\; \Delta mgl/NIS$

$$B = \frac{(60\times10^{-6})\times9.8\times0.3}{200\times22\times10^{-3}\times1\times10^{-4}} = 4\times10^{-1}\text{ T}$$

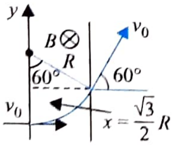
7. (2) Force exerted by air on the rod = $\left(\rho\frac{L}{2}2R\right)v^2 = \rho LRv^2$

Balancing torque about point O, $NI(\pi R^2)B = \rho LRv^2\,\dfrac{3L}{4}$

$$\Rightarrow\quad 300\pi IBR = \frac{3\rho v^2L^2}{4}$$

$$\Rightarrow\quad I = \frac{\rho L^2v^2}{400\pi BR} = \frac{1}{100}\left(\frac{Lv}{2}\sqrt{\frac{\rho}{\pi BR}}\right)^2 = 0.002\text{ A} = 2\text{ mA}$$

8. (4) The particle will come out of the magnetic field at an angle $\theta = 60°$ with the original direction.

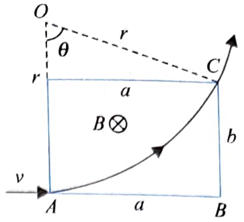


$$\Delta\vec{v} = \left(v_0\cos60\hat{i} + v_0\sin60\hat{j} - v_0\hat{i}\right) \Rightarrow |\Delta\vec{v}| = v_0$$

9. (5) Let speed of the particle is v (speed will remain constant)

$$r = AO = CO = \frac{mv}{qB}\ \text{also}\ \frac{a}{r} = \sin\theta,\ \frac{r-b}{r} = \cos\theta$$

Solve the above equation to get



$$v = \frac{qB(a^2+b^2)}{2mb} = \frac{10^{-6}\times1.2(4^2+3^2)}{2\times10^{-6}\times3} = 5\text{ m/s}$$

10. (2) $v_\perp = -\dfrac{v_0}{2}$ will contribute to circular motion.

$v_\parallel = \dfrac{v_0\sqrt{3}}{2}$ will contribute to helical path. $T = 2\pi\dfrac{m}{B_0q}$

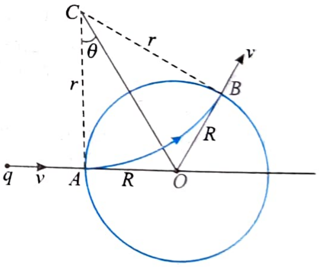
Pitch, $P = v_\parallel T = \dfrac{v_0\sqrt{3}}{2}T$

Number of turns $= \dfrac{R}{P} = \dfrac{v_0T\sqrt{3}(2)}{v_0\sqrt{3}T} = 2$

11. (60) Radius of circular path of the particle is

$$r = \frac{mv}{qB} = \frac{2\times10^{-3}\times0.3}{10^{-3}\times0.2} = 3\text{ m}$$

C is centre of the circular path.



Particle enters the field at A and leaves at B.

$$\tan\theta = \frac{R}{r} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

Deviation = $2\theta = 60°$.

12. (0.01) F_2 is in y-direction when velocity is along z-axis. Therefore magnetic field should be along x-axis. So let, $B = B_0\hat{i}$

Given $v_1 = \dfrac{10^6}{\sqrt{2}}\hat{i} + \dfrac{10^6}{\sqrt{2}}\hat{j}$

and $F_1 = -5\sqrt{2}\times10^{-3}\,\hat{k}$

From equation $F = q(v\times B)$

We have

$$(-5\sqrt{2}\times10^{-3})\hat{k} = (10^{-6})\left[\left(\frac{10^6}{\sqrt{2}}\hat{i} + \frac{10^6}{\sqrt{2}}\hat{j}\right)\times(B_0\hat{i})\right]$$

$$= -\frac{B_0}{\sqrt{2}}\hat{k}$$

$$\therefore\quad \frac{B_0}{\sqrt{2}} = 5\sqrt{2}\times10^{-3}$$

or $\quad B_0 = 10^{-2}$ T

Therefore, the magnetic field is $B = (10^{-2}\hat{i})$ T

$$F_2 = B_0 q v_2 \sin 90°$$

As the angle between B and v in this case is 90°.

$$F_2 = (10^{-2})(10^{-6})(10^6)$$

$$= 10^{-2}\text{ N} \approx 0.01\text{ N}$$

13. (1.6) From the equation, $r = \dfrac{mv}{Bq}$

We have $B = \dfrac{mv}{qr}$

Substituting the values, we have

$$B = \frac{(1.67\times10^{-27})(10^7)}{(1.6\times10^{-19})(6.4\times10^6)} = 1.6\times10^{-8}\text{ T}$$

14. (2) $F_e = F_m$ or $eE = eBv$

$$\therefore \quad v = \frac{E}{B} = \frac{120\times10^3}{50\times10^{-3}} = 2.4\times10^6\text{ m/s}$$

Let n be the number of protons striking per second.

Then, $ne = 0.8\times10^{-3}$

or $\quad n = \dfrac{0.8\times10^{-3}}{1.6\times10^{-19}} = 5\times10^{15}$ m/s

Force imparted = Rate of change of momentum

$$= nmv$$

$$= 5\times10^{15}\times1.67\times10^{-27}\times2.4\times10^6$$

$$= 2.0\times10^{-5}\text{ N}$$

15. (3.46) To graze at C

Using equation of trajectory of parabola,

$$y = x\tan\theta - \frac{ax^2}{2v^2\cos^2\theta} \quad \text{...(i)}$$

Here, $a = \dfrac{qE}{m} = \dfrac{10^{-6}\times10^{-3}}{10^{-10}} = 10\text{ m/s}^2$

Substituting in equation (i), we have

$$0.05 = 0.17\tan 30° - \frac{10\times(0.17)^2}{2v^2\times(\sqrt{3}/2)^2}$$

Solving this equation, we have $v = 2$ m/s

In magnetic field, $AC = 2r$

or $\quad 0.1 = 2r$

or $\quad r = 0.05\,m = \dfrac{mv\cos 30°}{Bq}$

$$\therefore \quad B = \frac{mv\cos30°}{(0.05)q} = \frac{(10^{-10})(2)(\sqrt{3}/2)}{(0.05)(10^{-6})}$$

$$= 3.46\times10^{-3}\text{ T} = 3.46\text{ mT}$$

Archives

JEE Main

Single Correct Answer Type

1. (2) Kinetic energy of a charged particle, $K = \dfrac{1}{2}mv^2$

Hence velocity of the charge particle, $v = \sqrt{\dfrac{2K}{m}}$

The radius of the circular path of a charged particle in the magnetic field, $r = \dfrac{mv}{Bq}$

$$\therefore \quad r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

As K and B are constants,

Hence $r \propto \dfrac{\sqrt{m}}{q}$

$$\Rightarrow \quad r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$= \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$$

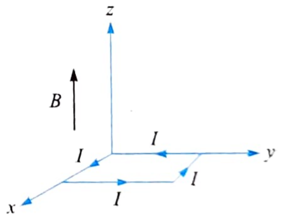
$$\Rightarrow \quad r_\alpha = r_p < r_d$$

2. (4) $P = \dfrac{\text{Work done}}{\text{Time}} = \dfrac{\int F dx}{t} = \dfrac{\int I\ell dBdx}{t}$

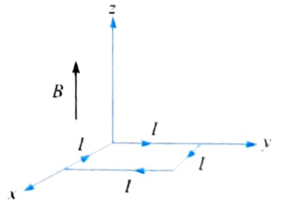
$$= \frac{\int_0^2 (10)(3)(3\times10^{-4}e^{-0.2x})dx}{5\times10^{-3}}$$

$$= \frac{9\times10^{-3}}{5\times10^{-3}}\left[\frac{e^{-0.2x}}{-0.2}\right]_0^2 = 9\left[1-e^{-0.4}\right] = 2.97\text{ W}$$

3. (3) Stable equilibrium $\vec{M} \parallel \vec{B}$



Unstable equilibrium $\vec{M} \parallel (-\vec{B})$



4. (3) Radius of circular path in magnetic field

$$R = \frac{mv}{qB} = \frac{\sqrt{2Km}}{qB}$$

where K = kinetic energy of particle

m = mass of particle

q = charge on particle

B = magnetic field intensity

R = radius of path

For electron $r_e = \dfrac{\sqrt{2Km_e}}{eB}$ ...(i)

For proton $r_p = \dfrac{\sqrt{2Km_p}}{eB}$ ...(ii)

For a particle

$$r_\alpha = \frac{\sqrt{2Km_\alpha}}{q_\alpha B} = \frac{\sqrt{2K\,4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB} \qquad \text{...(iii)}$$

as $m_e < m_p$ so $r_e < r_p = r_\alpha$

5. (4) We know magnetic dipole moment of a coils is; $m = IA$.

In our case current is kept constant and dipole moment is doubled, it means $m \propto A$

i.e., the area will be doubled as coil is circular it means the radius will become $\sqrt{2}$ times.

We know magnetic field at the centre of a circular coil is given by $B = \frac{\mu_0 I}{2R}$ of for constant current $B \propto \frac{I}{R}$

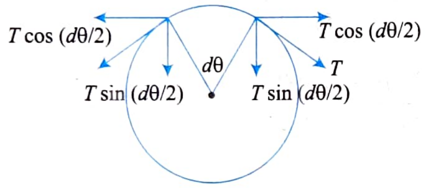
Hence magnetic field of the centre of loop will become $\frac{1}{\sqrt{2}}$ times.

Hence $\frac{B_1}{B_2} = \frac{R_2}{R_1} = \frac{\sqrt{2}R}{R}$ or $\frac{B_1}{B_2} = \sqrt{2}$

JEE Advanced

Single Correct Answer Type

1. (3) $2T\sin\frac{d\theta}{2} = BIRd\theta$



$$Td\theta = BIRd\theta \qquad \text{(for } \theta \text{ small)}$$

$$T = BIR = \frac{BIL}{2\pi}$$

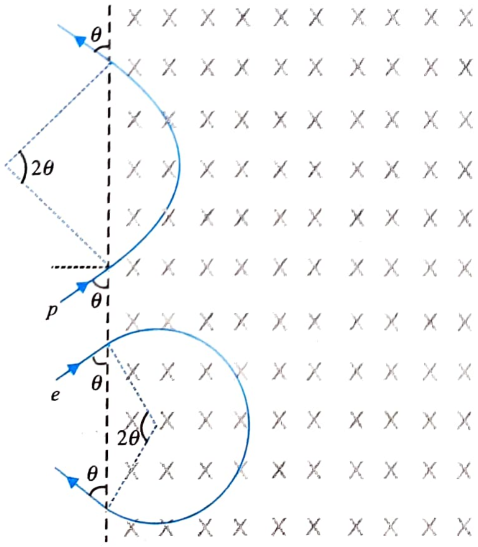
2. (2) Area $= a^2 + 4 \times \frac{\pi\left(\frac{a}{2}\right)^2}{2} = a^2 + \frac{\pi a^2}{2}$

$$A = \left(1 + \frac{\pi}{2}\right)a^2\hat{k}$$

Multiple Correct Answers Type

1. (2), (4)

$$t_p = \frac{2\theta \times R_P}{v} = \frac{2\theta \times m_P v}{eBv} = \frac{2\theta m_P}{eB}$$



$$t_e = \frac{(2\pi - 2\theta) \times R_e}{v} = \frac{(2\pi - 2\theta)m_e v}{eBv} = \frac{(2\pi - 2\theta)m_e}{eB}$$

$$t_e \neq t_p$$

2. (3), (4)

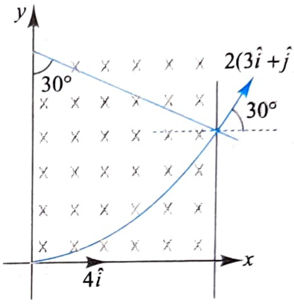
If $\theta = 0°$ then due to magnetic force path is circular but due to force qE_0 (↑) q will have accelerated motion along y-axis. So combined path of q will be a helical path with variable pitch so (1) and (2) are wrong.

If $\theta = 10°$, then due to $v\cos\theta$, path is circular and due to qE_0 and $v\sin\theta$, q has accelerated motion along y-axis so combined path is a helical path with variable pitch (3) is correct.

If $\theta = 90°$ then $F_B = 0$ and due to qE_0 motion is accelerated along y-axis. (4)

3. (1), (3)

So magnetic field is along –ve, z-direction.



Time taken in the magnetic field $= 10 \times 10^{-3} = \frac{\pi M}{6QB}$

$$B = \frac{\pi M}{6 \times 10^{-3} Q} = \frac{1000\pi M}{60Q} = \frac{50\pi M}{3Q}$$

4. (1), (2), (3)

As $\vec{B}$ is uniform

⇒ Wire can be replaced by a straight current carrying conductor.

⇒ $\vec{F} = i(\vec{l} \times \vec{B})$

$= (2(L + R)\hat{i} \times \vec{B})$

if $\bar{B}$ is along x-axis ⇒ $\bar{F} = 0$

otherwise $F \propto (L + R)$

5. (1, 2)

The particle will follow circular trajectory inside the magnetic field region. The magnetic field cannot change the magnitude of velocity and momentum.

For longest possible path, the radius of circular motion can be $\frac{3R}{2}$.

At farthest point from y-axis, the momentum is directed upwards.

$\therefore |\Delta \vec{p}| = \sqrt{2}p$

The radius and hence separation between p_1 and re-entry point is proportional to m, if Q, v, B are same. So option (3) is incorrect.

The particle will return to region only if it completes the half circle.

∴ Particle will not enter in region 3 and will re-enter region 1 charge in momentum $\sqrt{2}p$.

So option (4) is incorrect.

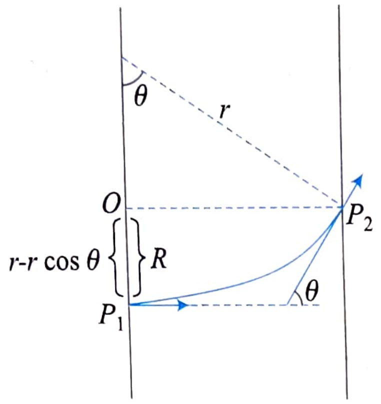
If $r \le \frac{3R}{2}$

$$\frac{mv}{QB} \le \frac{3R}{2} \Rightarrow \frac{p}{QB} \le \frac{3R}{2}$$

Or, $B \ge \frac{2p}{3QR}$

$\therefore$ Option (2) is correct.

If $B = \frac{8p}{13QR}$; $r = \frac{p}{QB} = \frac{13R}{8}$



It passes through point P_2 if $r - r\cos\theta = R$

Here $\sin\theta = \frac{3\frac{R}{2}}{r} = \frac{12}{13} \Rightarrow \cos\theta = \frac{\sqrt{(13)^2-(12)^2}}{13} = \frac{5}{13}$

$\Rightarrow r(1-\cos\theta) = R$ or $\frac{13R}{8}\left(1-\frac{5}{13}\right) = R$

$\therefore$ Particle will enter region 3 through point P_2

$\therefore$ Option (1) is correct.

Linked Comprehension Type

1. **(1)** Larger the magnetic field, smaller the critical temperature.
2. **(2)** If $0 < B < 7.5$ T, then 75 K $< T_c(B) <$ 100 K

Matrix Match Type

1. **(4)** For constant velocity, acceleration of particle should be zero. Hence net force should be zero.

$$qvB = qE \Rightarrow v = \frac{E}{B} \quad (\text{for } a = 0)$$

Electric field and magnetic field should be perpendicular.

Option (4): $\vec{v} = \frac{E_0}{B_0}\hat{y}$, $\vec{E} = -E_0\hat{x}$, $\vec{B} = -E_0\hat{z}$ (for electron)

$$\vec{F}_B = q(\vec{v}\times\vec{B}) = -e\left(\frac{E_0}{B_0}\hat{y}\times B_0\hat{z}\right) = -E_0\hat{x}$$

$$F_E = E_0\hat{x}$$

2. **(2)** $\vec{v} = 0$, $\vec{E} = -E_0\hat{y}$, $\vec{B} = B_0\hat{y}$ (Proton)

$F_B = 0$ only force along $-y$ axis is acting due to electric field alone.

3. **(4)** $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$; $\vec{E} = E_0\hat{z}$; $\vec{B} = B_0\hat{z}$ (Proton)

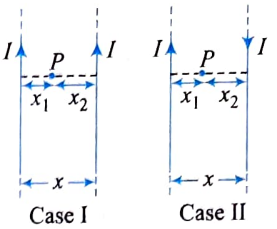
F_B = along $-y$ axis

F_E = along $+z$ axis

So condition for helical path is satisfied.

Numerical Value Type

1. **(3)** $X_1 = \frac{X_0}{3}$ and $X_2 = \frac{2X_0}{3}$



$$r = \frac{mu}{qB}$$

Hence ratio of radian

$$\frac{R_1}{R_2} = \frac{B_2}{B_1}$$

Case I

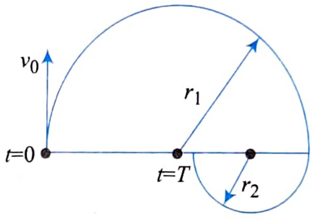
$$B_1 = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x_1} - \frac{1}{x_2}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{3}{x_0} - \frac{3}{2x_2}\right) = \frac{3\mu_0 I}{4\pi x_0}$$

Case II

$$B_2 = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x_1} + \frac{1}{x_2}\right) = \frac{9\mu_0 I}{4\pi x_0} \Rightarrow \frac{R_1}{R_2} = \frac{B_2}{B_1} = 3$$

2. **(2)** The path of the particle motion should be as follows.

We have, radius of a charged particle moving in magnetic field.



$$r_1 = \frac{mv}{qB_1}, \quad r_2 = \frac{mv}{qB_2}$$

Since $B_1 = \frac{B_2}{4}$ $\therefore$ $r_1 = 4r_2$

Time in $B_1 \Rightarrow \frac{\pi m}{qB_1} = t_1$

Time in $B_2 \Rightarrow \frac{\pi m}{qB_2} = t_2$

Average speed along x-axis $\langle v_x \rangle = \frac{\int |\vec{v}_x|\,dt}{\int dt} = \frac{d_1 + d_2}{t_1 + t_2}$

Total distance along x-axis

$$d_1 + d_2 = 2r_1 + 2r_2 = 2(r_1 + r_2) = 2(5r_2)$$

Total time $T = t_1 + t_2 = 5t_2$

$\therefore$ Average speed $= \frac{10r_2}{5t_2} = 2\frac{mv}{qB_2}\times\frac{qB_2}{\pi m} = 2$

3. **(5.55)** Number of turns $n = 50$ turns

Area $A = 2\times10^{-4}\ \text{m}^2$

Magnetic field $B = 0.02\,\text{T}$

Torsion constant of wire $K = 10^{-4}$

Full scale deflection angle, θ $Q_m = 0.2$ rad

Resistance of galvanometer $R_g = 50\Omega$

Range of ammeter $I_A = 0-1.0\text{ A}$

We know torque on the coil, $\tau = MB = C\theta$

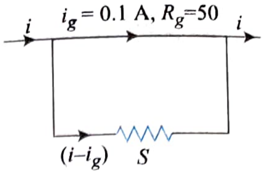
Magnetic moment of the coil $M = ni_g A$

It means $BINA = C\theta$

$$0.02\times i_g\times 50\times 2\times 10^{-4} = 10^{-4}\times 0.2\times 10$$

$$i_g = 0.1\text{ A}$$

To convert galvanometer into an ammeter, we need to apply a shunt resistance in parallel with the galvanometer.



From figure we can write $i_g\times R_g = (i-i_g)S$

or $$0.1\times 50 = (1-0.1)S$$

which gives $S = \frac{50}{9} = 5.55\ \Omega$

Hence $S = 5.55\Omega$ or $5.56\ \Omega$

Chapter 2

Concept Application Exercises

ercise 2.1

1. For a charge with velocity

$$\vec{v} = (8.00 \times 10^6 \text{ m/s})\,\hat{j},$$

the magnetic field produced at a position away from the particle is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}.$$

(a) $\vec{r} = (+0.500 \text{ m})\hat{i} \Rightarrow \hat{v} \times \hat{r} = -\hat{k},\ r_0^2 = \frac{1}{4}$

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{qv}{r_0^2}\hat{k}$$

$$= -\frac{\mu_0}{4\pi}\frac{(6.0 \times 10^{-6} \text{ C})(8.0 \times 10^6 \text{ m/s})}{(0.50 \text{ m})^2}\hat{k}$$

$$= -(1.92 \times 10^{-5} \text{ T})\hat{k}$$

(b) $\vec{r} = (-0.500 \text{ m})\hat{j} \Rightarrow \hat{v} \times \hat{r} = 0 \Rightarrow \vec{B} = 0$

2. The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero.

3. (a) Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$, respectively.

At A, $\vec{B}_1$ is ⊙ and $\vec{B}_2$ is ⊗

$$B_1 = \frac{\mu_0 2I}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$$\therefore \quad B_{\text{res}} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot$$

At C, $\vec{B}_1$ is ⊗ and $\vec{B}_2$ is also ⊗

$$\therefore \quad B_{\text{res}} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi \frac{a}{2}} + \frac{\mu_0 I}{2\pi \frac{a}{2}} = \frac{6\mu_0 I}{2\pi a}$$

$$= \frac{3\mu_0 I}{\pi a} \otimes$$

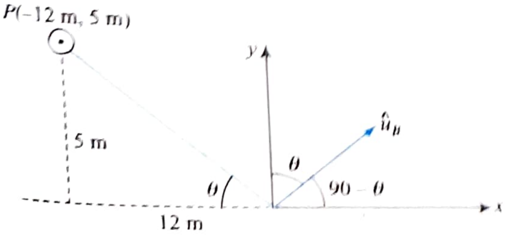
At D, $\vec{B}_1$ is ⊗ and $\vec{B}_2$ is ⊙ and both are equal in magnitude.

$$\therefore \quad B_{\text{res}} = 0$$

(b) It is clear from the above solution that $B = 0$ at point 'D'.

4. Unit vector of $\hat{u}_B$

$$\hat{u}_B = \hat{i}\cos(90 - \theta) + \hat{j}\cos\theta = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

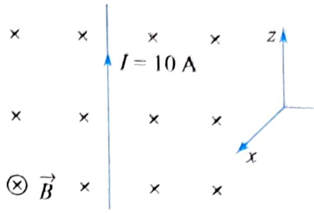


$$\sin\theta = \frac{5}{\sqrt{5^2 + 12^2}} = \frac{5}{13},\ \cos\theta = \frac{12}{\sqrt{5^2 + 12^2}} = \frac{12}{13}$$

$$\hat{u}_B = \frac{5}{13}\hat{i} + \frac{12}{13}\hat{j} = \frac{1}{13}(5\hat{i} + 12\hat{j})$$

5. Given, $\vec{B} = 2 \times 10^{-3}$ T

Now, direction of $\vec{B}$ due to the wire is ⊙ in the left half and ⊗ in the right half.



$$\therefore \quad B_{\text{net}} = 0 \text{ in the left half only}$$

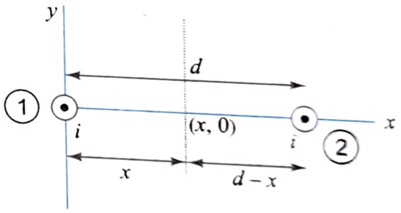
$|B_{\text{wire}}| = |\vec{B}|$ in the zy plane

$$\frac{\mu_0 10}{2\pi r} = 2 \times 10^{-3} \text{ T}$$

$$r = \frac{4 \times 10^{-7} \times 5}{\pi \times 2 \times 10^{-3}} = 10^{-3} \text{ m}$$

Thus, magnetic field is zero at $r = 1$ mm to the left of the wire in the indicated yz plane.

6. Let us consider a point P at a distance (x, y) from the wire 1.



$$0 \le x \le d$$

Hence $\vec{B}_1 = \frac{\mu_0 i}{2\pi x}\hat{j}$

The point is at a distance $d - x$ from wire 2.

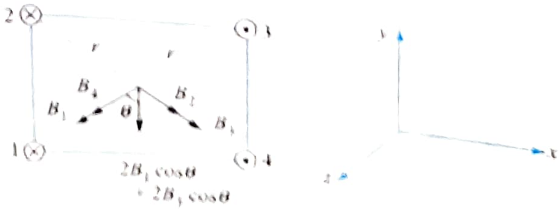
$$\therefore \quad \vec{B}_2 = -\frac{\mu_0 i}{2\pi(d - x)}\hat{j}$$

$$\therefore \quad \vec{B}_{\text{net}} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 i}{2\pi x}\hat{j} - \frac{\mu_0 i}{2\pi(d - x)}\hat{j}$$

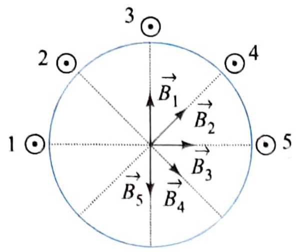
$$\vec{B}_{\text{net}} = \frac{\mu_0 i(d - 2x)}{2\pi x(d - x)}\hat{j}$$

7. $B_{\text{net}} = 4B\cos 45° = \frac{4}{\sqrt{2}}\frac{\mu_0 I}{2\pi r} = \frac{4}{\sqrt{2}}\frac{\mu_0}{2\pi}\frac{5}{\sqrt{2} \times 0.1}$

$$\Rightarrow \quad B_{\text{net}} = 2 \times 10^{-5}\,\text{T}(-\hat{j})$$



8. The magnetic field due to $\vec{B}_1 < \vec{B}_5$ will be



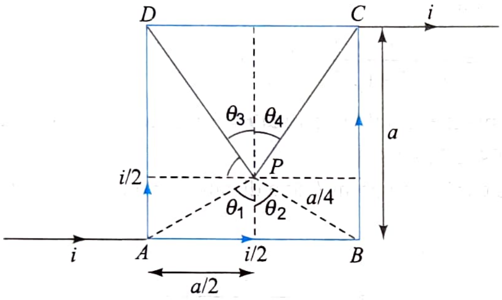
$$\vec{B}_{\text{net}} = \vec{B}_3 + 2\vec{B}_2 \cos 45^\circ = \frac{\mu_0 I}{2\pi r}\hat{i} + 2\frac{\mu_0 I}{2\pi r}\frac{1}{\sqrt{2}}\hat{i}$$

$$= \frac{\mu_0 I}{2\pi r}(1+\sqrt{2})\hat{i}$$

9. Magnetic field at point P due to part AD = −magnetic field due to part BC

$$\Rightarrow \quad \vec{B}_{AD} = -\vec{B}_{BC} \quad \Rightarrow \quad \vec{B}_{AD} + \vec{B}_{BC} = 0$$

$$\vec{B}_{AB} = \frac{\mu_0 (i/2)}{4\pi (a/4)}(\sin\theta_1 + \sin\theta_2) \odot$$



$$= \frac{\mu_0 i}{2\pi a}\left(\frac{a/2}{a\sqrt{5}/4} + \frac{a/2}{a\sqrt{5}/4}\right) \odot$$

$$= \frac{\mu_0 i}{2\pi a} \times 2 \times \frac{a}{2} \times \frac{4}{a\sqrt{5}} \odot = \frac{2\mu_0 i}{\sqrt{5}\pi a} \odot$$

$$\therefore \quad \vec{B}_{CD} = \frac{\mu_0 (i/2)}{4\pi (3a/4)}(\sin\theta_3 + \sin\theta_4) \otimes$$

$$= \frac{\mu_0 (i/2)}{3\pi a} \times 2 \times \frac{a}{2} \times \frac{4}{a\sqrt{13}} \otimes = \frac{2\mu_0 i}{3\sqrt{13}\pi a} \otimes$$

$$\therefore \quad \vec{B}_{\text{net}} = \frac{2\mu_0 i}{\pi a}\left(\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}}\right) \odot$$

10. Assume $PO = x$, so $PQ = x \sin(\alpha/2)$

Magnetic field B at P due to either segment of wire is

$$B = \frac{\mu_0 I}{4\pi x \sin(\alpha/2)}\left[1 + \cos\frac{\alpha}{2}\right]$$

Net magnetic field at P is

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi x \sin(\alpha/2)}[1 + \cos(\alpha/2)]$$

11. (a) At the point exactly midway between the wires, the two magnetic fields are in opposite directions and cancel.

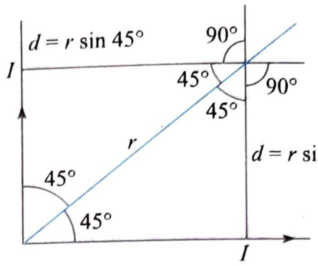
(b) At a distance a above the top wire, the magnetic fields are in the same direction and add up

$$\vec{B} = \frac{\mu_0 I}{2\pi r_1}\hat{k} + \frac{\mu_0 I}{2\pi r_2}\hat{k} = \frac{\mu_0 I}{2\pi a}\hat{k} + \frac{\mu_0 I}{2\pi(3a)}\hat{k} = \frac{2\mu_0 I}{3\pi a}\hat{k}$$

(c) At a distance a below the lower wire, magnitude of magnetic field is same as in part (b) but is in the opposite direction

$$\vec{B} = -\frac{2\mu_0 I}{3\pi a}\hat{k}$$

12. $B = \frac{\mu_0}{4\pi}\frac{I}{d}(\sin\theta_1 + \sin\theta_2)$

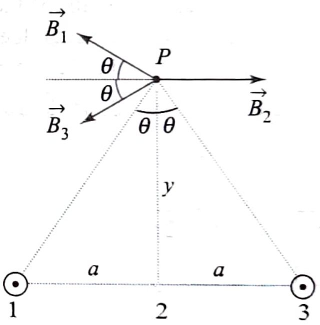


Here, $d = r \sin 45^\circ = \frac{r}{\sqrt{2}}$

$$B = 2 \times \frac{\mu_0}{4\pi}\frac{I}{\left(\frac{r}{\sqrt{2}}\right)}(\sin 45^\circ + \sin 90^\circ)$$

$$= 2\sqrt{2}\frac{\mu_0}{4\pi}\frac{I}{r}\left(\frac{1}{\sqrt{2}} + 1\right) = \frac{\mu_0}{2\pi}\frac{I}{r}(\sqrt{2} + 1)$$

13. For magnetic field to be zero at P,

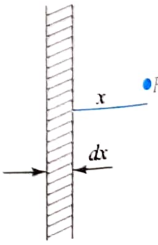


$$\vec{B}_P = 0; \quad 2B_1 \cos\theta = B_2$$

$$2\frac{\mu_0 I}{2\pi\sqrt{a^2 + y^2}}\frac{y}{\sqrt{a^2 + y^2}} = \frac{\mu_0 I}{2\pi y}$$

$$\Rightarrow \quad y = \pm a$$

14. To find 'B' at 'P' the sheet can be considered as collection of large nunber of infinitely long wires. Take a long wire at a distance 'x' from 'P' and of width 'dx'. Due to this the magnetic field at 'P' is 'dB'



$$dB = \frac{\mu_0\left(\frac{I}{w}dx\right)}{2\pi x} \otimes$$

due to each such wire magnetic field will be directed inwards

$$\therefore \quad B_{\text{res}} = \int dB = \frac{\mu_0 I}{2\pi w}\int_{x=a}^{a+w}\frac{dx}{x} = \frac{\mu_0 I}{2\pi w}\ell n\left(\frac{a+w}{a}\right)$$

Exercise 2.2

1. There is no contribution from the straight wires, and now we have two oppositely oriented contributions from the two semicircles

$$B=(B_1-B_2)=\frac{1}{2}\left(\frac{\mu_0}{2R}\right)|I_1-I_2|,$$

into the page. Note that if the two currents are equal, the magnetic field goes to zero at the centre of the loop.

2. $B=(B_a-B_b)=\frac{1}{2}\left(\frac{\mu_0 I}{2}\right)\left(\frac{1}{a}-\frac{1}{b}\right)$

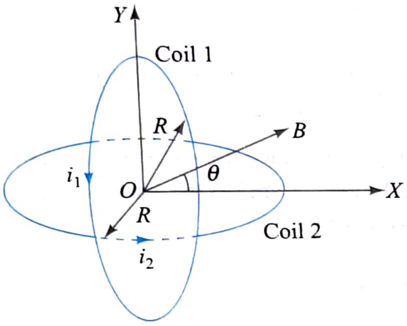
$$B=\frac{\mu_0 I}{4a}\left(1-\frac{a}{b}\right)\text{, out of the page.}$$

3. Current in the ring $I=\frac{\omega q}{2\pi}$

Magnetic field $B=\frac{\mu_0 I}{2r}=\frac{\mu_0\omega q}{2\pi\times 2r}$

$$\Rightarrow \quad B=\frac{\mu_0\omega q}{4\pi r}$$

4. Let current flowing through coil 1 be i_1 and through coil 2 be i_2.



Radius of each coil = R. Hence, magnetic field at O due to coil 1 is

$$\vec{B}_1=\frac{\mu_0 i_1}{2R}\hat{i}$$

and magnetic field at O due to coil 2 is

$$\vec{B}_2=\frac{\mu_0 i_2}{2R}\hat{j}$$

Therefore, net magnetic field $\vec{B}=\vec{B}_1+\vec{B}_2$

$$\Rightarrow \quad \vec{B}=\frac{\mu_0 i_1}{2R}\hat{i}+\frac{\mu_0 i_2}{2R}\hat{j}$$

$$\Rightarrow \quad |\vec{B}|=\frac{\mu_0}{2R}\sqrt{(i_1^2+i_2^2)}$$

5. $|B_P|=\frac{\mu_0 i}{4\pi d}$, $|B_R|=\frac{\mu_0 i}{4\pi d}$

At P and R, the field due to vertical part is zero and the points are equidistant from the horizontal wire. Similarly, the points S and Q are equidistant from the vertical wire and field due to horizontal wire is zero.

Hence $|B_Q|=|B_S|=\frac{\mu_0 i}{4\pi d}$

$$\Rightarrow \quad |B_P|=|B_Q|=|B_R|=|B_S|=\frac{\mu_0 i}{4\pi d}$$

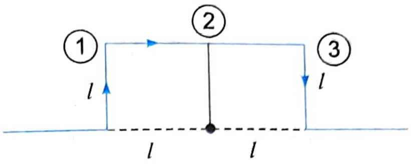
6. At the centre of the circular loop the current I_2 generates a magnetic field that is into the page. So the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude

$$\frac{\mu_0 I_1}{2\pi D}=\frac{\mu_0 I_2}{2R}$$

Thus $I_1=\frac{\pi D}{R}I_2$

7. **Case I:** $\vec{B}_{\text{net}}=\vec{B}_1+\vec{B}_2+\vec{B}_3$

$$=\left[\frac{\mu_0 I}{4\pi l}[0+\sin 45^\circ]+\frac{\mu_0 I}{4\pi l}[\sin 45^\circ+\sin 45^\circ]+\frac{\mu_0 I}{4\pi l}[0+\sin 45^\circ]\right]\otimes$$



$$\therefore \quad |\vec{B}_{\text{net}}|=\frac{\mu_0 I}{\sqrt{2}\,\pi l}=B_1$$

Case II: $2\pi r=4l \quad\Rightarrow\quad r=\frac{2l}{\pi}$

Magnetic field at the centre of semicircle,

$$B_{\text{semicircle}}=\frac{\mu_0 I}{4r}=\frac{\mu_0 I\pi}{8l}=B_2$$

$$\frac{B_1}{B_2}=\frac{\mu_0 I}{\sqrt{2}\pi l}\times\frac{8l}{\mu_0 I\pi}=\frac{4\sqrt{2}}{\pi^2}$$

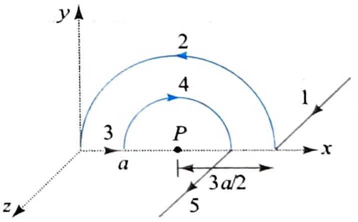
8. From the figure in question, $\vec{B}_0=\vec{B}_{AB}+\vec{B}_{BCD}+\vec{B}_{DE}$

$$\Rightarrow \quad \vec{B}_0=\left(\frac{\mu_0 I}{4\pi R}\right)(-\hat{j})+\left[\frac{\mu_0 I}{4R}+\frac{\mu_0 I}{4\pi R}\right](-\hat{i})$$

9. Consider figure

$$\vec{B}_p=(\vec{B}_1)_p+(\vec{B}_2)_p+(\vec{B}_3)_p+(\vec{B}_4)_P+(\vec{B}_5)_P$$

where $(\vec{B}_1)_p=\frac{\mu_0 i}{4\pi\left(\frac{3a}{2}\right)}(-\hat{j})$ (semi-infinite wire)



$$(\vec{B}_2)_p=\frac{\mu_0 i}{4\left(\frac{3a}{2}\right)}(+\hat{k})$$

$$(\vec{B}_3)_p=0$$

$$(\vec{B}_4)_p=\frac{\mu_0 i}{4\left(\frac{a}{2}\right)}(-\hat{k}) \quad\Rightarrow\quad (\vec{B}_5)_p=\frac{\mu_0 i}{4\pi\left(\frac{a}{2}\right)}(-\hat{j})$$

$$\vec{B}_p=\frac{\mu_0 i}{2a}\left[-\left(\frac{1}{3\pi}+\frac{1}{\pi}\right)\hat{j}-\left(1-\frac{1}{3}\right)\hat{k}\right]$$

$$\vec{B}_p=\frac{\mu_0 i}{3a}\left[\frac{2}{\pi}\hat{j}-\hat{k}\right]$$

10. Magnetic induction at the point O due to circular portion of the wire

$$\vec{B}_1=\frac{\mu_0 I\alpha}{4\pi R}=\frac{\mu_0 i}{4\pi a}\times\frac{3}{2}\pi\odot\text{ (out of the page) }\left(\alpha=\frac{3\pi}{2}\right)$$

Magnetic induction at O due to wire cd will be zero since O lies on the line cd itself when extended backward.

Magnetic induction at O due to infinitely long straight wire ae is

$$\therefore \quad B_2 = \frac{\mu_0 i}{4\pi r}[\sin\phi_1 + \sin\phi_2]$$

where $r = a,\ \phi_1 = 0,\ \phi_2 = \frac{\pi}{2}$

$$B_2 = \frac{\mu_0 i}{4\pi a}\left[\sin 0° + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\mu_0 i}{4\pi a}$$

Because both the fields are in same direction i.e. perpendicular to plane of paper and directed upwards, hence the resultant magnetic induction at O is

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 i}{4\pi a}\left(\frac{3\pi}{2} + 1\right)\odot$$

11. Field at O due to part 1 $\vec{B}_1 = \frac{\mu_0 i}{4\pi r}(-\hat{i})$

Field at O due to part 2 $\vec{B}_2 = \frac{1}{4}\left(\frac{\mu_0 i}{2r}\right)(-\hat{k})$

$\because$ Wire 3 passes through origin when it is extended backwards $\vec{B}_3 = 0$

$$\vec{B}_0 = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = -\frac{\mu_0 i}{4r}\left[\frac{\hat{i}}{\pi} + \frac{\hat{k}}{2}\right]$$

12. $\vec{B}_0 = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4;$

$$\vec{B}_1 = -\frac{\mu_0 i}{4\pi a}(-\hat{i})$$

$$\vec{B}_2 = \frac{\mu_0 i}{2a} \times \frac{1}{4}(-\hat{k}) = \frac{\mu_0 i}{8a}(-\hat{k})$$

$$\vec{B}_3 = \frac{\mu_0 i}{4\pi a}(\sin 0° + \sin 45°)(-\hat{j}) = \frac{\mu_0 i}{4\sqrt{2}\pi a}(-\hat{j})$$

$$\vec{B}_4 = \frac{\mu_0 i}{4\pi a}(\sin 45° + \sin 90°)(-\hat{j}) = \frac{\mu_0 i}{4\pi a}\left(\frac{1}{\sqrt{2}} + 1\right)(-\hat{j})$$

Then, $\vec{B}_0 = -\frac{\mu_0 i}{4\pi a}\left[-\hat{i} - (\sqrt{2}+1)\hat{j} - \left(\frac{\pi}{2}\right)\hat{k}\right]$

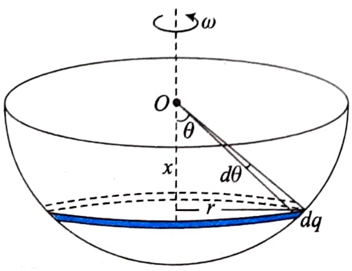
Exercise 2.3

1. $\vec{B}_C = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 i_1 a^2}{2(a^2+b^2)^{3/2}}(-\hat{i}) + \frac{\mu_0 i_2}{2b}\hat{k}$

$$= \frac{\mu_0}{2}\left[-\frac{i_1 a^2}{(a^2+b^2)^{3/2}}\hat{i} + \frac{i_2}{b}\hat{k}\right]$$

2. The magnetic field due to the ring is

$$dB = \frac{\mu_0 (di) r^2}{2R^3}$$



The total magnetic field is

$$B = \int dB = \frac{\mu_0}{2R^3}\int r^2 di,$$

where $di = (2\pi r \cdot R d\theta \sigma)\frac{\omega}{2\pi}$

or $\quad B = \frac{\mu_0}{2R^3}\int r^2 (2\pi r\, R\, d\theta\, \sigma)\frac{\omega}{2\pi}$

$$= \frac{\pi\mu_0\sigma\omega}{2R^2}\int r^3\, d\theta = \frac{\pi\mu_0\sigma\omega}{2R^2}\int_0^{\pi/2} R^3 \sin^3\theta\, d\theta$$

Evaluating the integration, we have $B = \frac{2\mu_0\sigma\omega}{3}$

3. $F_{\text{res}} = \frac{\mu_0 I_1 I_2}{2\pi a}a(-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)}a(\hat{i}) = \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$

4. Magnetic field due to ring will be parallel to wire, hence force on the wire will be zero.

5. (a) $\quad F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$

$$= \frac{\mu_0 (5.00\text{ A})(2.00\text{ A})(1.20\text{ m})}{2\pi(0.400\text{ m})} = 6.00 \times 10^{-6}\text{ N}$$

The force is repulsive since the currents are in opposite directions.

(b) Doubling the current makes the force increase by a factor of 4 to

$F = 2.40 \times 10^{-5}$ N

6. $\frac{\mu_0}{4\pi} \times \frac{2 I_A I_B l}{r} = mg$

where $r = 2.5$ cm

$m = (0.100\text{ g/cm})\, l$

Solve to get $I_B = \frac{250}{3}$ A

7. Net force on circular parts will be zero because magnetic field produced by I_1 on these parts will be tangent at any point.

On straight parts, forces will be in same direction.

Net force $= 2\left[\frac{\mu_0}{4\pi}\frac{2I_1I_2}{R}L\right] = \frac{\mu_0 I_1 I_2 L}{\pi R}$

8. Consider a thin strip at a distance x and of the thickness dx. It is equivalent to a long straight conductor carrying $(I_2 dx/b)$ current.

$$dF \text{ (force of attraction)} = \frac{\mu_0 I_1}{2\pi x} \times \frac{I_2 dx}{b} = \frac{\mu_0 I_1 I_2}{2\pi b} \times \frac{dx}{x}$$

$$\therefore \quad F = \frac{\mu_0 I_1 I_2}{2\pi b}\int_a^{a+b}\frac{dx}{x} = \frac{\mu_0 I_1 I_2}{2\pi b}\ln\frac{a+b}{a}$$

This force is per unit length.

9. We need the magnetic and gravitational forces to cancel.

$$\lambda L g = \frac{\mu_0 I^2 L}{2\pi h} \quad \Rightarrow \quad h = \frac{\mu_0 I^2}{2\pi\lambda g}$$

10. $F_{\text{mag}} = ma$

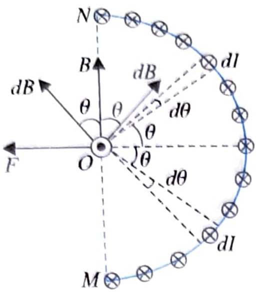
$$\frac{\mu_0 I^2}{2\pi d} = \lambda l a \text{ (where } l = \text{length of each rod)}$$

$$\frac{\mu_0}{2\pi d}\left(\frac{Q}{RC}e^{\frac{-t}{RC}}\right)^2 = \lambda l \frac{dv}{dt}$$

$$\frac{\mu_0}{2\pi d} \times \frac{Q^2}{(RC)^2} \times \int_0^\infty e^{-\frac{2t}{RC}}dt = \lambda l \int_0^v dv$$

$$\frac{\mu_0 Q^2}{2\pi dRC\cdot 2}=\lambda lv \Rightarrow v=\frac{\mu_0 Q^2}{4\pi dRC\lambda l}$$

Let us first find magnetic field at O due to 'I'.



$$dB=\frac{\mu_0 dl}{2\pi R}$$

where $dl=\frac{IRd\theta}{\pi R}=\frac{I}{\pi}d\theta$

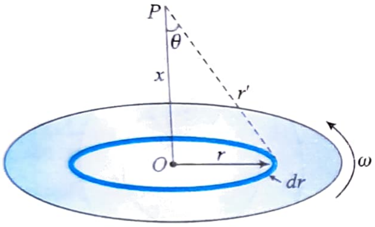
$$\Rightarrow \quad dB=\frac{\mu_0}{2\pi R}\frac{I}{\pi}d\theta$$

Net field will be along ON:

$$B=\int_{-\pi/2}^{\pi/2} dB\cos\theta=\int_{-\pi/2}^{\pi/2}\frac{\mu_0 I}{2\pi^2 R}\cos\theta\, d\theta=\frac{\mu_0 I}{\pi^2 R}$$

$$F=BIl=\frac{\mu_0 I^2}{\pi^2 R}l \Rightarrow \frac{F}{l}=\frac{\mu_0 I^2}{\pi^2 R}$$

Let us consider an elemental ring of radius r and thickness dr. The magnetic field due to the elemental ring is



$$dB=\frac{\mu_0(di)r^2}{2(r^2+x^2)^{3/2}}=\frac{\mu_0(di)r^2}{2r'^3};$$

The equivalent current due to rotation of the disc,

$$di=\sigma 2\pi r\,dr\left(\frac{\omega}{2\pi}\right)=\sigma\,\omega r\,dr$$

Then, $dB=\frac{\mu_0}{2(r^2+x^2)^{3/2}}\cdot(\sigma\omega r dr)r^2$

or $\quad B=\int dB=\frac{\sigma\omega\mu_0}{2}\int_0^R\frac{r^3\,dr}{(r^2+x^2)^{3/2}}$

Evaluating the integration, we have $B=\frac{\mu_0\sigma\omega x}{2}\left[\frac{2x^2+R^2}{\sqrt{R^2+x^2}}-2x\right]$

Exercise 2.4

1. For L_1, $\oint\vec{B}\cdot d\vec{l}=\mu_0(I_1-I_2)$. Here I_1 is taken positive because magnetic lines of force produced by I_1 is anticlockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $\vec{dl}$ is anticlockwise as seen from top.

   For $\quad L_2$: $\oint\vec{B}\cdot\vec{dl}=\mu_0(I_1-I_2+I_4)$

   For $\quad L_3$: $\oint\vec{B}\cdot\vec{dl}=0$

2. Direction of magnetic field due to both layers is opposite, as direction of current is opposite so

$$B_{net}=B_1-B_2=\mu_0 n_1 I_1-\mu_0 n_2 I_2$$

$$=\mu_0\frac{N_1}{l}I-\mu_0\frac{N_2}{l}I\,(\because I_1=I_2=I)$$

$$B_{net}=\frac{\mu_0 I}{l}(N_1-N_2)=\frac{4\pi\times10^{-7}\times3}{0\cdot1}(50-40)=12\pi\times10^{-5}\text{ T}$$

3. Magnetic field produced by a long solenoid is $B=\mu_0 nI$, where $n=N/l$

   $\because$ Same number of turns wound over half length

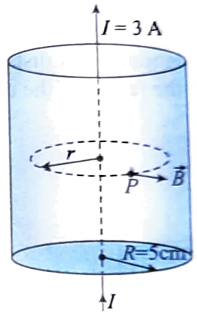
   $\therefore$ Magnetic field produced by new solenoid is

$$B'=\mu_0\left(\frac{N}{l/2}\right)I=2\left(\frac{\mu_0 NI}{l}\right)=2B$$

4. Imagine a circular path of radius r whose centre lies on the axis of solid conductor such that the point P lies on it.the current threading this closed path

$$I'=\frac{I}{\pi R^2}\times\pi r^2=\frac{Ir^2}{R^2}$$

   Magnetic field B acts tangential to the Amperian circular path at P and is same in magnitude at every point on circular path.

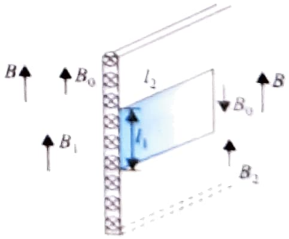


   Using Ampere circuital law $\oint\vec{B}.d\vec{l}=\mu_0\mu_r I'\Rightarrow B(2\pi r)$

$$=\mu_0\mu_r\left(\frac{Ir^2}{R^2}\right)\Rightarrow B=\frac{\mu_0\mu_r I r}{2\pi R^2}$$

$$B=\frac{4\pi\times10^{-7}\times1000\times3\times0.04}{2\pi\times(0.05)^2}=9.6\times10^{-3}\text{ T.}$$

5. Let external magnetic field is B and B_0 is the magnetic field due to plane.



   Then $B_1=B+B_0$, and $B_2=B-B_0$

   where $B_0=\frac{\mu_0 J}{2}$

   Solving we get $B=\frac{B_1+B_2}{2}$, $B_0=\frac{B_1-B_2}{2}$

   Force on plane $F=B(Jl_1)l_2$

   Force per unit area

$$\frac{F}{l_1l_2}=BJ=\frac{B_1+B_2}{2}\left(\frac{2B_0}{\mu_0}\right)=\frac{B_1^2-B_2^2}{2\mu_0}$$

6. From Ampere's law $B_\phi 2\pi r = \mu_0 \int_0^r j(r')2\pi r'\,dr'$

$j(r')$ = current density at a distance r'.

Given $B_\phi = br^\alpha$.

Subsituting in above, $br^{\alpha+1} = \mu_0 \int j(r')r'\,dr'$.

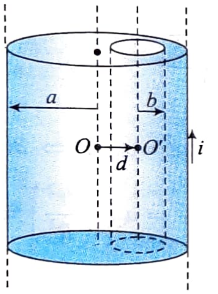
Differentiating, we obtain

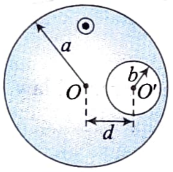
$$(\alpha+1)\,br^\alpha = \mu_0 j(r)r$$

$$\Rightarrow \quad j(r) = \frac{b(\alpha+1)}{\mu_0} r^{\alpha-1}$$

7. The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current i, which is uniformly distributed over a cross section, produces a magnetic field with magnitude at a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder.

$$B = \frac{\mu_0 ir}{2\pi R^2}$$





Top view

For the cylinder of this problem the current density is

$$j = \frac{i}{A} = \frac{i}{\pi(a^2 - b^2)},$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole.

The current in the cylinder without the hole is

$$i_1 = jA = j\pi a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance d from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 d}{2\pi a^2} = \frac{\mu_0 ida^2}{2\pi a^2(a^2 - b^2)} = \frac{\mu_0 id}{2\pi(a^2 - b^2)}.$$

The current in the cylinder that fills the hole is,

$$i_2 = j\pi b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r from the its axis, has magnitude

$$B_2 = \frac{\mu_0 i_2 r}{2\pi b^2} = \frac{\mu_0 irb^2}{2\pi b^2(a^2 - b^2)} = \frac{\mu_0 ir}{2\pi(a^2 - b^2)}.$$

At the center of the hole, this field is zero as $r = 0$ in the above expression for B_2.

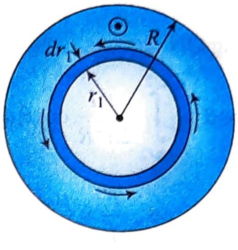
Hence the field at the center of the hole,

$$\vec{B} = \vec{B}_1 + \vec{B}_2 \Rightarrow B = \frac{\mu_0 id}{2\pi(a^2 - b^2)}$$

The field points upward in the diagram if the current is out of the page.

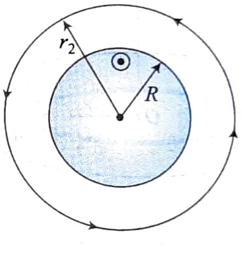
8. $i = \int_0^r (2\pi r\,dr)j = \int_0^r (2\pi r\,dr)(br) = \frac{2\pi br^3}{3}$

(a) For $r_1 > R$



$$B = \frac{\mu_0}{2\pi}\frac{i_{in}}{r_1} = \frac{\mu_0}{2\pi}\left(\frac{2\pi br_1^3/3}{r_1}\right) = \frac{\mu_0 br_1^2}{3}$$

(b) For $r_2 > R$

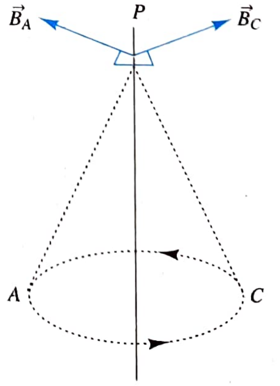


$$B = \frac{\mu_0}{2\pi}\frac{i_{in}}{r_2} = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{2\pi bR^3/\mathrm{b}}{r_2}\right) = \frac{\mu_0 bR^3}{3r_2}$$

Exercises

Single Correct Answer Type

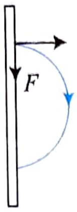
1. (1) The point charge moves in circle as shown in figure. The magnetic field vectors at a point P on the axis of circle are $\vec{B}_A$ and $\vec{B}_C$ at the instant the point charge is at A and C, respectively, as shown in figure.



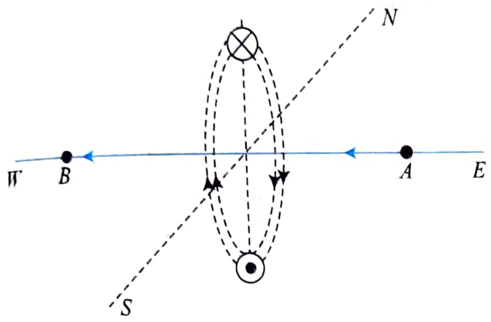
Hence, as the particle rotates in circle, only magnitude of magnetic field remains constant at P but its direction changes.

2. (2) Current due to a stream of electrons will be in a direction opposite to the direction of current in the straight conductor which is held parallel above the stream of electrons. The two currents are in opposite directions and hence they will repel. So, the stream of electrons is pulled downward.

3. (1) The initial force on the electron is downward. As the electron changes direction, the force on it remains in the xy plane, with a component directed toward the conductor.



4. (4) Using right hand rule, the direction of magnetic field is clearly toward west at A and B



5. (2) Using Ampere's circuital law, $B = 0$ for $r \le R_1$

$$B = \frac{\mu_0 i}{2\pi(R_2^2 - R_1^2)}\left(\frac{r^2 - R_1^2}{r}\right) \text{ for } R_1 \le r \le R_2$$

and $B = \dfrac{\mu_0}{2\pi}\dfrac{i}{r}$ for $r \ge R_2$

The corresponding B–r graph will be as shown in option (2).

6. (2) $B_{\text{axis}} = \dfrac{\mu_0}{4\pi} \times \dfrac{2\pi IR^2}{\left(R^2 + x^2\right)^{3/2}}$

At centre, $B_{\text{centre}} = \dfrac{\mu_0}{4\pi} \times \dfrac{2\pi I}{R}$

In the given problem,

$$\frac{\mu_0}{4\pi} \times \frac{2\pi IR^2}{\left(R^2 + x^2\right)^{3/2}} = \frac{1}{8}\left[\frac{\mu_0}{4\pi} \times \frac{2\pi I}{R}\right]$$

or $(R^2 + x^2)^{3/2} = 8R^3$

Solving, we get $x = R\sqrt{3}$

7. (3) Magnetic field at the centre due to either arm

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{(L/2)}[\sin 45° + \sin 45°]$$

$$= \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}\,I}{L}$$

Field at centre due to the four arms of the square

$$B = 4B_1 = 4 \times \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}\,I}{L}$$

i.e., $B \propto \dfrac{1}{L}$

8. (1) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I = 4\pi \times 10^{-7} \times \dfrac{1}{4\pi} = 10^{-7}\,\text{Wb m}^{-1}$

9. (3) Loop B: $\mu_0(2i - i) = \oint B \cdot dl$

loop C: $\mu_0(i - 2i) = \oint B \cdot dl$

Loop A: $\mu_0(3i - 3i) = \oint B \cdot dl$

Loop D: $\mu_0(0 - i) = \oint B \cdot dl$

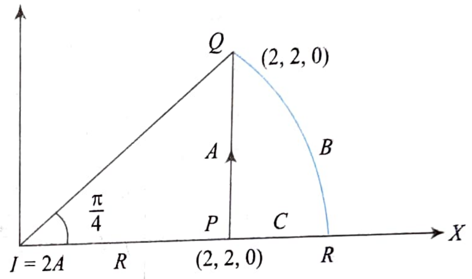
$B > A > C = D$

10. (4) No current will flow through section BC as the potentials of points B and C are same. Therefore magnetic field at centre O will be equal and opposite due to section AB and AC. Hence, they cancel out.

11. (1) $\displaystyle\int_{PAQ} \vec{B} \cdot d\vec{l} = \int_{RBQ} \vec{B} \cdot d\vec{l}$ $\qquad \left(\because \displaystyle\int_{PCR} \vec{B} \cdot d\vec{l} = 0\right)$

$$= \left(\frac{\mu_0 I}{2\pi R}\right)\left(\frac{\pi}{4}R\right)$$

$$= \frac{(4\pi \times 10^{-7})(2)}{16} = \frac{\pi \times 10^{-7}}{2} \text{ S.I. units}$$



12. (3) Net current is (20 – 6 + 12 – 7 + 18) A, i.e., 37 A.

$$r = \frac{10}{100}\,\text{m} = \frac{1}{10}\,\text{m}$$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 37 \times 10}{2\pi \times 1}\,\text{T} = 74 \times 10^{-6}\,\text{T} = 74\,\mu\text{T}$$

13. (1) To find the magnetic field outside a thick conductor, the current may be assumed to flow along the axis. As points 1, 2 and 3 are equidistant from the axis.

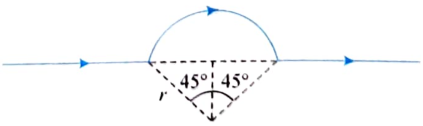
$B_1 = B_2 = B_3$

14. (2) $B = \dfrac{3}{4}\left[\dfrac{\mu_0 I}{2a}\right] + \dfrac{1}{4}\left[\dfrac{\mu_0 I}{2b}\right]$

$$B = \frac{3\mu_0 I}{8a} + \frac{\mu_0 I}{8b}$$

15. (4) To get magnetic field in resultant direction, current in X should be in and that in Z should be out; I in W should be in and that in Y should be out.

16. (2) Magnetic field due to straight wires



$$B_{\text{circular}} = \frac{1}{4}\frac{\mu_0 I}{2r} = \frac{\mu_0 I}{8r}$$

$$B_{\text{straight}} = 2\frac{\mu_0 I}{4\pi(r\cos 45°)}[\sin 90° - \sin 45°]$$

$$= \frac{\sqrt{2}\mu_0 I}{2\pi r}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$B = B_{\text{circular}} + B_{\text{straight}}$$

$$B = \frac{\mu_0 I}{2\pi r}\left[\frac{\pi}{4} + (\sqrt{2} - 1)\right]$$

17. **(3)** $B = \dfrac{\mu_0 ni}{2r}$

$\Rightarrow \quad l = 2\pi rn,\ r = \dfrac{l}{2\pi n}$

So, $B = \dfrac{\mu_0 \pi i n^2}{l} \quad \Rightarrow \quad B$ is minimum if $n = 1$

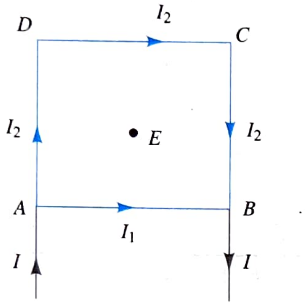
18. **(1)** $B_L = \dfrac{\mu_0 I}{2\pi R}$

If the radius is R/n; the number of turns will be n.

$$B_C = \frac{n\mu_0 I}{2\pi(R/n)} = n^2\,\frac{\mu_0 I}{2\pi R}$$

Hence, $\dfrac{B_L}{B_C} = \dfrac{1}{n^2}$

19. **(3)** Let current in AB is I_1 and in DC is I_2. Then



$\dfrac{I_1}{I_2} = \dfrac{3}{1}$. It is because resistance of AB will be one-third of that of $ADCB$.

Now, $\dfrac{B_1}{B_2} = \dfrac{I_1}{I_2} = 3$

20. **(1)** $F = \dfrac{4\pi \times 10^{-7} \times 20 \times 10 \times 10 \times 10^{-2}}{2\pi \times 2 \times 10^{-2}}$

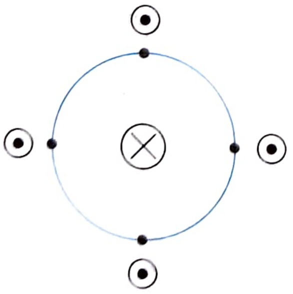
$- \dfrac{4\pi \times 10^{-7} \times 10 \times 30 \times 10 \times 10^{-2}}{2\pi \times 10 \times 10^{-2}}$

$= \dfrac{4\pi \times 10^{-7} \times 10 \times 10^{-2}}{2\pi \times 10^{-2}}[100 - 30]$

$= 20 \times 10^{-7} \times 70 = 1400 \times 10^{-7}$

$= 1.4 \times 10^{-4}$ N toward right.

21. **(4)** For a circular wire



22. **(3)** AB and DE do not give any magnetic field at O.

Magnetic field at O due to $BC = \dfrac{\mu_0 I}{4\pi a}$

It is directed 'up'.

Magnetic field at O due to

$EF = \dfrac{\mu_0 I}{4\pi a}$, it is also directed up.

So, $B = \dfrac{\mu_0 I}{2\pi a}$

23. **(1)** Using $B = \dfrac{\mu_0 I}{4\pi d}[\sin\theta_1 + \sin\theta_2]$

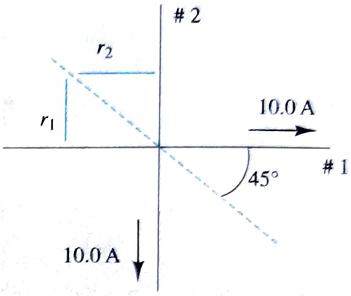
But $\theta_1 + \phi_1 = 90°$ or $\theta_1 = 90° - \phi$,

$\sin\theta_1 = \sin(90° - \phi_1) = \cos\phi_1$

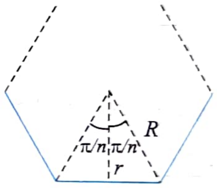
Similarly, $\sin\theta_2 = \cos\phi_2$

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi d}(\cos\phi_1 + \cos\phi_2)$$

24. **(3)** Along the dashed line, $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions. If the line has slope -1.00, then $r_1 = r_2$ and $B_1 = B_2$. So, $B_{\text{tot}} =$



25. **(3)** $B = \dfrac{\mu_0 I}{4\pi r}\left[2\sin\dfrac{\pi}{n}\right]$



But $\cos\dfrac{\pi}{n} = \dfrac{r}{R}$ or $r = R\cos\dfrac{\pi}{n}$

$\therefore \quad B = \dfrac{\mu_0 I}{4\pi R\cos\dfrac{\pi}{n}}\left[2\sin\dfrac{\pi}{n}\right] = \dfrac{\mu_0 I}{2\pi R}\left[\tan\dfrac{\pi}{n}\right]$

26. **(1)** $\vec{B}_1 = \dfrac{\mu_0 I}{4r}\odot;\ \vec{B}_2 = \dfrac{\mu_0 I}{4r}\otimes$

$\vec{B}_3 = \dfrac{\mu_0 I}{2r}\left(\dfrac{3}{4}\right)\otimes + \dfrac{\mu_0 I}{4\pi r}\odot = \dfrac{\mu_0 I}{4r}\left[\dfrac{3}{2}\otimes + \dfrac{1}{\pi}\odot\right]$

$= \dfrac{\mu_0 I}{4r}\left[\dfrac{3\pi - 2}{2\pi}\right]\otimes = \dfrac{\mu_0 I}{4r}\left(\dfrac{2}{\pi}\right)\left(\dfrac{3\pi}{4} - \dfrac{1}{2}\right)\otimes$

$B_1 : B_2 : B_3 = -\left(\dfrac{\pi}{2}\right) : \left(\dfrac{\pi}{2}\right) : \left(\dfrac{3\pi}{4} - \dfrac{1}{2}\right)$

27. **(3)** $\vec{B}_1 = \dfrac{\mu_0 I}{4\pi R}\odot,\ \vec{B}_2 = \dfrac{\mu_0 I}{4\pi R} + \dfrac{\mu_0 I/2}{4\pi R}\odot$

$\Rightarrow \quad B_2 = \dfrac{\mu_0 I}{4\pi R}\left(\dfrac{3}{2}\right)\odot \quad \Rightarrow \quad \left|\dfrac{B_1}{B_2}\right| = \dfrac{2}{3}$

28. **(3)** $B_{AB} = \dfrac{\mu_0 I}{4\pi(OC)}[2\sin\theta]$

But $OC = r\cos\theta$ or $B_{AB} = \dfrac{\mu_0 I}{2\pi r}\tan\theta$

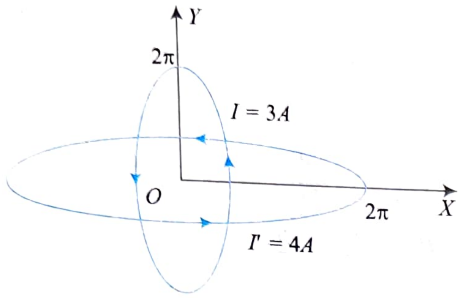
Magnetic field due to circular portion,

$$B_{AB} = \frac{\mu_0 I}{2r}\left(\frac{2\pi - 2\theta}{2\pi}\right) = \frac{\mu_0 I}{2\pi r}(\pi - \theta)$$

Total magnetic field

$$= \frac{\mu_0 I}{2\pi r}\tan\theta + \frac{\mu_0 I}{2\pi r}(\pi - \theta) = \frac{\mu_0 I}{2\pi r}[\tan\theta + \pi - \theta]$$

(1)



$$B_x = \frac{\mu_0}{2}\frac{I}{2\pi \times 10^{-2}} = \frac{\mu_0}{4\pi}3 \times 10^2 = 3 \times 10^{-5}\ \text{T}$$

$$B_y = \frac{\mu_0}{2}\frac{I'}{2\pi \times 10^{-2}} = 4 \times 10^{-5}\ \text{T}$$

$$B_{\text{net}} = \sqrt{B_x^2 + B_y^2} = \sqrt{25 \times 10^{-10}} = 5 \times 10^{-5}\ \text{T}$$

(1) The magnetic field on the axis of a coil carrying current I, having n turns, radius r and at a distance h from the centre of the coil, is given by

$$B = \frac{\mu_0}{4\pi} \times \frac{2\pi N I r^2}{(r^2 + h^2)^{3/2}} \quad \text{...(1)}$$

The field at the centre is given by

$$B_C = \frac{\mu_0}{4\pi} \times \frac{2\pi N I}{r} \quad \text{...(2)}$$

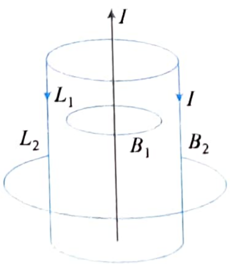
$$\therefore \quad \frac{B}{B_C} = \frac{r^3}{(r^2 + h^2)^{3/2}} = \frac{r^3}{r^3\left[1 + \frac{h^2}{r^2}\right]^{3/2}} = \left[1 - \frac{3}{2}\frac{h^2}{r^2}\right]$$

We have to find: $f = \frac{B_C - B}{B_C} = 1 - \frac{B}{B_C} = \frac{3}{2}\frac{h^2}{r^2}$

31. (3) Apply Ampere's circuital law to the coaxial circular loops L_1 and L_2. The magnetic field is B_1 at all points on L_1 and B_2 at all points on L_2. $\Sigma I \neq 0$ for L_1 and 0 for L_2.

Hence, $B_1 \neq 0$ but $B_2 = 0$

$$\left[\text{As} \oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I\right]$$



32. (2) $B_1 = \frac{\mu_0}{4\pi} \times \frac{2\pi I}{R} = \frac{\mu_0}{4\pi} \times \frac{2\pi I \times 2\pi}{L}$ ...(1)

($\because L = 2\pi R$, for circular loop)

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{1}{(a/2)}[\sin 45° + \sin 45°] \times 4$$

where $a = (L/4)$

$$\therefore \quad B_2 = \frac{\mu_0 I}{4\pi L} \times 8 \times 4 \times \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] = \frac{\mu_0 I}{4\pi L} \times \frac{64}{\sqrt{2}}$$

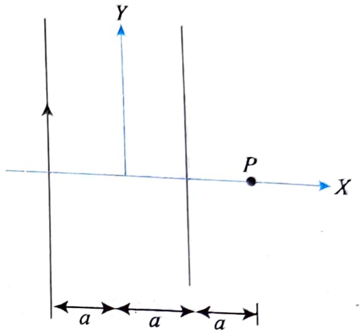
$$\therefore \quad \frac{B_1}{B_2} = \left(\frac{\mu_0}{4\pi}\right)\frac{4\pi^2 I}{L} \Big/ \frac{\mu_0 I}{4\pi L} \times \frac{64}{\sqrt{2}} = \frac{\pi^2}{8\sqrt{2}}$$

33. (1) As shown in figure,

$$\vec{B}_1 = -2\left(\frac{\mu_0}{2\pi}\frac{i}{a}\right)\hat{k} = -\frac{\mu_0 i}{\pi a}\hat{k} = \text{magnetic field at } O$$

$$\vec{B}_2 = \frac{\mu_0}{2\pi}\frac{i}{a}\hat{k} - \frac{\mu_0}{2\pi}\frac{i}{3a}\hat{k} = \frac{\mu_0}{3\pi}\frac{i}{a}\hat{k}$$

$$= \text{magnetic field at } P$$



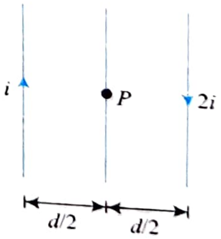
$\therefore \quad B_1/B_2 = 3$

34. (3) $\frac{\mu_0 I_1}{4\pi(a+x)} = \frac{\mu_0 I_2}{4\pi(a-x)}$

$$\frac{a-x}{a+x} = \frac{I_2}{I_1} \Rightarrow x = \left(\frac{I_1 - I_2}{I_1 + I_2}\right)a$$

35. (2) Magnetic field may be zero in first and third quadrants. $\frac{\mu_0 I_1}{2\pi y} = \frac{\mu_0 I_2}{2\pi x}$ represents a straight line passing through the origin.

36. (4) Magnetic field at P is perpendicular to paper inward due to both the wires. Charged particle is also projected in the same direction. So, force on the charged particle is zero as $\vec{v} \parallel \vec{B}$ and $\vec{F}_m = q(\vec{v} \times \vec{B})$



37. (1) Magnetic field at A due to q_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{q_2 v_2 r \sin 45°}{r^3}\hat{k} \text{ where } r = \sqrt{2}a$$

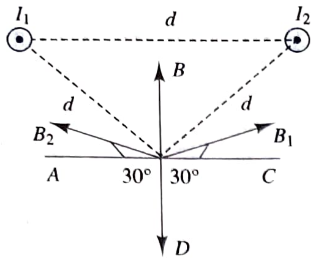
$$\Rightarrow \quad \vec{B}_2 = \frac{\mu_0}{4\pi}\frac{q_2 v_2}{2\sqrt{2}a^2}\hat{k}$$

Force on q_1: $\vec{F} = q_1\vec{v}_1 \times \vec{B}_2 = \frac{\mu_0}{4\pi}\frac{q_1 q_2 v_1 v_2}{2\sqrt{2}a^2}(-\hat{j})$

38. (2) $F = qvB = \frac{qv\mu_0 i}{2\pi r}$

$$\frac{mv^2}{R} = \frac{qv\mu_0 i}{2\pi r} \Rightarrow R = \frac{2\pi r m v}{q\mu_0 i}$$

39. **(2)**



40. **(3)** The field due to clockwise current cancels the field due to anticlockwise current.

41. **(3)** Due to current shown in figure, the magnetic field within loop is outwards. So field at O should be inward because magnetic field lines form closed loop. But if $\theta > \pi$, then point O will lie within loop and for a point within loop magnetic field is outward.

42. **(1)** $B_{\text{total}} = B + B' = \frac{\mu_0}{4\pi}\left(\frac{qv}{d^2} + \frac{q'v'}{d^2}\right)$

$$B = \frac{\mu_0}{4\pi}\left(\frac{(8.0\times10^{-6}\text{ C})(4.5\times10^{6}\text{ ms}^{-1})}{(0.120\text{ m})^2} + \frac{(3.0\times10^{-6}\text{ C})(9.0\times10^{6}\text{ ms}^{-1})}{(0.120\text{ m})^2}\right)$$

$B = 4.38\times10^{-4}$ T, into the page.

43. **(2)** $B \propto I$, because distance of centre for all wires is zero.

$$B_1 = kI_1 = 10k \otimes,\ B_2 = kI_2 = 8k \otimes$$

$$B_3 = kI_3 = 20k \odot,\ B_4 = kI_4$$

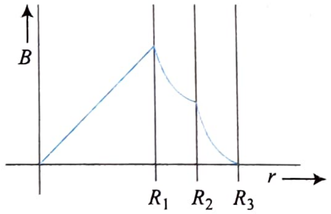
Now $B_1 + B_2 + B_3 + B_4 = 0$

$\Rightarrow$ $(10k + 8k - 20k) \otimes + kI_4 = 0$

$\Rightarrow$ $kI_4 = 2k \otimes$

$\Rightarrow$ $I_4 = 2$A towards the bottom.

44. **(3)**



From Ampere's law, the field at the axis is zero. From $x = 0$ to R_1, the field increases linearly as the current enclosed increases. From $x = R_1$ to R_2 and from $x = R_2$ to R_3, the field decreases hyperbolically but with different slopes as the media are different. Hence, the required graph is (c).

45. **(1)** $\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} = 2\hat{k},\ \hat{r} = \hat{k}$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \mu_0\varepsilon_0\left[\frac{q}{4\pi\varepsilon_0 r^2}\right]\left(\hat{i} + 3\hat{j}\right)\times\hat{k}$$

$$= \mu_0\varepsilon_0 2\left[-\hat{j} + 3\hat{i}\right] = \left(6\hat{i} - 2\hat{j}\right)\mu_0\varepsilon_0$$

46. **(2)** Magnetic field due to 'AB';

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi R}\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right]\otimes = \frac{\mu_0 I}{4\pi R}\left[1 - \frac{1}{\sqrt{2}}\right]\otimes$$

Magnetic field due to circular loop,

$$\vec{B}_2 = \frac{\mu_0 I}{2R}\odot$$

Magnetic field due to straight wire BC,

$$\vec{B}_3 = \frac{\mu_0 I}{4\pi R}\left[\sin\frac{\pi}{2} + \sin\frac{\pi}{4}\right]\odot = \frac{\mu_0 I}{4\pi R}\left[1 + \frac{1}{\sqrt{2}}\right]\odot$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$= \left(\frac{\mu_0 I}{2R} + \frac{2\mu_0 I}{4\pi R}\frac{1}{\sqrt{2}}\right)\odot = \frac{\mu_0 I}{2R}\left[1 + \frac{1}{\sqrt{2}\pi}\right]\odot$$

47. **(2)** $B_{\text{centre}} = 0$

$$\frac{\mu_0 I}{4\pi b}\odot + \frac{\mu_0 I}{4\pi b}\odot + \frac{\mu_0 I}{2b}\odot + \frac{\mu_0 I}{2a}\otimes = 0$$

$$\Rightarrow \quad \frac{\mu_0 I}{2\pi b} + \frac{\mu_0 I}{2b} - \frac{\mu_0 I}{2a} = 0 \ \Rightarrow\ \frac{1}{2\pi b} + \frac{1}{2b} = \frac{1}{2a}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{\pi}{\pi + 1}$$

48. **(3)** Magnetic field at the centre

$$B = \frac{\mu_0 I}{2c}\otimes + \frac{\mu_0 I}{2b}\left(\frac{3}{4}\right)\odot + \frac{\mu_0 I}{2a}\left(\frac{3}{4}\right)\otimes$$

As per problem:

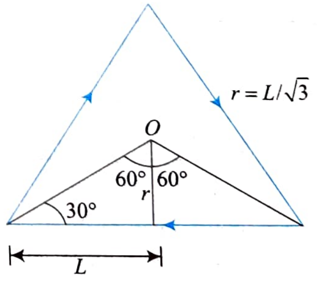
$$0 = \frac{\mu_0 I}{2}\left[\frac{1}{c} - \frac{3}{4b} + \frac{3}{4a}\right] \ \Rightarrow\ \frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{c}$$

If $c = 2a$,

$$\frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{2a} \ \Rightarrow\ a = \frac{5}{3}b$$

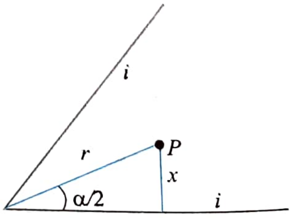
49. **(1)** Magnetic field at O is

$$B = 3\left[\frac{\mu_0}{4\pi}\frac{i}{r}(\sin 60° + \sin 60°)\right]$$



$$= 3\left(\frac{\mu_0}{4\pi}\right)(i)\left(\sqrt{3}\right)\frac{\left(\sqrt{3}\right)}{L} = \frac{9\mu_0 i}{4\pi L}$$

50. **(3)** $x = r\sin\frac{\alpha}{2}$



$$\therefore \quad B_P = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{x}\right)\left[\sin\left(90° - \frac{\alpha}{2}\right) + \sin 90°\right]$$

$$= \frac{\mu_0}{2\pi}\frac{i}{r}\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\sin\frac{\alpha}{2}}$$

(1) $\vec{B} = \frac{\mu_0 I}{2R}(\pm\hat{i}) + \frac{\mu_0 I}{2R}(\pm\hat{j}) + \frac{\mu_0 I}{2R}(\pm\hat{k})$

$$|\vec{B}| = \frac{\mu_0 I}{2R}\sqrt{3}$$

(2) $\vec{B} = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\cdots\right)\hat{k} - \left(\frac{1}{2}+\frac{1}{8}+\cdots\right)\hat{k}\right]$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k} - \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k}\right]$$

$$= \frac{2}{3}\frac{\mu_0}{2\pi}\hat{k} = \left(\frac{2}{3}\times 2\times 10^{-7}\right)\hat{k} = 1.33\times 10^{-7}\hat{k}$$

(1) Point (0, 0, $-a$) lies on z-axis, therefore magnetic field due to current along z-axis is zero and due to rest two wires is $\frac{\mu_0}{2\pi}\frac{1}{a}$ in mutually perpendicular directions along positive y direction and negative x direction.

$$\therefore \quad \vec{B} = \frac{\mu_0}{2\pi}\frac{i}{a}(\hat{j}-\hat{i})$$

4. (2) Applying Ampere's circuital law.

$$B2\pi r = \frac{\mu_0 I\pi r^2}{\pi a^2} \qquad \text{for } r < a$$

and $\quad B2\pi R = \mu_0 I \qquad$ for $R > a$

Taking the ratio

$$\frac{r}{R} = \frac{r^2}{a^2} \Rightarrow a = \sqrt{Rr}$$

55. (1) B at all the points lying on OP is zero so u will not be affected by the magnetic force. Hence, it moves on a straight line.

56. (4) $B = \frac{\mu_0 i}{2r} - \frac{\mu_0 i}{2(2r)} + \frac{\mu_0 i}{2(2^2 r)} + \cdots\infty$

$$B = \frac{\mu_0 i}{2r}\left[1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots\infty\right]$$

$$B = \frac{\mu_0 i}{2r}\left[\frac{1}{1-\left(-\frac{1}{2}\right)}\right] = \frac{\mu_0 i}{3r}$$

57. (2) $B_1 = \frac{\mu_0}{4\pi}\times\frac{2\pi I}{a}\times\frac{1}{2}$ (due to semicircular part)

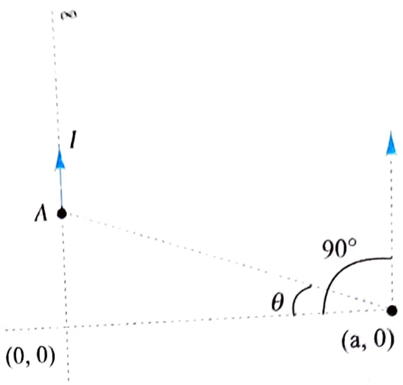
$B_2 = \frac{\mu_0}{4\pi}\times\frac{2I}{a}$ (due to parallel parts of currents)

These two fields are at right angles to each other.
Hence, resultant field

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0 I}{4\pi a}\sqrt{\pi^2+4}$$

58. (2) $B = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 90° + \sin(-\theta)] = \frac{\mu_0}{4\pi}\frac{I}{a}(1-\sin\theta)$

$$= \frac{\mu_0}{4\pi}\frac{I}{a}\left(1 - \frac{b}{\sqrt{a^2+b^2}}\right)$$

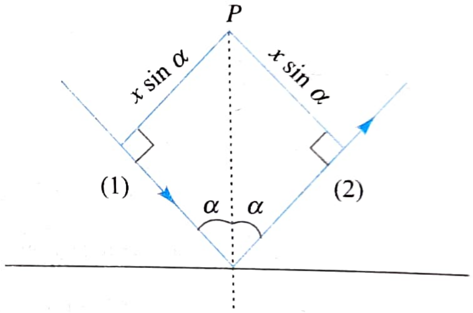


59. **(3)** Net magnetic field at I will be zero, because the magnetic field at centre P is cancelled by opposite pairs like AD–EF, etc.

60. **(3)** Magnetic field at 'P' due to wires (1) and (2) is

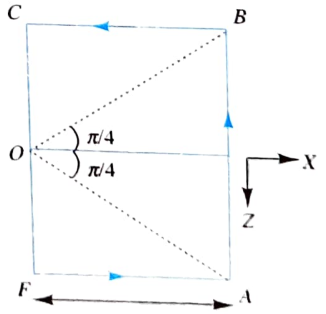
$$B_1 = \frac{\mu_0 I}{2\pi(x\sin\alpha)} + \frac{\mu_0 I}{2\pi(x\sin\alpha)} = \frac{2\mu_0 I}{2\pi(x\sin\alpha)}$$

(outside the paper)



Now, if a current of $\frac{2I}{\sin\alpha}$ is flowing in the third wire, then the magnetic field due to the same will be $B_2 = \frac{\mu_0}{2\pi x}\left(\frac{2I}{\sin\alpha}\right)$, which will cancel B_1 if it is in the paper which is possible if the current $\frac{2I}{\sin\alpha}$ in the third wire is from right to left.

61. **(3)** Due to $FABC$, the magnetic field at O is along y-axis and due to $CDEF$, the magnetic field is along x axis.



Hence, the field will be of the form $A\left[\hat{i}+\hat{j}\right]$

Calculating field due to $FABC$:
Due to AB:

$$\vec{B}_{AB} = \frac{\mu_0 i}{4\pi\left(\frac{l}{2}\right)}(\sin 45° + \sin 45°)\hat{j} = \sqrt{2}\frac{\mu_0 i}{2\pi l}\hat{j}$$

Due to BC:

$$\vec{B}_{AB} = \frac{\mu_0 i}{4\pi\left(\frac{l}{2}\right)}(\sin 0° + \sin 45°)\hat{j} = \frac{\mu_0 i}{2\sqrt{2}\pi l}\hat{j}$$

$$\vec{B}_{FA} = \frac{\mu_0 i}{2\sqrt{2}\pi l}\hat{j}$$

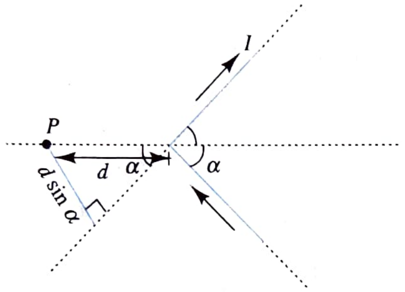
Hence, $\vec{B}_{FABC} = \frac{\mu_0 i}{\pi l}\left[\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{\sqrt{2}}{2}\right]\hat{i}$

$$\Rightarrow \quad \vec{E}_{FABC} = \frac{\sqrt{2}\mu_0 i}{\pi l}(\hat{j})$$

Similarly due to $CDEF$: $\vec{B}_{CDEF} = \frac{\sqrt{2}\mu_0 i}{\pi l}\hat{i}$

$$\Rightarrow \quad \vec{B}_{\text{net}} = \frac{\sqrt{2}\mu_0 i}{\pi l}(\hat{i} + \hat{j})$$

62. (2)



Let us compute the magnetic field due to any one segment:

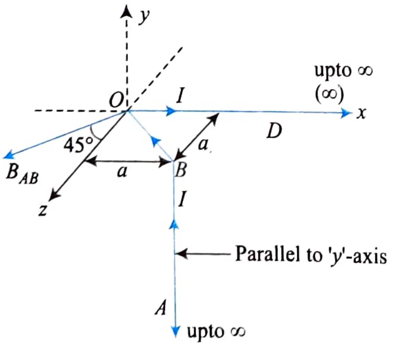
$$B = \frac{\mu_0 i}{4\pi(d\sin\alpha)}(\cos 0° + \cos(180° - \alpha))$$

$$= \frac{\mu_0 I}{4\pi(d\sin\alpha)}(1 - \cos\alpha) = \frac{\mu_0 I}{4\pi d}\tan\frac{\alpha}{2}$$

Resultant field will be

$$B_{\text{net}} = 2B = \frac{\mu_0 I}{2\pi d}\tan\frac{\alpha}{2} \quad \Rightarrow \quad K = \frac{\mu_0 I}{2\pi d}$$

63. (3)



$$B_{OD} = 0$$

$$B_{OB} = 0$$

$$B_{AB} = \frac{\mu_o I}{4\pi a\sqrt{2}}\left[\cos 45°(-\hat{i}) + \cos 45°\,\hat{k}\right]$$

$$= \frac{\mu_o I}{8\pi a}(-\hat{i} + \hat{k})$$

64. (2) Magnetic field strength at P due to I_1

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi(AP)}\hat{k} = \frac{4\pi\times10^{-7}\times2}{2\pi\times1\times10^{-2}}\hat{k} = \left(4\times10^{-5}\text{ T}\right)\hat{k}$$

Magnetic field strength at P due to I_2

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(BP)}\hat{j} = \frac{4\pi\times10^{-7}\times3}{2\pi\times2\times10^{-2}}\hat{j} = \left(3\times10^{-5}\text{ T}\right)\hat{j}$$

Hence, $\vec{B} = \left(3\times10^{-5}\text{ T}\right)\hat{j} + \left(4\times10^{-5}\text{ T}\right)\hat{k}$

65. (4) $\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_1 + I_3 + I_2 - I_3) = \mu_0(I_1 + I_2)$

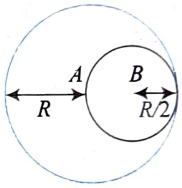
[Since for the given direction of circulation I_3 entering at $PSTU$ is positive while I_3 at $PQRS$ is negative]

Alternative solution:

$$\underset{ABCDA}{\oint}\vec{B}\cdot\vec{dl} = \underset{ABCA}{\oint}\vec{B}\cdot\vec{dl} + \underset{CDAC}{\oint}\vec{B}\cdot\vec{dl}$$

$$= \mu_0(I_1 + I_3) + \mu_0(I_2 - I_3) = \mu_0(I_1 + I_2)$$

66. (4) Current density: $\rho = \frac{I}{\pi R^2 - \pi(R/2)^2} \quad \Rightarrow \quad \rho = \frac{4I}{3\pi R^2}$



Current in smaller cylinder (if there were):

$$I_1 = \rho\pi\left(\frac{R}{2}\right)^2 = \frac{I}{3}$$

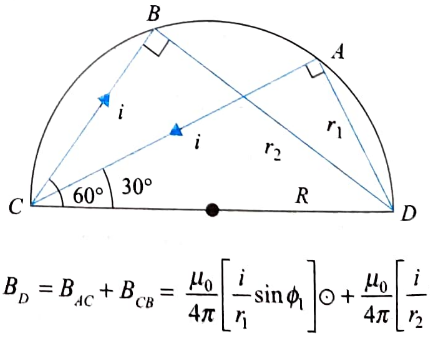
For A: $B_A = B_{\text{whole-cylinder}} - B_{\text{small-cylinder}}$

$$\Rightarrow \quad B_A = 0 - \frac{\mu_0(I/3)}{2\pi(R/2)} = \frac{-\mu_0 I}{3\pi R}$$

For B: $B_B = B_{\text{whole-cylinder}} - B_{\text{small-cylinder}}$

$$= \frac{\mu_0(I + I/3)(R/2)}{2\pi R^2} - 0 = \frac{\mu_0 I}{3\pi R}$$

67. (2) $r_1 = 2R\sin 30° = R$, $r_2 = 2R\sin 60° = R\sqrt{3}$



$$B_D = B_{AC} + B_{CB} = \frac{\mu_0}{4\pi}\left[\frac{i}{r_1}\sin\phi_1\right]\odot + \frac{\mu_0}{4\pi}\left[\frac{i}{r_2}\sin\phi_2\right]\otimes$$

$$= \frac{\mu_0}{4\pi}i\left[\frac{\sin 60°}{r_1} - \frac{\sin 30°}{r_2}\right]\odot = \frac{\mu_0 i}{4\pi R\sqrt{3}}\cdot$$

68. (3) Magnetic field at centre due to a square of side a:

$$B = 4\left[\frac{\mu_0}{4\pi}\frac{i}{(a/2)}[\sin 45° + \sin 45°]\right] = \frac{2\sqrt{2}\mu_0 i}{\pi a}$$

For the case in figure

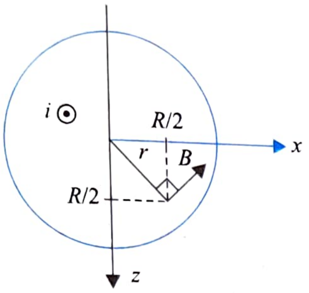
$$B_{\text{net}}=\frac{2\sqrt{2}\,\mu_0 i}{\pi}\left[\frac{1}{a}-\frac{1}{2a}+\frac{1}{3a}-\frac{1}{4a}+\dots\right]$$

$$=\frac{2\sqrt{2}\mu_0 i}{\pi a}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots\right]=\frac{2\sqrt{2}\mu_0 i}{\pi a}(\ln 2)$$

9. **(4)** $B=\mu nI=\mu_r\,\mu_0\,nI$

$$=4000\times 4\pi\times 10^{-7}\times 1000\times 5=8\pi\text{ T}$$

0. **(1)** $r=R\sqrt{2},\ B=\dfrac{\mu_0 I(R/\sqrt{2})}{2\pi R^2}=\dfrac{\mu_0 I}{2\sqrt{2}\pi R}$



$$\vec{B}=B\left[\cos 45^\circ\hat{i}+\sin 45^\circ(-\hat{k})\right]=\frac{\mu_0 I}{4\pi R}\left[\hat{i}-\hat{k}\right]$$

1. **(2)** Current within radius $r<R$:

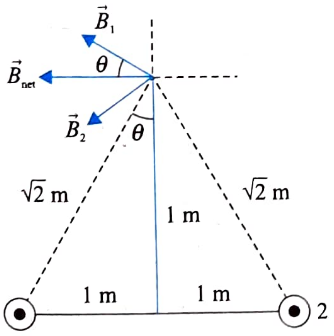
$$I_{\text{in}}=J\pi r^2=\frac{i}{\pi R^2}\pi r^2=\frac{ir^2}{R^2}$$

for $r<R$, $\displaystyle\oint\vec{B}\cdot d\vec{\ell}=\mu_0 I_{\text{in}}=\frac{\mu_0 ir^2}{R^2}$

for $r>R$, $\displaystyle\oint\vec{B}\cdot d\vec{\ell}=\mu_0 I_{\text{in}}=\mu_0 I\ \rightarrow$ Constant

2. **(1)** If a wire carries current along positive x-axis, then the magnetic field due to it along the line $y=x$ in first quadrant is upwards. If a wire carries current along positive y-axis, then the magnetic field due to it along the line $y=x$ in first quadrant is downwards. Along the line $y=x$, above two values of magnetic field are equal in magnitude but opposite in direction and thus, their resultant is zero.

3. **(4)** In three dimensional space, the point lies perpendicular to the plane containing wires.



Let θ be the half-angle made by position vector of particle with vertical. Then,

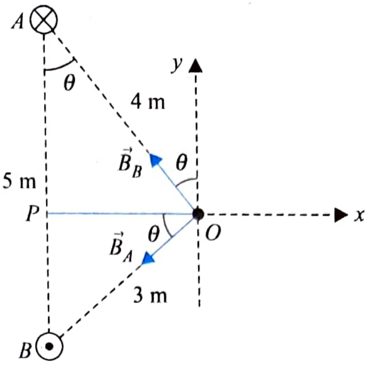
$$\sin\theta=\frac{1}{\sqrt{2}}\ \Rightarrow\ \theta=45^\circ$$

If field due to each wire is B,

$$B_{\text{net}}=2B\cos\theta$$

$$B_{\text{net}}=2\left(\frac{\mu_0 i}{2\pi\sqrt{2}}\right)\frac{1}{\sqrt{2}}=\frac{\mu_0 i}{2\pi}$$

74. **(2)** From trigonometry,



$AP=16/5$ and $PB=9/5$. Thus,

$\sin AOP=4/5$ and $\cos AOP=3/5$

$\sin BOP=3/5$ and $\cos BOP=4/5$

Magnetic field due to A is $B_A=\dfrac{\mu_0 i}{8\pi}$ which makes angle θ with negative x-axis and lies in third quadrant.

Magnetic field at O due to wire B is $B_B=\dfrac{\mu_0 i}{6\pi}$ which makes angle θ with y-axis and lies in second quadrant.

In component form,

$$\overline{B_A}=\frac{\mu_0 i}{8\pi}\left(-\frac{4}{5}i-\frac{3}{5}j\right)$$

$$\overline{B_B}=\frac{\mu_0 i}{6\pi}\left(-\frac{3}{5}i+\frac{4}{5}j\right)$$

Thus, $\overline{B_A}+\overline{B_B}=\dfrac{\mu_0 i}{5\pi}\left(-i+\dfrac{7}{24}j\right)$

75. **(1)** Since the rod is a rigid body, every point has same angular velocity $\dot{u}=1$ rad/s. For points at a distance of 1 m, $v_1=1$ m/s

For points at a distance of 2 m, $v_2=2$ m/s

If a charge rotating in a circle constitutes a current I,

$$B=\frac{\mu_0 i}{2r},\text{ where } i=\frac{q}{t}=\frac{qv}{2\pi r}$$

Since field due to rotation of all charges are in same direction,

$$B=\sum\frac{\mu_0 qv}{4\pi r^2}=\frac{\mu_0 q}{4\pi}\left(1+1+\frac{1}{2}+\frac{1}{2}+\frac{1}{3}+\frac{1}{3}\right)$$

76. **(1)** Given that wires kept at negative x values carry current successively in opposite directions. The fields due to wires kept at $x=\pm 2$m, ± 8m etc. cancel each other. Due to wires kept at $x=\pm 1$m, ± 4m, fields are in upward direction and net field is given by:

$$B=2\times\frac{\mu_0 i}{2\pi}\left(1+\frac{1}{4}+\frac{1}{16}+\dots\right)$$

$$\Rightarrow\quad B=\frac{4\mu_0 i}{3\pi}=\frac{2\mu_0 i}{3\pi}\ (\text{Since } i=0.5\text{ A})$$

This field is negated by field due to wire placed at $y=2$ m. This wire must carry current I along positive x-axis such that

$$\frac{\mu_0 I}{2\pi\times 2}=\frac{2\mu_0 i}{3\pi}$$

Thus, $I=\dfrac{8}{3}$ A along $+x$-axis.

77. (3) The magnetic field due to wire placed along x-axis will be zero. The point under consideration is semifinite position of the wires placed along y-axis and z-axis.

Magnetic field due to wire placed along y-axis $\vec{B}_y = \frac{\mu_0 I}{4\pi r}(\hat{k})$

Magnetic field due to wire placed along z-axis $\vec{B}_z = \frac{\mu I}{4\pi r}(-\hat{j})$

Hence net magnetic field $\vec{B} = \frac{\mu_0 I}{2\pi r}(\hat{k} - \hat{j})$

After substituting the values, we get

$$\vec{B} = \frac{\mu_0}{8\pi}(\hat{k} - \hat{j})\ \text{T}$$

78. (2) At the desired point, the magnetic fields due to the two wires must have equal magnitude but opposite directions. The point should be either to the left of P or to the right of Q. As the wire Q has smaller current, the point should be closer to Q. Let this point R be at a distance x from Q.

The magnetic field at R due to the current i_1 will have magnitude

$$B_1 = \frac{\mu_0 i_1}{2\pi(d + x)}$$

and will be directed in the plane of the figure. The field at the same point due to the current i_2 will be

$$B_2 = \frac{\mu_0 i_2}{2\pi x}$$

directed upward in the plane of the figure. If the resultant field at R is zero, we should have $B_1 = B_2$, so that $\frac{i_1}{d + x} = \frac{i_2}{x}$

Hence, $x = \frac{i_2 d}{i_1 - i_2} = \frac{(2\text{ A})(6\text{ cm})}{(3\text{ A})} = 4\text{ cm}$

79. (1) Magnetic field due to situation I and II

$$\vec{B}_{\text{net}} = \vec{B}_{\text{infinite wire}} + \vec{B}_{\text{circular}}$$

Case I: $\vec{B}_a = \frac{\mu_0 I}{2\pi R}\odot + \frac{\mu_0 I}{2R}\odot$

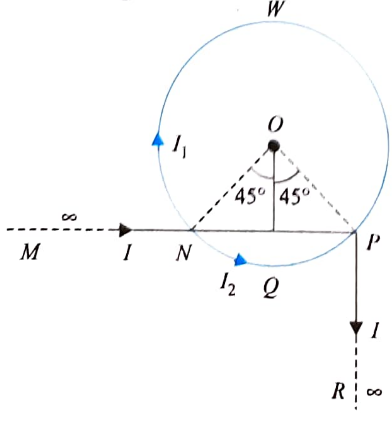
$$\vec{B}_a = \frac{\mu_0 I}{2R}\left[\frac{1+\pi}{\pi}\right]\odot$$

Case II: $\vec{B}_b = \frac{\mu I}{2\pi R}\odot + \frac{\mu_0 I}{2R}\otimes$

$$\vec{B}_b = \frac{\mu_0 I}{2R}\left[\frac{1-\pi}{\pi}\right]\odot$$

$$\frac{|\vec{B}_a|}{|\vec{B}_b|} = \frac{1+\pi}{1-\pi}$$

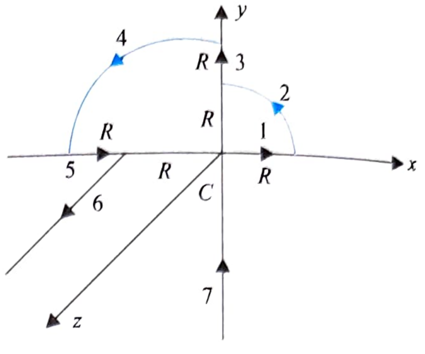
80. (3) Since length NWP is 3 times that of NQP, so resistance of NWP is 3 times that of NQP.



Hence $\frac{I_1}{I_2} = \frac{1}{3}$

Net field due to I_1 and I_2 at centre O will be zero. Field at O due to MN will be same as that due to PQ, but in opposite direction, so net field at centre will be zero.

81. (1) $B_c = B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + B_7$



$$= 0 + \frac{\mu_0}{4\pi}\frac{I}{R}\frac{\pi}{2}\hat{k} + 0 + \frac{\mu_0}{4\pi}\frac{I}{2R}\frac{\pi}{2}\hat{k} + 0 + \frac{\mu_0}{4\pi}\frac{I}{R}\hat{j}$$

82. (2) $\vec{B}_A = \frac{\mu_0 R}{4\pi R}\hat{k} + \frac{\mu_0 I}{2R}\hat{i} - \frac{\mu_0 I}{4\pi R}\hat{j}$

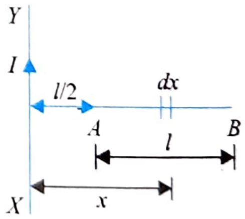
$$B_A = \frac{\mu_0 I}{4\pi R}\sqrt{(2\pi - 1)^2 + (1)^2} = \frac{\mu_0 I}{4\pi R}\sqrt{2(2\pi^2 - 2\pi + 1)}$$

83. (4) $B = \frac{\mu_0 i}{2r} - \frac{\mu_0 i}{2(2r)} + \frac{\mu_0 i}{2(2^2 r)} \ldots \infty$

$$B = \frac{\mu_0 i}{2r}\left[1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \ldots\infty\right]$$

$$B = \frac{\mu_0 i}{2r}\left[\frac{1}{1 - \left(-\frac{1}{2}\right)}\right] \Rightarrow B = \frac{\mu_0 i}{3r}$$

84. (2) Consider an element of length dx on AB at a distance x from XY.



Force on this element, $dF = \frac{\mu_0 I}{2\pi x} i dx$

Total force on $dF = \frac{\mu_0 I}{2\pi x} i dx$

$$= \frac{\mu_0 Ii}{2\pi}\int_{\ell/2}^{3\ell/2}\frac{1}{x}dx = \frac{\mu_0 Ii}{2\pi}\log_e 3$$

85. (1) The moving string behaves like a current,

$i = (40 \times 10^{-6}\text{ C/m})(300\text{ m/sec})$

$= 1.2 \times 10^{-2}$ ampere

Then $B = \frac{\mu_0 i}{2\pi r} = \frac{2 \times 10^{-7} \times 1.2 \times 10^{-2}}{0.005}$

$= 4.8 \times 10^{-7}$ tesla

Multiple Correct Answers Type

1. **(2), (3)**

If a current passes through any conductor, net charge through any section is zero. Hence, electric field in the vicinity of the conductor is zero. We can observe using Biot and Savart law that the magnetic field at the axis of the cylindrical wire will be zero. Electric field at the axis of the wire cannot be zero otherwise the current through the wire cannot flow.

2. **(2), (4)**

In first and third quadrants, direction of magnetic field is opposite due to the two wires. So, magnetic field can be zero only in first and third quadrants not in second and fourth.

Hence, (2) and (4) are correct.

3. **(1), (2), (3)**

For B_1 and B_4, the contributions due to the different sections add up. For B_2 and B_3, the contributions due to the outer sections oppose the contributions due to the inner sections. Thus, B_1 and B_4 are greater than B_2 and B_3. For B_4, there is a section with radius $< b$ and hence, it contributes more than the semicircular section of radius b for B_1. Thus, $B_4 > B_1$.

For B_3, there is a section with radius $> b$ and hence it contributes less than the semicircular section of radius b for B_2. Thus, $B_3 < B_2$.

Hence, $B_4 > B_1 > B_2 > B_3$

4. **(1), (3)**

Using $B = \dfrac{\mu_0 NI}{2R}$

$$|\vec{B}_1| = |\vec{B}_2| = 4\pi \times 10^{-4}\ \text{T}$$

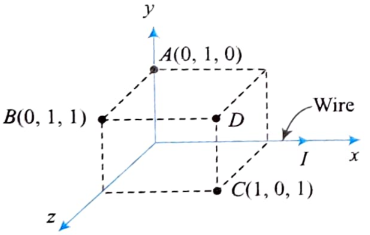
If current in same sense $B_{\text{net}} = 8\pi \times 10^{-4}$ T

If current in opposite sense $B_{\text{net}} = 0$

5. **(2), (4)**

Distance of A and C is same from wire. Hence, magnetic field at A and C is same in magnitude.

Similarly, for points B and D.



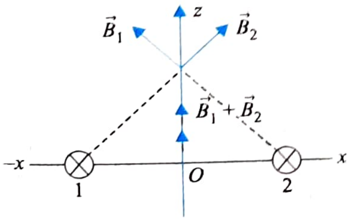
6. **(1), (3)**

Field due to each plate $= \dfrac{1}{2}\mu_0 K = 2\ \mu\text{T}$

At A, fields add up, being in the same directions whereas at B, fields cancel out due to opposite directions.

7. **(2), (4)**

From the diagram it is clear the correct choices are (2) and (4).



8. **(1), (4)**

Use the right hand rule to verify the options.

9. **(2), (3), (4)**

The direction of the magnetic field is perpendicular to the plane containing the point P and the current element I, which is the z-direction. Hence choice (1) is wrong and choice (2) is correct. Also the fields to the left and to the right of the current element are oppositely directed. Hence choice (4) is correct. Further, the lines of force of the magnetic field are concentric circles with their common centre at the wire. Hence choice (3) is also correct.

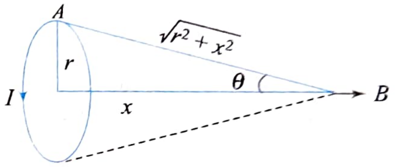
10. **(1), (4)**

A and B observe electrostatic fields. But B observes magnetic field due to moving charge.

11. **(1), (2)**

The magnetic field due to straight wires will be zero as both are directed towards the centre of the circle. The magnetic field due to circular arc will be cancelled out. Hence magnetic field due to loop will be zero.

12. **(2), (4)**



$$B = \frac{\mu_0 NIr^2}{2(r^2+x^2)^{3/2}} \text{ along } x\text{-axis}$$

$$B = \frac{\mu_0 NI}{2r}\frac{r^3}{(r^2+x^2)^{3/2}}$$

$$B = \frac{\mu_0 NI}{2r}\sin^3\theta$$

13. **(1), (2), (3)**

Magnetic field due to wire 1

- At position P and R is zero
- At position Q and S are equal and opposite $\dfrac{\mu_0 I}{4d}$, ⊙ at S and ⊗ at Q.

Magnetic field due to wire 2

- At positions S and Q is zero
- At positions P and R is $\dfrac{\mu_0 I}{4d}$ ⊙ at P and ⊗ at R.
- Hence net magnetic field at points P, Q, R and S are same in magnitude.
- Direction of magnetic field at Q and R and S and P are same.
- Direction of magnetic field at R and S and P and Q are opposite.

14. **(3), (4)**

$B_{\text{net}} = B_1 - B_2 \Rightarrow B_1 - B_2 = 0 \Rightarrow B_1 = B_2$

$\Rightarrow \quad B \propto ni$. So $n_1 i_1 = n_2 i_2$ or $n_1 = n_2$ and $i_1 = i_2$

15. **(2), (3)**

Applying the Ampere's circuital law, we have

$\oint B \cdot dl = \mu_0(I - I) = 0$ (because current is in opposite sense.)

Also, there may be a point on C where B and dl are perpendicular and hence, $\oint_C B \cdot dl = 0$.

16. **(1), (2)**

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{l} \times \vec{r}}{r^3} \text{ for } B_1 \quad \vec{r} = (-\hat{i} - \hat{j})$$

$$\therefore \quad \vec{B}_1 = \frac{\mu_0}{4\pi}\frac{i}{2\sqrt{2}}\hat{k}\times(-\hat{i}-\hat{j}) \qquad \text{...(1)}$$

for B_2 $\quad \vec{r} = \hat{i}+\hat{j}$

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{i\hat{k}\times(\hat{i}+\hat{j})}{2\sqrt{2}} \qquad \text{...(2)}$$

From (1) and (2)

$$\vec{B}_1 = -\vec{B}_2 \text{ and } \left|\vec{B}_1\right| = \left|\vec{B}_2\right|$$

17. (3), (4)

For cylinder;

$$B = \frac{\mu_0 i r}{2\pi R^2}\ ; \quad r < a$$

$$= \frac{\mu_0 i}{2\pi r}; \ r \geq a$$

We can consider the given cylinder as a combination of two cylinders. One of radius R carrying current I in one direction and other of radius $R/2$ carrying current $I/3$ in both directions.

At point A:

$$B = \frac{\mu_0 (I/3)}{2\pi (R/2)} + 0 = \frac{\mu_0 I}{3\pi R}$$

At point B:

$$B = \frac{\mu_0}{2}\left(\frac{4I/3}{\pi R^2}\right)\left(\frac{R}{2}\right) + 0 = \frac{\mu_0 I}{3\pi R}$$

18. (2), (3)

The radius of the double loop $R = R/2$.

Now $\quad B = \dfrac{\mu_0 I}{2R}$

Magnetic field due to loop of radius R at the centre of the loop is

$$B_1 = \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{2R} = 2B \qquad (\because \ r = R/2)$$

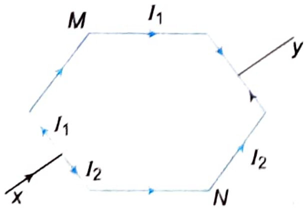
Similarly for the second loop of the double loop,

Since the currents in the two loops are in the same direction, the net magnetic field at the centre = $B_1 + B_2 = 4B$.

Since the currents in the two loops are in opposite directions, fields B_1 and B_2 are equal and opposite. Therefore, the net magnetic field at the Centre of the double loop = $B_1 - B_2 = 0$.

19 (2), (3), (4)

Let current divides in two parts I_1 and I_2. Magnetic field at center will be zero if $I_1 = I_2$. This is possible if both currents face same resistance in both the paths XMY and XNY.



20. (2), (4)

At points P and R, the angle between velocity of charged particle and direction of magnetic field is π and 0° respectively. Hence, force on charge at P and R is zero. Force on charged particle at Q and S is same in magnitude.

Linked Comprehension Type

For Problems 1–3

1. (3) **2. (1)** **3. (1)**

Inside the wire

$$B(r) = \frac{\mu_0}{2\pi}\frac{I}{R^2}r \ \Rightarrow \ \frac{dB}{dr} = \frac{\mu_0}{2\pi}\frac{I}{R^2}$$

i.e., $\quad$ Slope $\propto \dfrac{I}{\pi R^2} \propto$ current density

It can be seen that slope of curve for wire a is greater than wire c.

For Problems 4–6

4. (2) **5. (4)** **6. (1)**

Magnetic field at any point on Ampere's loop can be due to all currents passing through inside or outside the loop. But net contribution in the left hand side will come from inside current only.

For $r < a$, current passing through within the cylinder of radius r is given by

$$\int_0^r J dA = \int_0^r kr^2 2\pi r dr = 2\pi k\int_0^r r^3 dr$$

$$= k\pi r^4/2$$

Now using Ampere's law:

$$B \times 2\pi r = \mu_0 I = \mu_0 k\pi r^4/2 \ \Rightarrow \ B = \frac{\mu_0 k r^3}{4}$$

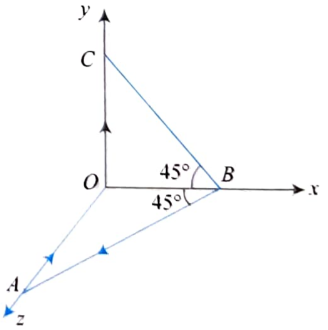
For $r > a$, $I = k\pi a^4/2$ (by putting $r = a$)

$$\therefore \quad B = \frac{\mu_0 k a^4}{4r}$$

For Problems 7–8

7. (2) **8. (3)**

At B, magnetic field will be zero due to CB and AB parts.



$$\vec{B}_{AO} = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin 0° + \sin 45°\right]\left(-\hat{j}\right) = \frac{-\mu_0 I}{4\pi\sqrt{2}a}\hat{j}$$

$$\vec{B}_{OC} = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin 0° + \sin 45°\right]\left(-\hat{k}\right) = \frac{-\mu_0 I}{4\pi\sqrt{2}a}\hat{k}$$

Net magnetic field at B

$$= \vec{B}_{AO} + \vec{B}_{OC} = -\frac{\mu_0 I}{4\pi\sqrt{2}a}(\hat{j}+\hat{k})$$

At O, magnetic field due to AO and OC will be zero.

Due to BC: $r = a\cos 45° = \dfrac{a}{\sqrt{2}}$

$$\vec{B}_{CB} = \frac{\mu_0}{4\pi}\frac{I}{r}\left[\sin 45° + \sin 45°\right]\left(-\hat{k}\right) = \frac{-\mu_0 I}{2\pi a}\hat{k}$$

Similarly, due to BA: $\vec{B}_{BA} = \dfrac{-\mu_0 I}{2\pi a}\hat{j}$

Net magnetic field = $\vec{B}_{CB} + \vec{B}_{BA} = \dfrac{-\mu_0 I}{2\pi a}\left(\hat{j}+\hat{k}\right)$

For Problems 9–10

9. (3) **10. (4)**

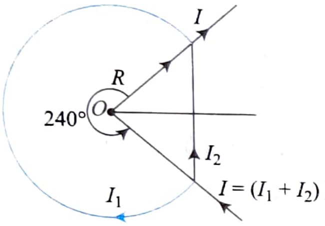
If I_1 and I_2 be the currents in circular and straight parts, respectively, and B_1, B_2 the magnetic fields due to them, then

$$B_1 = \frac{\mu_0 I_1}{2R} \times \frac{2}{3} = \frac{\mu_0 I_1}{3R}$$

$$B_2 = \frac{\mu_0 I_2}{4\pi\left[R\cos 60^\circ\right]}\left[2\sin 60^\circ\right] = \frac{\sqrt{3}\mu_0 I_2}{2\pi R}$$

For the total field at 'O' to be zero,

$$\frac{\mu_0 I_1}{3R} = \frac{\sqrt{3}\mu_0 I_2}{2\pi R} \Rightarrow \frac{I_1}{I_2} = \frac{3\sqrt{3}}{2\pi}$$



From $R = \dfrac{\rho l}{A}$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}\frac{A_2}{A_1} \Rightarrow \frac{V/I_1}{V/I_2} = \frac{l_1}{l_2}\frac{(\pi/4)d_2^2}{(\pi/4)d_1^2} \Rightarrow \frac{d_1}{d_2} = \sqrt{\frac{l_1}{l_2}\frac{I_1}{I_2}}$$

where $l_1 = 2\pi R\left(\dfrac{2}{3}\right) = \dfrac{4}{3}\pi R$ and $l_2 = 2R\sin 60^\circ = \sqrt{3}R$

$$\Rightarrow \frac{d_1}{d_2} = \sqrt{\frac{4\pi R}{3\sqrt{3}R}\frac{3\sqrt{3}}{2\pi}} = \sqrt{2}$$

For Problems 11–12

11. (1) **12. (2)**

Magnetic field produced by the conductor along z-axis will be parallel to MN. Hence, force on MN is zero.

Current from B to A (through NM or QP):

$$I = \frac{V}{R/2} = \frac{2V}{R}$$

Force due to I_0 on $N'N$ and $Q'Q$ will be out of the paper and on $M'M$ and $P'P$ will be in the paper. This will produce the torque on loop along negative x-axis.

Torque on $M'M$ (along negative x-axis):

$$\tau = \int_a^b \left(\frac{\mu_0 I_0}{2\pi r} I dr\right) r\sin\left(\frac{\pi}{6}\right) = \frac{\mu_0 I_0}{4\pi} I(b-a)$$

$$= \frac{\mu_0 I_0 2V}{4R\pi}(b-a)$$

Total torque $= 4\tau = 2\dfrac{\mu_0 I_0 V}{\pi R}(b-a)$ (along negative x-axis.)

For Problems 13–15

13. (2) **14. (2)** **15. (4)**

Current within radius r: $I = \displaystyle\int_0^r J\,dA$

$$\Rightarrow \quad I = \int_0^r J_0\left[1-\frac{r}{R}\right]2\pi r\,dr = 2\pi J_0\left[\frac{r^2}{2} - \frac{r^3}{3R}\right]$$

Apply ampere's law: $B\,2\pi r = \mu_0 I$

$$B 2\pi r = \mu_0 2\pi J_0\left(\frac{r^2}{2} - \frac{r^3}{3R}\right) \Rightarrow B = \mu_0 J_0\left[\frac{r}{2} - \frac{r^2}{3R}\right]$$

To find max B: $\dfrac{dB}{dr} = 0 \Rightarrow r = 3R/4$

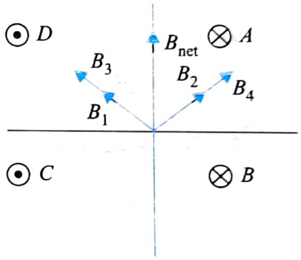
$$B_{\max} = \mu_0 J_0\left[\frac{3R/4}{2} - \frac{(3R/4)^2}{3R}\right] = \frac{3R}{16}\mu_0 J_0$$

For Problems 16–17

16. (3)

17. (1)

i.

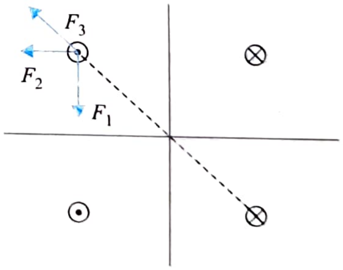


$$B = B_1 = B_2 = B_3 = B_4 = \frac{\mu_0 I}{2\pi\sqrt{2}a}$$

Net field $= 4B\cos 45^\circ$

$$= \frac{4\mu_0 I}{2\pi\sqrt{2}a}\frac{1}{\sqrt{2}} = \frac{\mu_0 I}{\pi a}$$

ii.



$$F_1 = \frac{\mu_0}{2\pi}\frac{I^2}{(2a)},\quad F_2 = \frac{\mu_0}{2\pi}\frac{I^2}{2a}$$

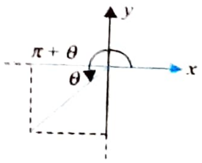
$$F_3 = \frac{\mu_0}{2\pi}\frac{I^2}{2\sqrt{2}a}$$

$$F_x = -(F_2 + F_3\cos 45^\circ) = -\frac{\mu_0}{4\pi}\frac{I^2}{2a}3$$

$$F_y = -(F_1 - F_3\cos 45^\circ) = -\frac{\mu_0}{4\pi}\frac{I^2}{2a}$$

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \frac{\mu_0}{4\pi}\frac{I^2}{2a}\sqrt{10}\text{ N/m}$$

$$\tan\theta = \frac{F_y}{F_x} = \frac{1}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right)$$



For Problems 18–20

18. (1) The current path *abcdefgha* can be treated as superposition of three loops *fghaf*, *fabef* and *ebcde*, each carrying current *I*.

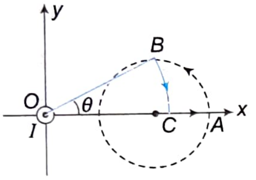
19. (2) The magnetic field at centre of cube cancels due to two square loops *fghaf* and *ebcde* each carrying current *I*. The magnetic field at centre of cube is only due to loop *fabef*. Hence the magnetic field at centre of cube is in negative *y*-direction.

20. (4) The current path *abcdefgha* can be treated as superposition of three loops *fghaf*, *fabef* and *ebcde* each carrying current *I*. The dipole moment two square loops *fghaf* and *ebcde* each carrying current *I* cancels out. Hence the dipole moment is only due to loop *fabef*. Now the dipole moment of loop is in negative *y*-direction and direction of external magnetic field is in positive *y*-direction. Therefore torque on loop is zero.

For Problems 21–23

21. (4) Since there is no current passing through circular path, the integral along the dotted circle is zero.

22. (2) Let segment $OB = OC$ and arc BC is a circular arc with centre at origin. Since the shown closed path $ABCA$ encloses no current, the path integral of magnetic field over this path is zero.



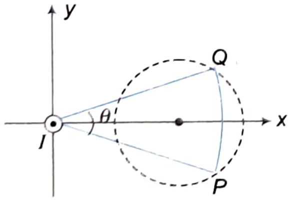
Hence, $\int_A^B \vec{B}\cdot\vec{dl} + \int_B^C \vec{B}\cdot\vec{dl} + \int_C^A \vec{B}\cdot\vec{dl} = 0$

Because $\vec{B}$ is perpendicular to segment AC at all points,

therefore $\int_C^A \vec{B}\cdot\vec{dl} = 0$.

Hence, $\int_A^B \vec{B}\cdot\vec{dl} = \int_C^B \vec{B}\cdot\vec{dl} = \frac{\mu_o I}{2\pi}\frac{OB(\theta)}{OB} = \frac{\mu_o I}{2\pi}\tan^{-1}\frac{1}{2}$

23. (3) Consider two points P and Q lying on dotted circle and equidistant from origin O. We draw a circular arc QP with centre at origin O. The path integral of magnetic field, that is, $\int \vec{B}\cdot d\vec{l}$ along the dotted circle between two points P and Q is also is equal to path integral $\int \vec{B}\cdot d\vec{l}$ along the arc QP whose centre is at origin.



Therefore the path integral of magnetic field $\int \vec{B}\cdot d\vec{l}$ along the dotted circle between two points P and Q

$$= \frac{\mu_o I}{2\pi}\frac{OP(\theta)}{OP} = \frac{\mu_o I}{2\pi}\theta$$

The value of θ will be maximum when chord OQ and chord OP will be tangent to the dotted circle, that is, $\theta = \frac{\pi}{3}$.

Hence, the required maximum value $= \frac{\mu_0 \mathrm{I}}{6}$.

For Problems 24–26

24. (1) When a uniform field is used,

$$\tau = BINA\cos\theta = k\theta$$

$$\Rightarrow \quad \frac{\theta}{I} = BNA\cos\theta$$

$\therefore$ In a uniform field, torque decreases as θ increases.

So scale will be non-linear in case (1).

25. (1) $\tau_1 = NBIA\cos\theta$

$$\tau_2 = NBIA\cos(90^\circ + \theta) = NBIA\sin\theta$$

$$\Rightarrow \quad \tau_1^2 + \tau_2^2 = (NBIA)^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow \quad B = \frac{\sqrt{\tau_1^2 + \tau_2^2}}{NIA}$$

26. (4) A non-linear instrument's scale is difficult to read and calibrate e.g. when the pointer is between 1 and 2, it can be read as 1.5 but the true value may be 1.4 mA.

Matrix Match Type

1. i. → b., d.; ii. → a., d.; iii. → c., d.; iv. → c., d.

Work done by magnetic force on a charge = 0 in any part of its motion.

'*d*' is matching for all parts (i), (ii), (iii),(iv).

For loop 1: $\sum I_{\text{in}} = -i + i - i = -i$

$$\therefore \quad \oint \vec{B}\cdot d\vec{\ell} = \mu_0(-i)$$

For loop 2: $\sum I_{\text{in}} = i - i + i = i$

$$\therefore \quad \oint \vec{B}\cdot d\vec{\ell} = \mu_0(i)$$

For loop 3: $\sum I_{\text{in}} = -i + i = 0$

$$\therefore \quad \oint \vec{B}\cdot d\vec{\ell} = 0$$

For loop 4: $\sum I_{\text{in}} = +i - i = 0$

$$\therefore \quad \oint \vec{B}\cdot d\vec{\ell} = 0$$

(Note that current will be taken as positive which produces lines of magnetic field in the same sense in which $d\vec{\ell}$ is taken.)

2. i. → b., c.; ii. → b., d.; iii. → a.; iv. → c.

i. $M = NIA = 100 \times 5 \times \pi(0.1)^2 = 5\pi$ Am

As the current in *xy* plane is along anticlockwise direction, so moment will be along *z*-axis by right hand thumb rule.

$$\vec{M} = 5\pi\hat{k}$$

ii. $\vec{\tau} = \vec{M}\times\vec{B} = 5\pi\hat{k}\times(-\hat{i}+\hat{k}) = -5\pi\hat{j}$

iii. Net force on a closed current carrying loop in a uniform magnetic field is zero.

iv. Magnetic field at centre of loop due to current in loop will be along positive *z*-axis from right hand thumb rule.

3. i. → a., b., c.; ii. → a., b., c., d.; iii. → c.; iv. → a., b., c., d.

The magnetic field is along negative *y*-direction in cases *a*, (b) and (c). In case (d) there will be component of magnetic field along negative *x*-axis.

Hence, i. → a., b., c.

z-component of magnetic field is zero in all cases.

Hence, ii. → a., b., c., d.

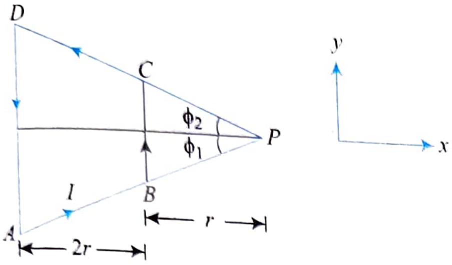
The magnetic field at P is $\frac{\mu_0}{4\pi}\frac{i}{d}$ in case (c) only.

Hence, iii. → c.

The magnetic field at P is less than $\frac{\mu_0}{2\pi}\frac{i}{d}$ for all cases.

Hence, iv. → a., b., c., d.

4. **i. → d.; ii. → a., b., c.; iii. → b., c.; iv. → a., b., c.**



At P, magnetic field due to AB and CD will be zero.

$$\vec{B}_{BC} = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\phi_1 + \sin\phi_2)(-\hat{k})$$

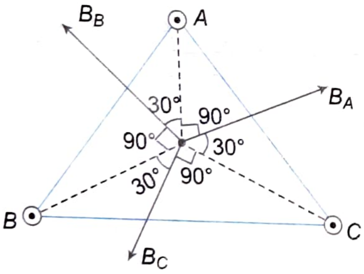
$$\vec{B}_{DA} = \frac{\mu_0}{4\pi}\frac{I}{3r}(\sin\phi_1 + \sin\phi_2)(\hat{k})$$

Net field at P:

$$\vec{B} = \vec{B}_{BC} + \vec{B}_{DA} = -\frac{2\mu_0}{4\pi}\frac{I}{3r}(\sin\phi_1 + \sin\phi_2)\hat{k}$$

Hence, we can find the answers given below.

5. (1)



For the given situation, net field at M will be zero because magnetic field due to all the three wires will cancel each other. If current in A is reversed, then B_A will also be reversed, so that net field is along CB. If current in A is halved, then B_A is also halved. So that net field becomes along CB. If net current in A is doubled, then B_A is also doubled so that net field becomes along BC.

6. **(2)** From symmetry magnetic field is zero at all points on line EF. Hence, magnetic field at origin and Q is zero.

At point P, field is non zero along +ve z-direction. At point R, field is non zero and along –ve z-direction.

7. **(4)** For a DC ammeter or voltmeter, a moving coil galvanometer is used. So, deflection of coil $\theta = \frac{BNA}{k}I$

or $\theta \propto I$

Also $\theta = \frac{BNA}{kR}IR$

$$= \frac{BNA}{kR}V \text{ or } \theta \propto V$$

For an AC ammeter or voltmeter, a hot wire deflection instrument is used.

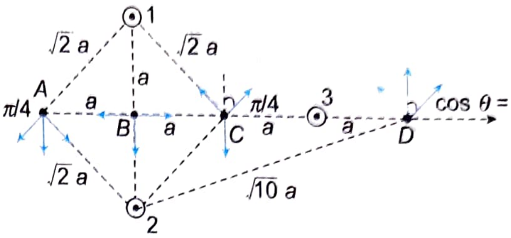
$\therefore \quad \theta = kI^2R$

$\Rightarrow \quad \theta \propto I^2$ and $\theta = k\frac{V^2}{R}$

$\Rightarrow \quad \theta \propto V^2$

Hence, (i) → (s), (ii) → (r), (iii) → (q), (iv) → (p)

8. **(3)** For A, $B_A = 2.\left(\frac{\mu_0 i}{2\pi.\sqrt{2}\,a}\right)\cos\frac{\pi}{4} + \frac{\mu_0 i}{2\pi.3a} = \frac{4}{3}\frac{\mu_0 i}{2\pi a}$



At B, $B_B = \frac{1}{2}\frac{\mu_0 i}{2\pi a}$

At C, $B_C = 2.\left(\frac{\mu_0 i}{2\pi.\sqrt{2}\,a}\right)\cos\frac{\pi}{4} - \frac{\mu_0 i}{2\pi a} = 0$

At D, $B_D = 2.\left(\frac{\mu_0 i}{2\pi.\sqrt{10}\,a}\right)\frac{3}{\sqrt{10}} + \frac{\mu_0 i}{2\pi.a}$

$$= \left(\frac{3}{5}+1\right)\frac{\mu_0 i}{2\pi a} = \frac{8}{5}\frac{\mu_0 i}{2\pi a}$$

Numerical Value Type

1. **(4)** When the coils are connected in parallel, the ratio of currents flowing in these coils will be equal to reciprocal of the ratio of their resistances.

$$\therefore \quad \frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{\rho(l_2/A)}{\rho(l_1/A)}$$

(as both the coils are made of similar wires)

$$\Rightarrow \quad \frac{\ell_2}{\ell_1} = \frac{2\pi r_2}{2\pi r_1} = \frac{r_2}{r_1} \quad \therefore \quad \frac{B_1}{B_2} = \frac{(\mu_0 i_1)/2r_1}{(\mu_0 i_2)/2r_2}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{i_1}{i_2} \times \frac{r_2}{r_1}\left[\because \quad \frac{i_1}{i_2} = \frac{r_2}{r_1}\right]$$

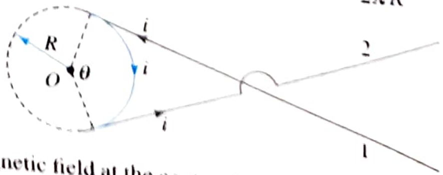
$$\therefore \quad \frac{B_1}{B_2} = \frac{r_2^2}{r_1^2} = \left(\frac{40}{20}\right)^2 = \frac{4}{1} = 4$$

2. **(2)** Magnetic field at O due to wire 1 is,

$$\vec{B}_1 = \frac{\mu_0 i}{4\pi R} \odot$$

and that due to wire 2 is

$$\vec{B}_2 = \frac{\mu_0 i}{4\pi R} \odot \Rightarrow \vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 i}{2\pi R} \odot$$



Magnetic field at the centre due to circular arc subtending angle θ at the centre is given by

$$B_{arc} = \frac{\mu_0 i}{2R}\left(\frac{\theta}{2\pi}\right) \otimes$$

If $\vec{B}_{\text{net}} = 0 \Rightarrow \vec{B} + \vec{B}_{\text{arc}} = 0$

$$\Rightarrow \vec{B} = -\vec{B}_{\text{arc}}$$

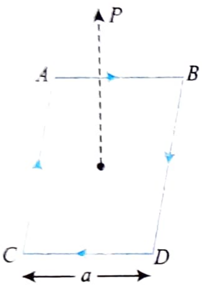
$$\Rightarrow \frac{\mu_0 i}{2\pi R} \odot = -\frac{\mu_0 i}{2R}\left(\frac{\theta}{2\pi}\right) \otimes$$

$$\Rightarrow \frac{\mu_0 i}{2\pi R} = \frac{\mu_0 i}{2R}\left(\frac{\theta}{2\pi}\right) \Rightarrow \frac{1}{\pi} = \frac{\theta}{2\pi} \Rightarrow \theta = 2 \text{ rad}$$

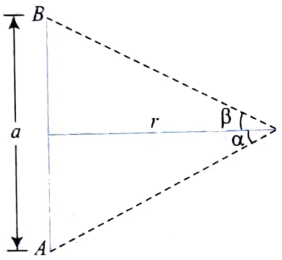
3. (2) $B_y = \frac{\mu_0}{4\pi} \frac{2\pi I (2r)^2}{[(2r)^2 + (2d)^2]^{3/2}}$, $B_x = \frac{\mu_0}{4\pi} \frac{2\pi I r^2}{[r^2 + d^2]^{3/2}}$

solve to get $\frac{B_x}{B_y} = 2$

4. (9) Axis of the loop means a line passing through centre of loop and normal to its plane. Since distance of the point P is x from centre of loop and side of square loop is a as shown in Fig. (a).



(a)



(b)

Therefore, perpendicular distance of P from each side of the loop is, $r = \sqrt{x^2 + \left(\frac{a}{2}\right)^2} = 4$ cm

Now, consider only one side AB of the loop as shown in Fig. (b).

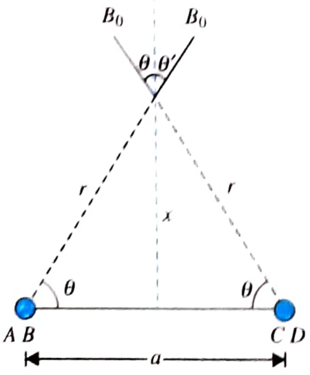
$$\tan\alpha = \tan\beta = \frac{(a/2)}{r} = \frac{3}{4}$$

$$\alpha = \beta = \tan^{-1}\left(\frac{3}{4}\right) = 37°$$

Magnitude of magnetic induction at P, due to current in this side AB, is

$$B_0 = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta) = 3 \times 10^{-6} \text{ T}$$

Now, consider magnetic inductions, produced by currents in two opposite sides AB and CD as shown in figure.



Components of these magnetic inductions, parallel to plane of loop neutralise each other. Hence, resultant of these two magnetic inductions is $2B_0 \cos\theta$ (along the axis).

Similarly, resultant of magnetic inductions produced by currents in remaining two opposite sides BC and AD will also be equal to $2B_0 \cos\theta$ (along the axis in same direction).

Hence, resultant magnetic induction,

$$B = 4B_0 \cos\theta$$

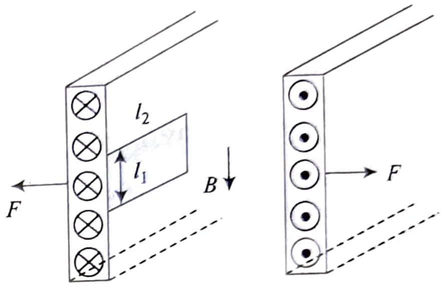
$$B = 4 \times (3 \times 10^{-6}) \frac{(a/2)}{r} = 9 \times 10^{-6} \text{ T} = 9\ \mu\text{T}$$

5. (6) According to Biot–Savart law,

$$B = \frac{\mu_0}{4\pi} \frac{qv}{r^2} \sin\theta = \frac{10^{-7} \times 0.5 \times 5 \times 10^3}{5^2}\left(\frac{3}{5}\right) = 6\ \mu\text{T}$$

6. (2) Let J be the current per unit length.

$$B = 2\left[\frac{\mu_0 J}{2}\right] = \mu_0 J$$



Magnetic field due to one plane = $B/2$

Force on other plane $F = \frac{B}{2}(Jl_1)l_2$

This force is on the shaded part.

Force per unit area: $\frac{F}{l_1 l_2} = \frac{B}{2} J = \frac{B}{2} \frac{B}{\mu_0} = \frac{B^2}{2\mu_0}$

Hence $N = 2$.

7. (2) $10 \text{ mT} = \frac{\mu_0}{2\pi(r/2)}(i_1 - i_2)$

$$30 \text{ mT} = \frac{\mu_0}{2\pi(r/2)}(i_1 + i_2)$$

Dividing : $i_1/i_2 = 2$

8. (3) $B_1 = \frac{\mu_0 I}{4\pi R}$

$$B_2 = \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 (I/2)}{4\pi R} = \frac{3}{4} \frac{\mu_0 I}{4\pi R}$$

$$\frac{B_1}{B_2} = \frac{3}{4} \Rightarrow B_1 = \frac{3}{4} B_2 = \frac{3}{4} \times 4 = 3 \text{ mT}$$

9. (3) Repulsion by wire D, $F_1 = \frac{\mu_0}{2\pi} \frac{i_1 i_2 l}{r}$ [towards right]

$$= \frac{(2 \times 10^{-7})(30 \times 10)}{3 \times 10^{-2}}(0.25) = 5 \times 10^{-4} \text{ N}$$

Repulsion by wire G,

$$= F_2 = \frac{(2 \times 10^{-7})(20 \times 10)}{5 \times 10^{-2}}(0.25) = 2 \times 10^{-4} \text{ N}$$

$$\therefore F_{\text{net}} = F_1 - F_2 = 3 \times 10^{-4} \text{ N}$$

10. (16) The magnetic induction inside the tore of the toroid is given as

$$B = \frac{\mu_r \mu_0 NI}{2\pi R}$$

$$B=\frac{100\times 4\pi\times 10^{-7}\times 1000\times 2}{2\pi\times 0.25}$$

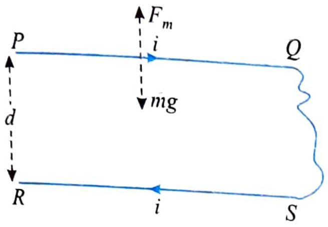
$$B=16\times 10^{-2}\,T$$

(50) In equilibrium magnetic force F_m will balance weight mg.

So $\quad mg=F_m \Rightarrow mg=\dfrac{\mu_0 i^2 l}{2\pi d}$

$$\Rightarrow \quad i=\sqrt{\frac{2\pi mgd}{\mu_0 l}}=\sqrt{\frac{2\pi\times 2.0\times 10^{-3}\times 10\times 0.50\times 10^{-2}}{4\pi\times 10^{-7}\times 20\times 10^{-2}}}$$

$$=\sqrt{2500}=50\,\text{A}$$

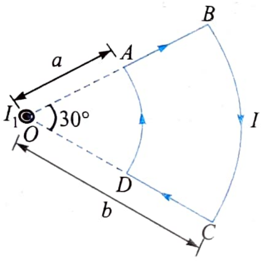


Archives

E Main

ngle Correct Answer Type

(2) The direction of current in the wires AB and CD pass through O, hence these wires do not contribute to the magnetic induction at O. The field due to wire DA is out of the paper and that due to wire BC is into the paper.



Let us take the direction out of the paper as positive and the direction into the paper as negative.

The total magnetic field due to loop $ABCD$ at O,

$$B=B_{AB}+B_{BC}+B_{CD}+B_{DA}$$

$$\Rightarrow \quad B=0-\frac{\mu_0 I}{4\pi b}\times\frac{\pi}{6}+0+\frac{\mu_0 I}{4\pi a}\times\frac{\pi}{6}$$

$$\Rightarrow \quad B=\frac{\mu_0 I}{24ab}(b-a)\text{, out of the paper or positive.}$$

2. (2) The forces on AD and BC are zero because the magnetic field due to a straight wire on AD and BC is parallel to the elementary length of the loop.

3. (1) The magnetic field in between because of each will be in the opposite direction.

$$B_{\text{in between}}=\frac{\mu_0 i}{2\pi x}\hat{j}-\frac{\mu_0 i}{2\pi(2d-x)}-(-\hat{j})$$

$$=\frac{\mu_0 i}{2\pi}\left[\frac{1}{x}-\frac{1}{2d-x}\right](\hat{j})$$

At $x=d$, $B_{\text{in between}}=0$

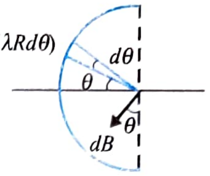
For $x<d$, $B_{\text{in between}}=\hat{j}$

For $x>d$, $B_{\text{in between}}=-\hat{j}$

Towards x, the net magnetic field will add up and direction will be $-\hat{j}$

Towards x', the net magnetic field will add up and direction will be $\hat{j}$

4. (1) Current per unit length: $\lambda=\dfrac{I}{\pi R}$

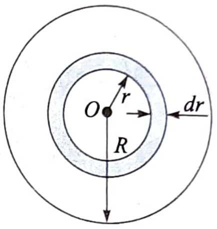


$$dB=\left(\frac{\mu_0}{2\pi}\right)\frac{dI}{R}, \qquad dI=\lambda R\,d\theta$$

$$\Rightarrow \quad B=\int_{-\pi/2}^{\pi/2} dB\cos\theta=\frac{\mu_0\lambda}{2\pi}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$=\frac{\mu_0\lambda}{\pi}=\frac{\mu_0 I}{\pi^2 R}$$

5. (1) Let us consider an elementary ring of radius R and a disc of thickness dr.



Charge on the ring, $dq=\dfrac{Q}{\pi R^2}(2\pi r dr)=\dfrac{2Qr}{R^2}dr$

Current due to rotation of charge on ring,

$$I=\frac{dq\omega}{2\pi}=\frac{Qr\omega dr}{\pi R^2}$$

Magnetic field at the centre due to elementary ring,

$$dB=\frac{\mu_0 I}{2r}=\frac{\mu_0 Qr\omega dr}{2\pi R^2 r}\frac{\mu_0 Q\omega dr}{2\pi R^2}$$

Magnetic field at the centre due to the complete disc,

$$B=\int dB=\frac{\mu_0 Q\omega}{2\pi R^2}\int_0^R dr=\frac{\mu_0 Q\omega R}{2\pi R^2}=\frac{\mu_0 Q\omega}{2\pi R}$$

Since, Q and ω are constants, $B\propto\dfrac{1}{R}$

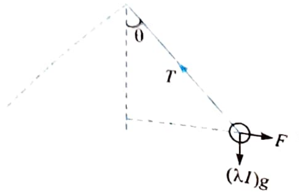
Hence, variation of B with R should be a rectangular hyperbola as represented in option (1).

6. (2) $T\cos\theta=\lambda gl$

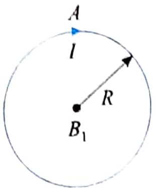
$$T\sin\theta=\frac{\mu_0}{2\pi}\cdot\frac{I\times Il}{(2L\sin\theta)} \qquad \text{...(i)}$$

...(ii)

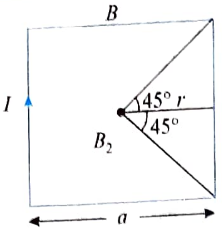
$$\Rightarrow \quad I=2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$$



7. (4)



$l = 2\pi R$



$l = 4a$

$$R = \frac{l}{2\pi} \qquad a = \frac{l}{4}$$

$$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2 \times \frac{l}{2\pi}} = \frac{\mu_0 \pi I}{l}$$

$$B_B = \frac{\mu_0 I}{4\pi r}[\sin\alpha + \sin\beta] \times 4$$

$$= \frac{\mu_0 I}{4\pi \frac{a}{2}}[\sin 45° + \sin 45°] \times 4$$

$$\Rightarrow B_B = 2\sqrt{2}\frac{\mu_0 I}{\pi a} = 2\sqrt{2}\frac{\mu_0 I}{\frac{\pi l}{4}} = 8\sqrt{2}\frac{\mu_0 I}{\pi l}$$

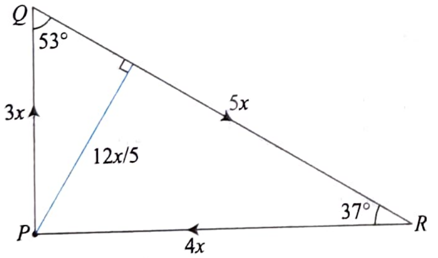
$$\frac{B_A}{B_B} = \frac{\frac{\mu_0 \pi I}{l}}{8\sqrt{2}\frac{\mu_0 I}{\pi l}} = \frac{\pi^2}{8\sqrt{2}}$$

JEE Advanced

Single Correct Answer Type

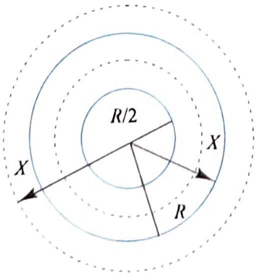
1. (3) $B = \dfrac{\mu_0 I}{4\pi \frac{12x}{5}}[\cos 53° + \cos 37°] = 7\left(\dfrac{\mu_0 I}{48\pi x}\right)$

$\therefore \quad k = 7$



2. (1) $B = \displaystyle\int \frac{\mu_0 (dN) I}{2x} = \int_a^b \frac{\mu_0\left(\frac{N}{b-a}dx\right)I}{2x} = \frac{\mu_0 N I}{2(b-a)} \ln \frac{b}{a}$

3. (4) **Case-I:** $x < \dfrac{R}{2}$



$|B| = 0$

Case-II: $\dfrac{R}{2} \le x < R$

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$|B| 2\pi x = \mu_0\left[\pi x^2 - \pi\left(\frac{R}{2}\right)^2\right]J$$

$$|B| = \frac{\mu_0 J}{2x}\left(x^2 - \frac{R^2}{4}\right)$$

Case-III: $x \ge R$

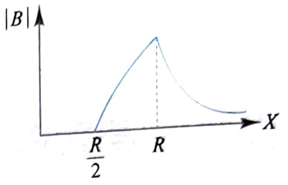
$$\int \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$|B| 2\pi x = \mu_0\left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right]J$$

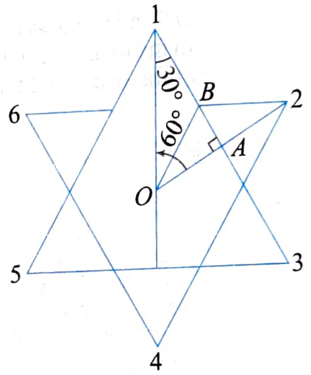
$$|B| = \frac{\mu_0 J}{2x}\frac{3}{4}R^2$$

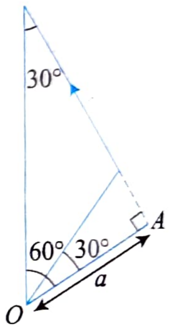
$$|B| = \frac{3\mu_0 J R^2}{8x}$$

so



4. (2) The given points (1, 2, 3, 4, 5, 6) make 360° angle at 'O'.





Hence angle made by vertices 1 and 2 with 'O' is 60°. Direction of magnetic field at 'O' due to each segment is same. Since it is a symmetric star shaped, magnitude will also be same.

Magnetic field due to section BC.

$$(B_1) = \frac{ki}{a}(\sin 60° - \sin 30°) = \frac{ki}{2a}(\sqrt{3} - 1)$$

$$B_{net} = 12 \times B_1 = \frac{6ki}{a}(\sqrt{3} - 1) \text{ and } \left(k = \frac{\mu_0}{4\pi}\right)$$

Multiple Correct Answers Type

1. (1), (4)

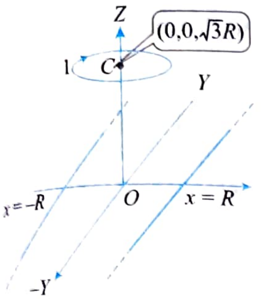
Due to field of solenoid is non zero in region $0 < r < R$ and non zero in region $r > 2R$ due to conductor.

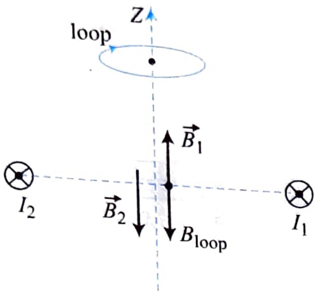
2. (1), (2), (4)

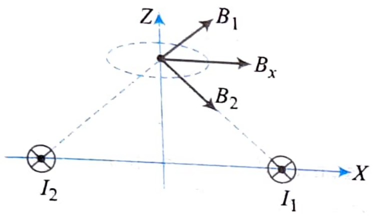
At origin, $\vec{B} = 0$ r due to two wires $I_1 = I_2$, if we consider current flowing towards positive y-direction. Hence $(\vec{B}_{net})$ or net $\vec{B}$ at origin is equal to B r due to ring, which is non-zero. Hence option (1) is correct.

If $I_1 > 0$ and $I_2 < 0$, $\vec{B}$ at origin due to wires will be along $+\hat{k}$ direction and $\vec{B}$ due to ring is along $-\hat{k}$ direction and hence $\vec{B}$ can be zero at origin. Hence option (2) is correct.

If $I_1 < 0$ and $I_2 > 0$, $\vec{B}$ at origin due to wires is along $-\hat{k}$ and also along $-\hat{k}$ due to ring, hence $\vec{B}$ cannot be zero. Hence option (3) is not correct.







At centre of ring, $\vec{B}$ due to wires is along x-axis, hence z-component is only because of ring

$$\vec{B} = \frac{\mu_0 i}{2R}(-\hat{k})$$

Hence option (4) is correct.

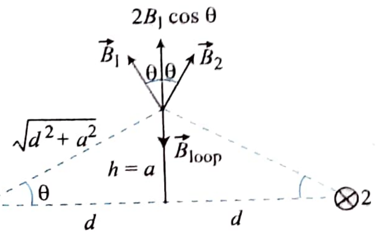
nked Comprehension Type

(3) The net magnetic field at the given point will be zero if. $|\vec{B}_{\text{wires}}| = |\vec{B}_{\text{loops}}|$

The direction of magnetic field at the given point due to the loop is normally out of the plane. Therefore, the net magnetic field due the both wires should be into the plane. For this current in wire I should be along PQ and that in wire RS should be along SR.

$$B_{\text{loop}} = \frac{\mu_0}{2}\frac{ia^2}{(h^2+a^2)^{3/2}}$$

$$B_{\text{wires}} = 2 \times \frac{\mu_0 i}{2\pi\sqrt{d^2+h^2}}\cos\theta$$



$$\therefore \quad \frac{\mu_0}{2}\cdot\frac{ia^2}{(h^2+a^2)^{3/2}} = \frac{2\mu_0 i}{2\pi}\frac{d}{d^2+h^2}$$

$$\Rightarrow \quad \frac{a^2}{2(h^2+a^2)^{3/2}} = \frac{a}{\pi(a^2+h^2)} \Rightarrow \pi a = 2\sqrt{h^2+a^2}$$

$$\Rightarrow \quad a = \frac{2h}{\sqrt{(\pi^2-4)}} \Rightarrow h \simeq 1.2a$$

2. (2) $B = 2\left(\dfrac{\mu_0 i}{2\pi d}\right)$

$$\therefore \quad \tau = i\pi a^2\frac{\mu_0}{\pi d}\sin(30°) = \frac{\mu_0 I^2 a}{2d}$$

3. (1), (4) $qVB = Eq$

$E = VB \qquad i = n_0 eAVd$

$$E = \frac{i}{n_0 eA}\cdot B$$

$$E = \frac{iB}{n_0 e(Wd)}$$

Potential difference : $V_{KM} = W.E.$

$$\therefore \quad V_1 = \frac{iB}{n_0 e}\left(\frac{1}{d_1}\right)$$

$$V_2 = \frac{iB}{n_0 e}\left(\frac{1}{d_2}\right)$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{d_2}{d_1}$$

Hence, (1) and (4).

4. (1), (3) As done in the above question

$$V_1 = \frac{iB}{n_1 e}\left(\frac{1}{d_1}\right) \text{ and } V_2 = \frac{iB}{n_2 e}\left(\frac{1}{d_2}\right)$$

In this case, $\dfrac{V_1}{V_2} = \dfrac{n_2 B_1}{n_1 B_2}$

Hence (1) and (3).

Matrix Match Type

1. **i.** → **a., c., d.; ii.** → **c., d.; iii.** → **a., b., e.; iv.** → **c., d.**

Numerical Value Type

1. (5) $B_1 = \dfrac{\mu_0 J\pi a^2}{2\pi a} = \dfrac{\mu_0 Ja}{2}$, $B_2 = \dfrac{\mu_0 J\pi(a/2)^2}{2\pi(3a/2)} = \dfrac{\mu_0 Ja}{12}$

$$B_1 - B_2 = \frac{\mu_0 Ja}{2} - \frac{\mu_0 Ja}{12}$$

$$= \left(\frac{\mu_0 Ja}{2}\right)\left(1-\frac{1}{6}\right) = \frac{5}{6}\left(\frac{\mu_0 Ja}{2}\right) = \frac{5\mu_0 aJ}{12}$$

$$= \frac{N}{12}\mu_0 aJ$$

$\therefore \; N = 5$

Chapter 3

Concept Application Exercises

Exercise 3.1

1. Here, $F = 5\text{ gf} = 5\times10^{-3}\text{ kgf} = 5\times10^{-3}\times9.8\text{ N}$; $r = 10\text{ cm} = 0.1\text{ m}$

Let m and $4m$ be the strengths of two poles.

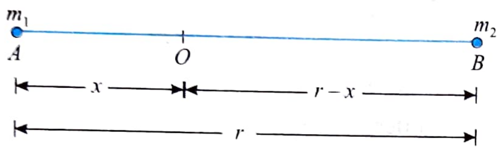
Now, $F = \frac{\mu_0}{4\pi}\cdot\frac{m\times4m}{r^2}$

Or $5\times10^{-3}\times9.8 = \frac{10^{-7}\times m\times4m}{(0.1)^2}$

Or $m = 35\text{ Am}$

and $4m = 4\times35 = 140\text{ Am}$

2. Here, $m_1 = 25$ Am; $m_2 = 64$ Am and $r = 1.0$ m



Let the magnetic field be zero at the point O, such that $AO = x$.

Then, $\frac{\mu_0}{4\pi}\cdot\frac{m_1}{x^2} = \frac{\mu_0}{4\pi}\cdot\frac{m_2}{(r-x)^2}$

Or $\frac{25}{x^2} = \frac{64}{(1.0-x)^2}$

Or $64x^2 = 25\,(1.0-x)^2$

or $8x = \pm5(1.0-x)$

or $8x = 5(1.0-x)$ or $-5(1.0-x)$

or $x = 0.385\text{m}$ or -1.667 m

3. Since lengths of both the magnets are quite large as compared to separation between them, the interaction is effective only between their N-poles.

Let m be the strength of each pole of the two magnets. Then,

$$\frac{\mu_0}{4\pi}\cdot\frac{m\times m}{r^2} = \text{weight of the magnet}$$

Here, weight of the magnet $= 50\text{ gf} = 50\times10^{-3}\text{kgf}$
$= 50\times10^{-3}\times9.8\text{ N}$

and $r = 3\text{ mm} = 3\times10^{-3}\text{ m}$

$\therefore\quad \frac{10^{-7}\times m^2}{(3\times10^{-3})^2} = 50\times10^{-3}\times9.8$

or $m = 6.64\text{ Am}$

4. Period of revolution of electron,

$$T = \frac{2\pi r}{v} = \frac{2\pi\times0.53\times10^{-10}}{2.3\times10^6} = 1.448\times10^{-16}\text{ s}$$

The orbital motion of electron is equivalent to current,

$$I = \frac{e}{T} = \frac{1.6\times10^{-19}}{1.448\times10^{-16}} = 1.105\times10^{-3}\text{ A}$$

Therefore, magnetic moment of the revolving electron,

$$M = IA = I\times\pi r^2 = 1.105\times10^{-3}\times\pi\times(0.53\times10^{-10})^2$$

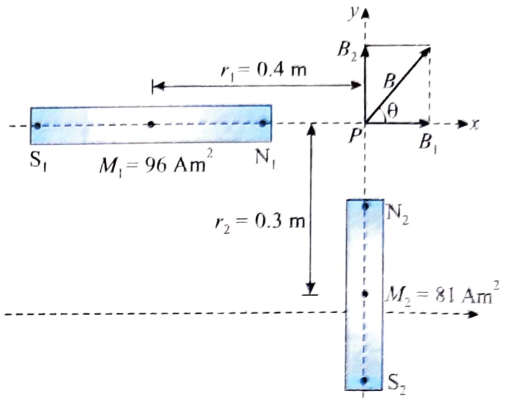
$$= 9.75\times10^{-24}\text{ Am}^2$$

5. $M_1 = 4\times0.21 = 0.84\text{ Am}^2$ and $M_2 = 7\times0.12 = 0.84\text{ Am}^2$

Let M be the magnetic moment of the combination of two magnets placed at right angles to each other. Then,

$$M = \sqrt{M_1^2+M_2^2} = \sqrt{(0.84)^2+(0.84)^2} = 1.19\text{ Am}^2$$

6. The two magnets are placed as shown in figure.



The point P lies on axial line of both the magnets.

Let B_1 and B_2 be the magnetic fields at the point P due to the magnets N_1S_1 and N_2S_2 respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2M_1}{r_1^3} = \frac{10^{-7}\times2\times96}{(0.4)^3}$$

$$= 3\times10^{-4}\text{ T} \qquad \text{(along } PX\text{)}$$

and $B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2M_2}{r_2^3} = \frac{10^{-7}\times2\times81}{(0.3)^3}$

$$= 6\times10^{-4}\text{ T} \qquad \text{(along } PY\text{)}$$

Therefore, resultant magnetic field at the point P,

$$B = \sqrt{B_1^2+B_2^2} = \sqrt{(3\times10^{-4})^2+(6\times10^{-4})^2}$$

$$= 3\sqrt{5}\times10^{-4}\text{ T}$$

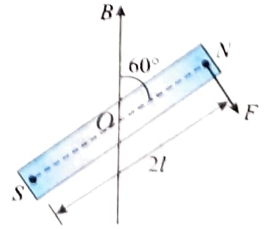
If the direction of B makes an angle θ with the X-axis (or N_1S_1) then

$$\tan\theta = \frac{B_2}{B_1} = \frac{6\times10^{-4}}{3\times10^{-4}} = 2$$

or $\theta = \tan^{-1}2$

7. Here, $m = 14.4$ Am; $B = 0.25$ T; $2l = 25\text{ cm} = 0.25$ m

The magnet is held in equilibrium at 60° to the direction of the magnetic field B by applying force F as shown in figure.



For the equilibrium of the magnet,

$$F\times ON = MB\sin\theta$$

Or $F\times l = (m\times2l)\,B\sin60^\circ$

Or $F = 2mB\sin60^\circ$

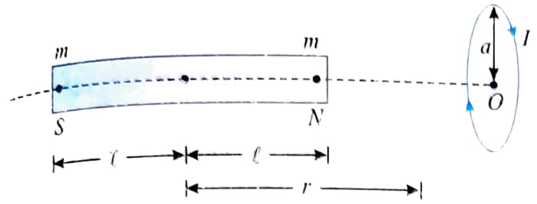
$$= 2\times14.4\times0.25\times\sin60^\circ = 6.24\text{ N}$$

If the force is removed, magnet will tend to align itself along the magnetic field.

The circular coil carrying current is placed on the axis of the magnet as shown in figure.

Here, $a = 0.002$ m, $I = 2$ A; $2l = 0.1$ m; $r = 0.15$ m

and $M = 10^5\ \mathrm{JT^{-1}}$



Therefore, pole strength of the magnet,

$$m = \frac{M}{2l} = \frac{10^5}{0.1} = 10^6\ \mathrm{Am}$$

Magnetic field at a distance x from the centre of a current carrying coil of radius a,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I a^2}{x^3} \qquad (a << x)$$

If B_1 and B_2 are magnetic fields due to the coil at the north and south poles of the magnet, then force on the magnet due to the coil,

$$F = m\,B_2 - m\,B_1$$

According to Newton's third law, magnet will exert an equal and opposite force on the coil. Therefore, force on the coil due to the magnet,

$$F = m\,B_1 - m\,B_2 = m(B_1 - B_2)$$

$$= m\left[\frac{\mu_0}{4\pi}\cdot\frac{2\pi\,Ia^2}{(2-l)^3} - \frac{\mu_0}{4\pi}\cdot\frac{2\mu I\,a^2}{(r+l)^3}\right]$$

$$= \frac{\mu_0}{4\pi}\cdot 2\pi I a^2\, m\left[\frac{1}{(r-l)^3} - \frac{1}{(r+l)^3}\right]$$

On substituting the corresponding values in above expression, we get, $F = 4.4\times10^{-3}$ N

ercise 3.2

1. Here, $B_H = \sqrt{3}\,B_V$

Now, $\delta = \dfrac{B_V}{B_H} = \dfrac{B_V}{\sqrt{3}\,B_V} = \dfrac{1}{\sqrt{3}}$

Or $\delta = 30°$

2. Here, $\delta = 60°$; $B_V = 0.3\times10^{-4}$ T

Now, $B_H = B\cos\delta$

Or $B = \dfrac{B_H}{\cos\delta} = \dfrac{0.3\times10^{-4}}{\cos 60°} = \dfrac{0.3\times10^{-4}}{0.5}$

$= 0.6\times10^{-4}$ T

3. At 52° of magnetic meridian, the apparent value of the horizontal component of earth's magnetic field,

$$B_H' = B_H\cos 52°$$

Apparent angle of dip δ′ (= 45°) is given by

$$\tan\delta' = \frac{B_V}{B_H'}$$

Or $\tan 45° = \dfrac{B_V}{B_H\cos 52°}$

Or $\dfrac{B_V}{B_H} = \tan 45° \times \cos 52° = 1\times0.6157 = 0.6157$

But $\dfrac{B_V}{B_H} = \tan\delta$

where δ is the true angle of dip.

Or $\delta = \tan^{-1}0.6157$

4. **In first case:** At the neutral point on axial line of a short magnet,

$$\frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3} = B_H$$

Or $\dfrac{10^{-7}\times2\times5}{r^3} = 0.38\times10^{-4}$

Or $r^3 = 0.0263$

Or $r = 29.7\times10^{-2}\ \mathrm{m} = 29.7$ cm

In second case: At the neutral point on equatorial line of a short magnet,

$$\frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = B_H$$

Or $\dfrac{10^{-7}\times5}{r^3} = 0.38\times10^{-4}$

Or $r^3 = 0.\,0132$

Or $r = 23.6\times10^{-2}\ \mathrm{m} = 23.6$ cm

5. At the neutral point on equatorial line of a short magnet.

$$\frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = B_H$$

Or $\dfrac{10^{-7}\times M}{(12.5\times10^{-2})^3} = 0.38\times10^{-4}$

Or $M = \dfrac{0.38\times10^{-4}\times(12.5\times10^{-2})^3}{10^{-7}} = 0.74$

(a) Now $B_{\text{axial}} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2M}{r^3} = \dfrac{10^{-7}\times2\times0.74}{(12.5\times10^{-2})^3}$

$= 0.76\times10^{-4}\ \mathrm{T} = 0.76$ G

Since at a point on the axial line, B_H and B_{axial} are in the same direction, total magnetic field,

$$B = B_{\text{axial}} + B_H = 0.76 + 0.38 = 1.14\ \mathrm{G}$$

(b) When the magnet is turned through 180°, neutral points will be obtained on axial line.

Now, at the neutral point on axial line of a short magnet,

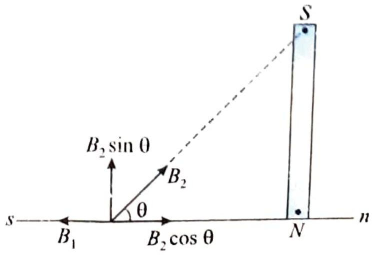
$$\frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3} = B_H$$

Or $\dfrac{10^{-7}\times2\times0.74}{r^3} = 0.38\times10^{-4}$

Or $r^3 = 3.895\times10^{-3}$

Or $r = 0.157\,\mathrm{m} = 15.7$ cm

6. The neutral point will be obtained at the point P_2 on the horizontal table as shown in figure.



At the neutral point, the horizontal component of earth's magnetic field (B_H) is balanced by the horizontal component of the magnetic field due to the magnet. Let m (= 1 Am) be the pole strength of the magnet. Magnetic field due to N-pole of the magnet at the neutral point P_2,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{m}{(NP_2)^2} = \frac{10^{-7} \times 1}{(0.06)^2}$$

$$= 2.78 \times 10^{-5}\ \text{T} \qquad \text{(along } P_2S\text{)}$$

Magnetic field due to S-pole of the magnet at the neutral point P_2,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m}{(SP_2)^2}$$

Now, $SP_2 = \sqrt{(NS)^2 + (NP_2)^2}$

$$= \sqrt{(0.08)^2 + (0.06)^2} = 0.1\ \text{m}$$

$$\therefore \quad B_2 = \frac{10^{-7} \times 1}{(0.1)^2} = 10^{-5}\ \text{T} \qquad \text{(along } P_2S\text{)}$$

The magnetic field B_2 makes an angle θ with the horizontal table as shown in the figure.
Therefore, component of field B_2 along the horizontal

$$= B_2 \cos\theta = B_2 \times \frac{NP_2}{SP_2} = 10^{-5} \times \frac{0.06}{0.1}$$

$$= 0.6 \times 10^{-5}\ \text{T} \qquad \text{(along } P_2N\text{)}$$

Therefore, net horizontal component of the field of magnet

$$= B_1 - B_2 \cos\theta = 2.78 \times 10^{-5} - 0.6 \times 10^{-5}$$

$$= 2.18 \times 10^{-5}\ \text{T} \qquad \text{(along } P_2S\text{)}$$

At the neutral point,

$$B_H = B_1 - B_2 \cos\theta = 2.18 \times 10^{-5}\ \text{T}$$

Exercise 3.3

1. If V is volume of the magnet, then $I = \frac{M}{V}$.

Now, $V = \frac{\text{mass}}{\text{density}} = \frac{75 \times 10^{-3}}{7.5 \times 10^3} = 10^{-5}\ \text{m}^3$

$$\therefore \quad I = \frac{2 \times 10^{-4}}{10^{-5}} = 20\ \text{Am}^{-1}$$

2. Here, $B = 1.0\ \text{Wb m}^{-2}$; $H = 2 \times 10^3\ \text{Am}^{-1}$

(a) Magnetic permeability,

$$\mu = \frac{B}{H} = \frac{1.0}{2 \times 10^3} = 5 \times 10^{-4}\ \text{TmA}^{-1}$$

(b) Relative permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} = 397.9$$

(c) Magnetic susceptibility,

$$\chi_m = \mu_r - 1 = 396.9 - 1 = 396.9$$

(d) Intensity of magnetization,

$$I = \chi_m H = 396.9 \times 2 \times 10^3 = 7.94 \times 10^5\ \text{A m}^{-1}$$

3. Here, $H = 1600\ \text{A m}^{-1}$; $\phi = 2.4 \times 10^{-5}\ \text{Wb}$,

$$A = 0.2\ \text{cm}^2 = 0.2 \times 10^{-4}\ \text{m}^2$$

Now, magnetic induction,

$$B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-5}} = 1.2\ \text{T}$$

Also, $\mu = \frac{B}{H} = \frac{1.2}{1600} = 7.5 \times 10^{-4}\ \text{TmA}^{-1}$

Therefore, relative permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{7.5 \times 10^{-4}}{4\pi \times 10^{-7}} = 596.8$$

Again, $B = \mu_0(H + I)$

$$\therefore \quad I = \frac{B}{\mu_0} - H = \frac{1.2}{4\pi \times 10^{-7}} - 1600$$

$$= 9.55 \times 10^5 - 1600 = 9.534 \times 10^5\ \text{Am}^{-1}$$

Finally, $\chi_m = \mu_r - 1 = 596.8 - 1 = 595.8$

4. Mean radius of toroid,

$$r = \frac{11 + 12}{2} = 11.5\ \text{cm} = 11.5 \times 10^{-2}\ \text{m}$$

Therefore, number of turns per unit length of the toroid,

$$n = \frac{\text{total number of turns}}{2\pi r} = \frac{3000}{2\pi \times 11.5 \times 10^{-2}}$$

$$= 4.152 \times 10^3\ \text{m}^{-1}$$

Now, magnetic field in the core of solenoid,

$$B = \mu_0 \mu_r n I$$

$$\text{Or} \quad \mu_r = \frac{B}{\mu_0 n I}$$

$$\text{Or} \quad \mu_r = \frac{2.5}{4\pi \times 10^{-7} \times 4.152 \times 10^3 \times 0.70} = 684.5$$

5. Here, $n = 10^3\ \text{m}^{-1}$; $I = 2\ \text{A}$; $B = 1.5\ \text{Wbm}^{-2}$

(a) Magnetic induction due to the solenoid,

$$B = \mu_0 \mu_r n I$$

$$\text{Or} \quad \mu_r = \frac{B}{\mu_0 n I}$$

$$\text{Or} \quad \mu_r = \frac{1.5}{4\pi \times 10^{-7} \times 10^3 \times 2} = 596.8$$

(b) $\chi_m = \mu_r - 1 = 596.8 - 1 = 595.8$

(c) $H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r} = \frac{1.5}{4\pi \times 10^{-7} \times 596.8}$

$$= 2000\ \text{Am}^{-1}$$

(d) $I = \chi_m H = 595.8 \times 2000 = 1.192 \times 10^6\ \text{Am}^{-1}$

6. According to Curie's law

$$\chi_m = \frac{C}{T}$$

$$\therefore \quad \frac{\chi_m}{\chi_m'} = \frac{T'}{T}$$

$$\text{Or} \quad T' = T \times \frac{\chi_m}{\chi_m'} = \frac{300 \times 1.2 \times 10^{-5}}{1.44 \times 10^{-5}} = 250\ \text{K}$$

7. Here, hysteresis loss per unit volume per cycle

$$= 300\ \text{Jm}^{-3}\ \text{cycle}^{-1}$$

Time, t = 1 hour = 3,600 s, frequency, f = 50 cycles s^{-1}
Therefore, number of hysteresis cycle in 1 hour,

$$N = f \times t = 50 \times 3600 = 1.8 \times 10^5$$

Volume of the iron specimen,

$$V = \frac{\text{mass}}{\text{density}} = \frac{12}{7500} = 1.60 \times 10^{-3}\ \text{m}^3$$

Hence, loss of energy in one hour,

W = hysteresis loss per unit volume per cycle × $V \times N$

$$= 300 \times 1.6 \times 10^{-3} \times 1.8 \times 10^{5} = 8.64 \times 10^{4}\ \text{J}$$

8. (a) Here, number of atoms per unit volume in iron,

$$n = 8.52 \times 10^{28}\ \text{m}^{-3}$$

Magnetic moment of each iron atom $= 2\mu_B$

$$= 2 \times 9.27 \times 10^{-24} = 1.854 \times 10^{-23}\ \text{Am}^2$$

Now, intensity of magnetization is equal to magnetic moment per unit volume.

Therefore, the maximum value of magnetization,

$$I_{\max} = n \times 2\mu_B = 8.52 \times 10^{28} \times 1.854 \times 10^{-23}$$

$$= 1.58 \times 10^{6}\ \text{Am}^{-1}$$

(b) Now, $B = \mu_0 (H + I)$

Since no magnetic field is applied, $H = 0$

$$\therefore \quad B = \mu_0 I = 4\pi \times 10^{-7} \times 1.58 \times 10^{6} = 1.985\ \text{T}$$

Exercises

Single Correct Answer Type

1. **(3)** Pole strength becomes half due to change in breadth. Change in length does not affect pole strength.

2. **(4)** New magnetic moment,

$$M' = \frac{2M\sin(\theta/2)}{\theta} = \frac{2M\sin(\pi/6)}{\theta/3} = \frac{2M \times \frac{1}{2}}{\pi/3} = \frac{3M}{\pi}$$

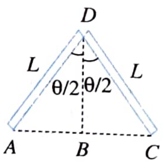
3. **(4)** When the magnet is bent, the length of the magnet is:

$$AC = AB + BC$$

$$= L\sin\frac{\theta}{2} + L\sin\frac{\theta}{2}$$

$$= 2L\sin(\theta/2) = 2L\sin(60°/2)$$

$$= 2L\ \sin 30° = L$$



4. **(1)** All the magnitude moment vectors can be represented both in magnitude and direction by the three sides of a triangle taken in the same order.

5. **(3)** The hysteresis loss for soft iron is low. On the other hand, hysteresis loss for steel is high.

6. **(2)** $\tau = MB\sin\theta$

τ and B are constants.

Now, $\sqrt{3}\sin\theta = 1 \times \sin(90° - \theta)$

or $\tan\theta = \dfrac{1}{\sqrt{3}}$ or $\theta = 30°$.

7. **(2)** $\tau = MB\sin\theta$

$$\tau = \tau_0 \sin\theta$$

$$\frac{1}{2}\tau_0 = \tau_0\sin\theta \quad \text{or} \quad \sin\theta = \frac{1}{2} \quad \text{or} \quad \theta = 30°$$

8. **(3)** $\tau = (m \times 2l)\,B\sin\theta$

$$= 48 \times 25 \times 10^{-2} \times 0.15 \times \sin 30°\ \text{N-m}$$

$$= 0.9\ \text{N-m.}$$

9. **(3)** $B_1 = \dfrac{\mu_0}{4\pi}\dfrac{2M \times 10}{(10^2 - l^2)^2}$

$$B_2 = \frac{\mu_0}{4\pi}\frac{2M \times 20}{(20^2 - l^2)^2}$$

$$\frac{B_1}{B_2} = \frac{10}{(10^2 - l^2)^2} \times \frac{(20^2 - l^2)^2}{20}$$

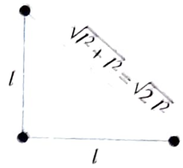
or $\dfrac{125}{10} = \dfrac{(20^2 - l^2)^2}{2(10^2 - l^2)^2}$, on solving we get

or $l = 5$ cm

Length $= 2l = 10$ cm.

10. **(3)** Here, distance between two poles $= \sqrt{2}l$

Hence, $M = m \times \sqrt{2}l = \sqrt{2}\,ml$



11. **(4)** $W = mB\,[\cos 0° - \cos 60°] = \dfrac{mB}{2}$

$$\tau = mB\sin 60°,\ \tau = mB \times \frac{\sqrt{3}}{2} = W \times \sqrt{3}$$

12. **(2)** $\tau = MH\sin\theta$

$$\therefore \quad \sin\theta = \frac{\tau}{MH} = \frac{1}{2} \quad \therefore \quad \theta = 30°$$

13. **(1)** $\tau = MB\sin 90° = 1.42$ N-m

14. **(4)** $F = \dfrac{\mu_0}{4\pi}\cdot\dfrac{6M_1M_2}{r^4}$

i.e., $\propto \dfrac{1}{r^4}$ $\quad\therefore\ F' = \dfrac{F}{16} = \dfrac{8}{16} = 0.5$ N

15. **(4)** $W = MB(\cos\theta_1 - \cos\theta_2)$

When the magnet is rotated from 0° to 60°, then work done is 0.8 J.

$$0.8 = MB(\cos 0° - \cos 60°) = \frac{MB}{2}$$

$$MB = 1.6\ \text{N-m}$$

In order to rotate the magnet through an angle of 30°, i.e., from 60° to 90°, the work done is

$$W' = MB(\cos 60° - \cos 90°)$$

$$= MB\left(\frac{1}{2} - 0\right) = \frac{MB}{2} = \frac{1.6}{2} = 0.8\ \text{J} = 0.8 \times 10^7\ \text{ergs}$$

16. **(2)** $2 = 2\pi\sqrt{\dfrac{I}{MH}}$

$$1 = 2\pi\sqrt{\frac{I}{M(F-H)}}$$

Dividing, $\dfrac{2}{1} = \sqrt{\dfrac{F-H}{H}} \Rightarrow \dfrac{H}{F} = \dfrac{1}{3}$

17. **(1)** $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{MB_H}{I}}$

$$\nu' = \frac{1}{2\pi}\sqrt{\frac{M(B_H + B)}{I}}$$

$$\nu'' = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$$

$$\left(\frac{\nu'}{\nu}\right) = \frac{B_H + B}{B_H} = \frac{14 \times 14}{10 \times 10} = \frac{B_H + B}{B_H}$$

or $\quad 1.96\,B_H = B_H + B \quad$ or $\quad 0.96\,B_H = B$

Now $\left(\dfrac{v''}{v}\right)^2 = \dfrac{B_H - B}{B_H} \quad$ or $\quad \dfrac{(v'')}{100} = \dfrac{B_H - 0.96\,B_H}{B_H}$

or $\quad (v'')^2 = 4$ or $v'' = 2$

18. (1) $\tau = MB \sin\theta$

or $\quad \sin\theta = \dfrac{\tau}{MB} = \dfrac{1.2\times10^{-3}}{60\times40\times10^{-6}} = 0.5$

or $\quad \theta = 30°$

19. (2) $T = 2\pi\sqrt{\dfrac{I_1+I_2}{(2M+M)H}} = 3 \qquad \text{...(i)}$

$T' = 2\pi\sqrt{\dfrac{I_1+I_2}{(2M-M)H}} \qquad \text{...(ii)}$

$\dfrac{T'}{T} = \sqrt{3}$ or $T' = 3\sqrt{3}$ s

20. (3) In the first case, it is vibrating in the total field of earth $n \propto \sqrt{B}$. In the second case, it is vibrating in vertical component only as the horizontal component is zero.

$\therefore \quad n' \propto \sqrt{V}$ but $V < B \quad \therefore\ n' < n.$

21. (1) $n \propto \sqrt{H}$ or $\dfrac{9}{16} = \dfrac{H}{35\times10^{-5}}$

or $\quad H = \dfrac{31.5\times10^{-5}\times9}{16} = 1.98\times10^{-5}\text{ T}$

22. (2) Here, $\dfrac{M_1}{M_2} = \dfrac{T_2^2+T_1^2}{T_2^2-T_1^2} = \dfrac{n_1^2+n_2^2}{n_1^2-n_2^2} = \dfrac{400+225}{400-225} = \dfrac{625}{175} = \dfrac{25}{7}$

23. (2) Since $T \propto \sqrt{I}$ and $I \propto m$, when mass is made 4 times, the time period becomes $2T$.

24. (1) $T = 2\pi\sqrt{\dfrac{I}{MB}}$

$T_1 = 2\pi\sqrt{\dfrac{I_1}{M_1 B}} \qquad \left(\because I_1 = \dfrac{I}{4} \text{ and } M_1 = \dfrac{M}{2}\right)$

$\therefore \quad \dfrac{T_1}{T_2} = \dfrac{1}{\sqrt{2}} \Rightarrow T_1 = \dfrac{T_0}{\sqrt{2}}$

25. (2) $T = 2\pi\sqrt{\dfrac{I}{MB\cos\delta}}$

$v = \dfrac{1}{2\pi}\sqrt{\dfrac{MB\cos\delta}{I}}$

$v = \sqrt{B\cos\delta}$

or $\quad B \propto \dfrac{v^2}{\cos\delta}$

$\dfrac{B_1}{B_2} = \dfrac{400}{\cos30°}\times\dfrac{\cos60°}{225} = \dfrac{16\times2}{9\times\sqrt{3}}\times\dfrac{1}{2} = 16:9\sqrt{3}$

26. (1) Theory based

27. (4) $B_0 = V_0$ also total intensity

$B = \sqrt{B_0^2+V_0^2} \Rightarrow B = \sqrt{2}B_0$

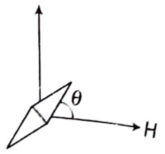
28. (1) Here, $\tan\delta = \dfrac{V}{H}$ and $\tan\delta' = \dfrac{V}{H\cos\theta}$

$\therefore \quad \dfrac{\tan\delta'}{\tan\delta} = \dfrac{V}{H\cos\theta}\times\dfrac{H}{V} = \dfrac{1}{\cos\theta}$

29. (1) In given case, H and H_0 are perpendicular to each other.

From figure $\tan\theta = \dfrac{H_0}{H}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{H_0}{H}\right)$



30. (1) Let the real dip be ϕ, then $\tan\varphi = \dfrac{B_V}{B_H}$.

For apparent dip,

$\tan\varphi' = \dfrac{B_V}{B_H\cos\beta} = \dfrac{B_V}{B_H\cos30°} = \dfrac{2B_V}{\sqrt{3}B_H}$

or $\quad \tan45° = \dfrac{2}{\sqrt{3}}\cdot\tan\varphi$ or $\varphi = \tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

31. (2) For a diamagnetic substance χ is small, negative and independent of temperature.

32. (1) Susceptibility of a paramagnetic substance is independent of magnetising field.

33. (1) Susceptibility of a ferromagnetic substance falls with rise of temperature $\left(\chi = \dfrac{C}{T-T_c}\right)$ and the substance becomes paramagnetic above Curie temperature, so magnetic susceptibility becomes very small above Curie temperature.

34. (1) $X = C\times\dfrac{1}{T} = \dfrac{0.4}{7\times10^{-3}} = 57$ K

35. (2) With rise in temperature their magnetic susceptibility decreases i.e.

$\chi_m \propto \dfrac{1}{T}$

36. (3) Diamagnetic substances are repelied by magnetic field.

37. (3) If the temperature is decreased, the thermal vibrations will be reduced. So, there would not be negative effect on magnetisation.

38. (2) By Curie's law, $\chi_m \propto \dfrac{1}{T}$

$0.5\,T = K\,(273+27)$ or $T = 600$ K

$\therefore \quad t = (600-273)°\text{C} = 327°\text{C}$

39. (2) Steel of an alloy Alnico (Al + Ni + Co) is used for making permanent magnets. Hysteresis loss is high. Moreover, coercivity is high and retentivity is low. Soft iron is used for electromagnets. Retentivity is high. Coercivity is low.

40. (4) $M_1B\sin30° \propto (180°-30°)$

and $\quad M_2B\sin30° \propto (270°-30°)$

$\therefore \quad \dfrac{M_1}{M_2} = \dfrac{150°}{240°} = \dfrac{5}{8}$.

41. (3) In equilibrium, $\tau_1 = \tau_2$

$MB\sin\theta = \sqrt{3}\,MB\sin(90°-\theta)$

$\therefore \quad \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

42. (3) In the given figure, two magnets are perpendicular to each other.

$B = \dfrac{\mu_0}{4\pi r^3}\sqrt{4+1} = \dfrac{\mu_0 M}{4\pi d^3}\sqrt{5}$

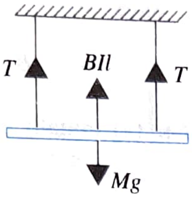
43. **(2)** $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 18}{2\pi \times 0.2}\,\text{T} = 18\ \mu\text{T}$

Now, $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$ and $T' = 2\pi\sqrt{\dfrac{I}{M(B_H - B)}}d$

Dividing $\dfrac{T'}{T} = \sqrt{\dfrac{B_H}{B_H - B}}$ or $\dfrac{T'}{T} = \sqrt{\dfrac{24}{24-18}} = 2$

$$T' = 2 \times 0.1\ \text{s} = 0.2\ \text{s}$$

44. **(4)** $2T + BIl = Mg$



or $\quad T = \dfrac{Mg - BIl}{2}$

45. **(1)** Here, $\tau = iAB \sin\theta$; $i = 0.1$ A

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

or $\quad A = \dfrac{1}{2} \times a \times \dfrac{a\sqrt{3}}{2} = \dfrac{\sqrt{3}\,a^2}{4} = \dfrac{\sqrt{3} \times (0.02)^2}{4}$

$$= \sqrt{3} \times 10^{-4}\ \text{m}^2\ ;\ \theta = 90°$$

$$\tau = 0.1 \times \sqrt{3} \times 10^{-4} \times 5 \times 10^{-2} \sin 90°$$

$$= 5\sqrt{3} \times 10^{-7}\ \text{N–m}$$

46. **(2)** As magnets are perpendicular to each other the resultant magnetic moment

$$= \sqrt{M^2 + M^2} = \sqrt{2}\,M$$

$\therefore \quad T_1 = 2\pi\sqrt{\dfrac{2I}{\sqrt{2}\,MH}}$ ...(i)

In the second case $T_2 = 2\pi\sqrt{\dfrac{I}{MH}}$

$$\frac{T_2}{T_1} = \frac{1}{(2)^{1/4}}$$

$\therefore \quad T_2 = \dfrac{4}{(2)^{1/4}} = 3.36$ s

47. **(2)** Since B and H are ⊥ to each other and the resultant field is inclined at an angle 45° with

So, $\quad B = H$

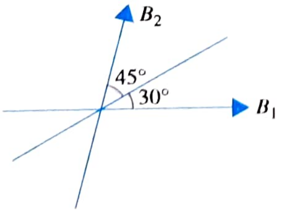
$$\frac{\mu_0}{4\pi}\frac{2M}{r^3} = H$$

$\therefore \quad r^3 = \dfrac{\mu_0}{4\pi}\dfrac{2M}{H} = 0.5$ m

48. **(2)** $B_1 = 1.5 \times 10^{-2}$ T

$B_2 = ?$

In equilibrium position,



$\tau_1 = \tau_2$

$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$

$1.5 \times 10^{-21} \times \sin 30°$

$= B_2 \times \sin 45°$

Then $B = \dfrac{1.5}{\sqrt{2}} \times 10^{-2}$ T

49. **(2)** $\tan\delta = \dfrac{B_V}{B_H}$ and $\tan\delta_1 = \dfrac{B_V}{B_H \cos\theta}$

or $\quad \tan\delta_1 = \dfrac{\tan\delta}{\cos\theta} = \tan\delta \sec\theta$

50. (1) $I = \dfrac{M}{V} = \dfrac{\mu N}{V} = \dfrac{1.5 \times 10^{-23} \times 2 \times 10^{26}}{1} = 3 \times 10^3$ A/m

51. (3) The number of atoms per unit volume in a specimen,

$$n = \frac{\rho N_A}{A}$$

For iron, $\rho = 7.8 \times 10^{-3}$ kg m^{-3},

$N_A = 6.02 \times 10^{26}$/ kg mol, $A = 56$

$\Rightarrow \quad n = \dfrac{7.8 \times 10^3 \times 6.02 \times 10^{26}}{56} = 8.38 \times 10^{28}\ \text{m}^{-3}$

Total number of atoms in the bar is

$N_0 = nV = 8.38 \times 10^{28} \times (5 \times 10^{-2} \times 1 \times 10^{-2} \times 1 \times 10^{-2})$

$N_0 = 4.19 \times 10^{23}$

The saturated magnetic moment of bar

$= 4.19 \times 10^{23} \times 1.8 \times 10^{-23} = 7.54$ Am^2

52. (4) We have, $B = \mu_0 H + \mu_0 I$

or $\quad I = \dfrac{B - \mu_0 H}{\mu_0}$ or $I = \dfrac{\mu H - \mu_0 H}{\mu_0} = \left(\dfrac{\mu}{\mu_0} - 1\right)H$

$I = (\mu_r - 1)H$

For a solenoid of n-turns per unit length and current i

$H = ni$

$\therefore \quad I = (\mu_r - 1)ni = (1000 - 1) \times 500 \times 0.5$

$I = 2.5 \times 10^5$ Am^{-1}

$\therefore \quad$ Magnetic moment $M = IV$

$M = 2.5 \times 10^5 \times 10^{-4} = 25$ Am^2

53. (4) The bar magnet coercively 4×10^3 Am^{-1} i.e., it requires a magnetic intensity $H = 4 \times 10^3$ Am^{-1} to get demagnetized. Let i be the current carried by solenoid having n number of turns per metre length, then by definition $H = ni$. Here $H = 4 \times 10^3$ Amp turn metre^{-1}

$$n = \frac{N}{l} = \frac{60}{0.12} = 500\ \text{turn metre}^{-1}$$

$\Rightarrow \quad i = \dfrac{H}{n} = \dfrac{4 \times 10^3}{500} = 8.0$ A

54. (2) $I = \dfrac{M}{V} = \dfrac{M}{\text{mass/density}}$, given mass = 1 gm = 10^{-3} kg,

and density $= 5\ \text{gm/cm}^3 = \dfrac{5 \times 10^{-3}\,\text{kg}}{(10^{-2})^3\,\text{m}^3} = 5 \times 10^3\ \text{kg/m}^3$

Hence $I = \dfrac{6 \times 10^{-7} \times 5 \times 10^3}{10^{-3}} = 3$ A/m

55. (1) $E = nAVt = nA\dfrac{m}{d}t = \dfrac{50 \times 250 \times 10 \times 3600}{7.5 \times 10^3} = 6 \times 10^4$ J

56. (2) $T = 2\pi\sqrt{\dfrac{I}{MB_H}} \Rightarrow T \propto \dfrac{1}{\sqrt{M}} \Rightarrow \dfrac{T_1}{T_2} = \sqrt{\dfrac{M_2}{M_1}}$

If $M_1 = 100$ then $M_2 = (100 - 36) = 64$

So $\quad \dfrac{T_1}{T_2} = \sqrt{\dfrac{64}{100}} = \dfrac{8}{10} \Rightarrow T_2 = \dfrac{10}{8}T_1 = 1.25\,T_1$

So % increase in time period = 25%

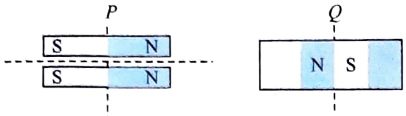
57. (1) $W = MB(\cos\theta_1 - \cos\theta_2) = MB(\cos 0° - \cos 60°)$

$$= MB\left(1-\frac{1}{2}\right) = \frac{MB}{2}$$

and $\tau = MB\sin\theta = MB\sin 60° = MB\frac{\sqrt{3}}{2}$

$$\therefore \quad \tau = \left(\frac{MB}{2}\right)\sqrt{3} \Rightarrow \tau = \sqrt{3}W$$

58. (3) If pole strength, magnetic moment and length of each part are m', M' and L' respectively then



$$\Rightarrow \quad m' = \frac{m}{2}$$

$$\Rightarrow \quad m' = m$$

$$\Rightarrow \quad L' = L$$

$$\Rightarrow \quad L' = \frac{L}{2} \quad \Rightarrow \quad M' = \frac{M}{2} \quad \Rightarrow M' = \frac{M}{2}$$

59. (3) $\tau = MB\sin\theta \Rightarrow \tau \propto \sin\theta$

$$\Rightarrow \quad \frac{\tau_1}{\tau_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \frac{\tau}{\tau/2} = \frac{\sin 90°}{\sin\theta_2}$$

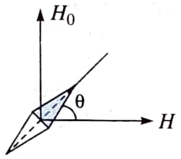
$$\Rightarrow \quad \sin\theta_2 = \frac{1}{2} \Rightarrow \theta_2 = 30°$$

$\Rightarrow$ Angle of rotation $= 90° - 30 = 60°$

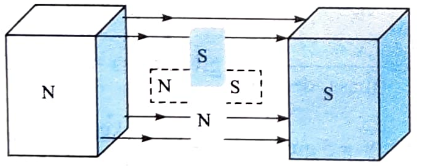
60. (1) In given case H and H_0 are perpendicular to each other.

From figure $\tan\theta = \frac{H_0}{H}$

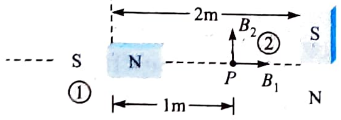
$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{H_0}{H}\right)$$



61. (1) A diamagnetic rod set itself perpendicular to the field if free to rotate between the poles of a magnet as in this situation the field is strongest near the poles.



62. (2) With respect to 1st magnet, P lies in end side-on position



$$\therefore \quad B_1 = \frac{\mu_0}{4\pi}\left(\frac{2M}{d^3}\right) \qquad \text{(RHS)}$$

With respect to 2nd magnet, P lies in broad sideon position.

$$\therefore \quad B_2 = \frac{\mu_0}{4\pi}\left(\frac{M}{d^3}\right) \qquad \text{(Upward)}$$

$$B_1 = 10^{-7} \times \frac{2\times 1}{1} = 2\times 10^{-7}T,\ B_2 = \frac{B_1}{2} = 10^{-7}\ \text{T}$$

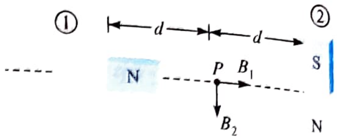
As B_1 and B_2 are mutually perpendicular, hence the resultant magnetic field

$$B_R = \sqrt{B_1^2 + B_2^2} = \sqrt{(2\times 10^{-7})^2 + (10^{-7})^2} = \sqrt{5}\times 10^{-7}\ \text{T}$$

63. (4) At point P net magnetic field $B_{\text{net}} = \sqrt{B_1^2 + B_2^2}$

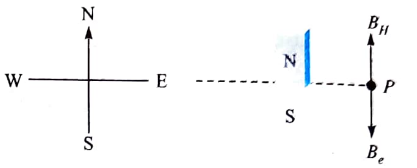
where $B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3}$

and $B_2 = \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3}$

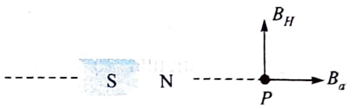


$$\Rightarrow \quad B_{\text{net}} = \frac{\mu_0}{4\pi}\cdot\frac{\sqrt{5}M}{d^3}$$

64. (4) Initially neutral point obtained on equatorial line and at neutral point $|B_H| = |B_e|$



where B_H = Horizontal component of earth's magnetic field, B_e = Magnetic field due to bar magnet on its equatorial line.

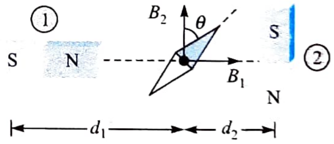


Finally, point P comes on axial line of the magnet and at P, net magnetic field

$$B = \sqrt{B_a^2 + B_H^2}$$

$$= \sqrt{(2B_e)^2 + (B_H)^2} = \sqrt{(2B_H)^2 + B_H^2} = \sqrt{5}\,B_H$$

65. (3) In equilibrium $B_1 = B_2\tan\theta$



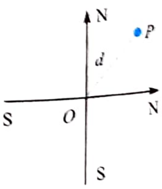
$$\Rightarrow \quad \frac{\mu_0}{4\pi}\cdot\frac{2M}{d_1^3} = \frac{\mu_0}{4\pi}\cdot\frac{M}{d_2^3}\tan\theta$$

$$\Rightarrow \quad \frac{d_1}{d_2} = (2\cot\theta)^{1/3}$$

66. (3) Resultant magnetic moment of the two magnets is

$$M_{\text{net}} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

Imagine a short magnet lying along OP with magnetic moment equal to $M\sqrt{2}$. Thus point P lies on the axial line of the magnet.



$\therefore$ Magnitude of magnetic field at P is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}M}{d^3}$$

67. (1) On passing current through the coil, it acts as a magnetic dipole. Torque acting on magnetic dipole is counter balanced by the moment of additional weight about position O.

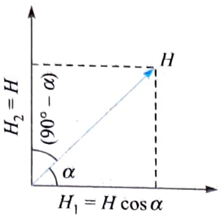
Torque acting on a magnetic dipole $\tau = MB\sin\theta = (NiA)B\sin 90^\circ$ $= NiAB$.

Again τ = force × Lever arm $= \Delta mg \times l$

$$\Rightarrow \quad NiAB = \Delta mgl$$

$$\Rightarrow \quad B = \frac{\Delta mgl}{NiA} = \frac{60\times10^{-3}\times9.8\times30\times10^{-2}}{200\times22\times10^{-3}\times1\times10^{-4}} = 0.4\ \text{T}$$

68. (4) Let α be the angle which one of the planes make with the magnetic meridian the other plane makes an angle $(90^\circ - \alpha)$ with it. The components of H in these planes will be $H\cos\alpha$ and $H\sin\alpha$ respectively. If ϕ_1 and ϕ_2 are the apparent dips in these two planes, then



$$\tan\phi_1 = \frac{V}{H\cos\alpha} \text{ i.e. } \cos\alpha = \frac{V}{H\tan\phi_1} \quad \text{...(i)}$$

$$\tan\phi_2 = \frac{V}{H\sin\alpha} \text{ i.e., } \sin\alpha = \frac{V}{H\tan\phi_2} \quad \text{...(ii)}$$

Squaring and adding (i) and (ii), we get

$$\cos^2\alpha + \sin^2\alpha = \left(\frac{V}{H}\right)^2\left(\frac{1}{\tan^2\phi_1} + \frac{1}{\tan^2\phi_2}\right)$$

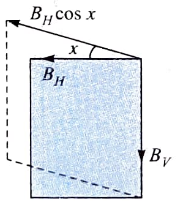
i.e. $\quad 1 = \dfrac{V^2}{H^2}(\cot^2\phi_1 + \cot^2\phi_2)$

or $\quad \dfrac{H^2}{V^2} = \cot^2\phi_1 + \cot^2\phi_2$ i.e.

$$\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

This is the required result.

69. (1) In first case $\tan\theta = \dfrac{B_V}{B_H}$ ...(i)



Second case $\tan\theta' = \dfrac{B_V}{B_H\cos x}$ ...(ii)

From Eqs. (i) and (ii), $\dfrac{\tan\theta'}{\tan\theta} = \dfrac{1}{\cos x}$

70. (3) $\tan\theta = \dfrac{B_V}{B_H}$ ... (i)

If apparent dip is θ' then

$$\tan\theta' = \frac{B'_V}{B'_H} = \frac{B_V}{B_H\cos 30^\circ} = \frac{B_V}{B_H \times \frac{\sqrt{3}}{2}}$$

$$\Rightarrow \quad \tan\theta' = \left(\frac{2}{\sqrt{3}}\right)\tan\theta \Rightarrow \tan\theta' > \tan\theta \Rightarrow \theta' > \theta$$

71. (2) In the given figure OQ refers to retentivity while OR refers to coercivity, for permanents both retentivity and coercivity should be high.

Archives

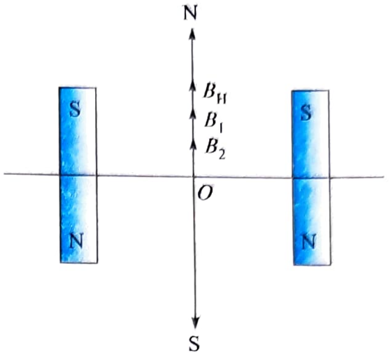
JEE Main

Single Correct Answer Type

1. (3) For ammeter, $S = \dfrac{I_g G}{I - I_g}$

So for I to increase, S should decrease, so additional S can be connected across it.

2. (1) The situation is as shown in the figure. As the point O lies on broad-side position with respect to both the magnets. Therefore,



The net magnetic field at point O is

$$B_{\text{net}} = B_1 + B_2 + B_H$$

$$B_{\text{net}} = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} + \frac{\mu_0}{4\pi}\frac{M_2}{r^3} + B_H = \frac{\mu_0}{4\pi r^3}(M_1 + M_2) + B_H$$

Substituting the given values, we get

$$B_{\text{net}} = \frac{4\pi\times10^{-7}}{4\pi\times(10\times10^{-2})^3}[1.2+1] + 3.6\times10^{-5}$$

$$= \frac{10^{-7}}{10^{-3}}\times2.2 + 3.6\times10^{-5}$$

$$= 2.2\times10^{-4} + 0.36\times10^{-4}$$

$$= 2.56\times10^{-4}\ \text{Wb/m}^2$$

3. (1) Here, $L = 10$ cm $= 0.1$ m, $N = 100$, $I = ?$

As, $\quad B = \mu_0 nI = \mu_0\dfrac{N}{L}I$

$$\Rightarrow \quad I = \frac{B}{\mu_0}\times\frac{L}{N} = 3\times10^3\times\frac{0.1}{100} = 3\ \text{A}$$

4. (4) For electromagnet and transformers, we require the core that can be magnetised and demagnetised quickly when subjected to alternating current. Transformer core and electromagnets are made by soft magnetic materials, which are easy to magnetise and demagnetise whereas for electric generators we use hard magnetic materials.

Fig. (A) → Hard magnetic material; Fig. (B) → soft magnetic material. Hence graph B is suitable.

5. (3) $T = 2\pi\sqrt{\dfrac{I}{MB}}$

$$I = 7.5\times10^{-6}\ \text{kg-m}^2$$

$$M = 6.7\times10^{-2}\ \text{Am}^2$$

By substituting value in the formula

$$T = 0.665\ \text{sec}$$

For 10 oscillations, time taken will be

$$\text{Time} = 10T = 6.65\ \text{sec}$$

Option (3) is correct.

Chapter 4

Concept Application Exercises

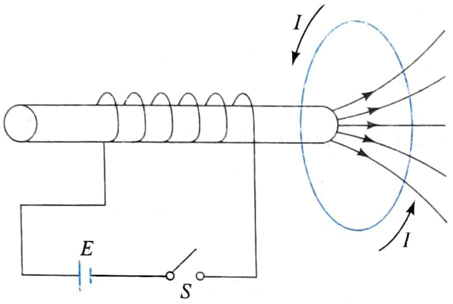
Exercise 4.1

1. (a) $\vec{A} = \ell^2\hat{i}$

$$\phi = \vec{B}.\vec{A} = (5\hat{i} + 4\hat{j} + 3\hat{k}).(2.5 \times 10^{-2})^2\hat{i} = 3.125 \times 10^{-3}\text{ Wb}$$

(b) As magnetic field is uniform no net flux through the closed structure (zero)

2. (a) As the switch is closed, the situation changes from one in which no magnetic flux passes through the ring to the one in which flux passes through the ring in the direction shown. The induced current flows in the anticlockwise sense as seen from the side on which the solenoid lies.



(b) When the switch has been closed for a long time, no change in the magnetic flux through the loop occurs; hence, no current is induced in the loop.

(c) When the switch is opened, flux decreases in the direction shown above. Hence current will be opposite to the direction of situation (a), i.e., in clockwise direction.

3. (a) Induced current should be in anticlockwise direction, so that upper face becomes north pole.

(b) Induced current should be clockwise so that upper face becomes south pole.

(c) Flux is increasing in the inward direction, so induced current should be in anticlockwise direction. It will oppose the increase of flux.

(d) Flux is decreasing in upward direction, so induced current will be in anticlockwise direction. It will oppose the decrease in flux.

4. (a) When the switch is opened, the magnetic field to the right decreases. Therefore, the second coil's induced current produces its own field to the right. That means the current must pass through the resistor from point a to point b.

(b) If coil B is moved closer to coil A, more flux passes through it towards the right. Therefore, the induced current must produce its own magnetic field to the left to oppose the increased flux. That means the current must pass through the resistor from point b to point a.

(c) If the variable resistor R is decreased, then more current flows through coil A, and so a stronger magnetic field is produced, leading to more flux to the right through coil B. Therefore, the induced current must produce its own magnetic field to the left to oppose the increased flux. That means the current must pass through the resistor from point b to point a.

5. (a) When current is passing from a to b and is increasing, the magnetic field becomes stronger to the left, so the induced field points towards right, and the induced current must flow from right to left through the resistor.

(b) If the current passes from b to a, and is decreasing, then there is less magnetic field pointing right, so the induced field points towards right, and the induced current must flow from right to left through the resistor.

(c) If current passes from b to a and is increasing, then there is more magnetic field pointing right, so the induced field points left, and the induced current must flow from left to right through the resistor.

6. (a) Φ_B is ⊙ and increasing so the flux Φ_{ind} of the induced current is clockwise.

(b) The current reaches a constant value so Φ_B is constant $d\Phi_B/dt = 0$ and there is no induced current.

(c) Φ_B is ⊙ and decreasing, so Φ_{ind} is ⊙ and current is counterclockwise.

7. $\phi = BA$

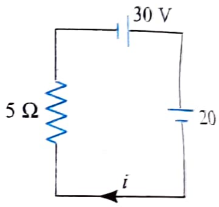
$$\text{emf} = A\frac{dB}{dt} = 2 \times 10 = 20\text{ V}$$

$\therefore i = 20/5 = 4$ A.

From lenz's law, direction of current will be anticlockwise.

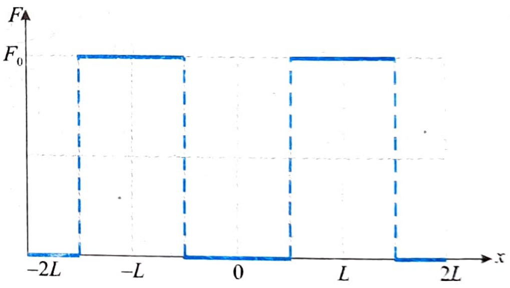
8. Induce emf = 20 V

The equivalent circuit can be drawn as

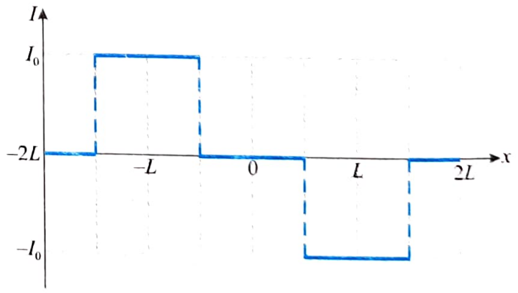


$$\text{current } i = \frac{30 - 20}{5} = 2\text{ A clockwise.}$$

9. (a)



(b)



$$\text{Here, } \varepsilon = vBL = I_0R \Rightarrow I_0 = \frac{vBL}{R}$$

$$\text{and } F_0 = I_0LB = \frac{vB^2L^2}{R}$$

10. $$e_{\text{av}} = \frac{\Delta\phi}{\Delta t} = \frac{B(\text{change in area})}{\Delta t} = \frac{0.3\left[2\left(\frac{\sqrt{3}}{4}L^2\right)\right]}{0.10} = 5.84\text{ V}$$

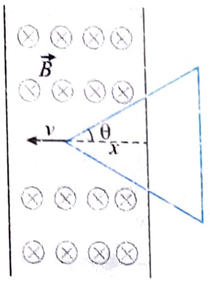
11. Area $A = a^2 = (0.20)^2 = 0.04\text{ m}^2$

$$\phi = AB = 15 \times 10^{-6} \times 0.04 = 6 \times 10^{-7}\text{ Wb}$$

$$q = \frac{\Delta\phi}{R} = \frac{6 \times 10^{-7}}{0.50} = 12 \times 10^{-7}\text{ C} = 1.2\ \mu\text{C}$$

12. (a) The area exposed to the magnetic field,

$$A = (2)\left(\frac{1}{2}x\right)(x\tan\theta) = x^2\tan\theta$$



The magnitude of induced EMF at any time 't',

$$|\varepsilon| = B\frac{dA}{dt} = 2Bx\tan\theta \cdot \frac{dx}{dt}, \text{ where } \frac{dx}{dt} = v$$

or $\quad |\varepsilon| = 2Bvx\tan\theta = 2(Bv^2\ \tan\theta)t$

(b) Induced current $i = \dfrac{|\varepsilon|}{R} = \dfrac{2Bv^2\tan\theta}{R}t$,

From Lenz law current in the loop will be in counter clockwise sense.

13. Flux: $\phi = B \cdot \pi r^2$

Induced emf: $\varepsilon = \left|\dfrac{d\phi}{dt}\right| = 2\pi Br\dfrac{dr}{dt} = 2\pi Brn$

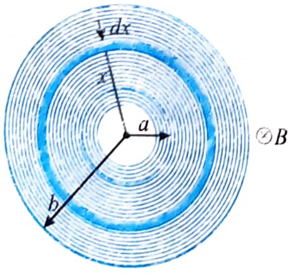
Induced current: $I = \dfrac{\varepsilon}{R} = \dfrac{2\pi Brn}{R}$

Rate of work done = Rate of heat dissipated in Joule heating

$$= I^2R = \frac{4\pi^2B^2r^2n^2}{R}$$

14. Consider a circular strip of radius x and width dx.

Number of turns in the strip $dN = \dfrac{Ndx}{(b-a)}$



Flux linked with a circular loop of radius x is $\phi = B \cdot \pi x^2$

$$\Rightarrow \quad \frac{d\phi}{dt} = \pi x^2\frac{dB}{dt}$$

Emf induced in coil of width dx will be

$$d\varepsilon = (dN)\left(\frac{d\phi}{dt}\right) = \frac{N\,dx}{b-a} \cdot \pi x^2\frac{dB}{dt}$$

∴ Emf induced in the complete coil is

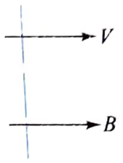
$$\varepsilon = \int d\varepsilon = \frac{\pi N}{(b-a)}\frac{dB}{dt}\int_a^b x^2dx$$

$$= \frac{\pi N}{(b-a)}\frac{dB}{dt} \cdot \frac{(b^3-a^3)}{3} = \frac{\pi N\alpha}{3}(a^2+b^2+ab)$$

Exercise 4.2

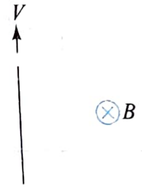
1. In loop (a), no emf will be induced because there is no flux change. In loop (b) emf will be induced because the coil is moving in a region of decreasing magnetic field in inward direction. Therefore, to oppose the decrease in flux in inward direction, current will be induced such that its magnetic field will be inwards. For this, direction of current should be clockwise.

2. (a)



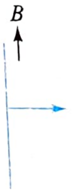
emf = 0 as $\vec{B} \parallel \vec{v}$

(b)



emf = 0 as $\vec{\ell} \parallel \vec{v}$

(c)



emf = 0 as $\vec{\ell} \parallel \vec{B}$

3. Consider rod AB, which is a part of the coil. emf induced in the rod $= BLv$.

Suppose the emf induced in part ACB is E, as shown.

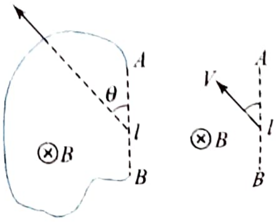
Since the emf in the coil is zero, emf (in ACB) + emf (in BA) = 0

or $\quad -E + vBL = 0$

or $\quad E = vBL$

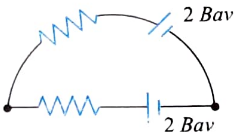
Thus, emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also, the equivalent emf between A and B is BLv (here the two emf's are in parallel).

4. The same emf will be induced in the straight imaginary wire joining A and B, which is $Bv\ell\sin\theta$.

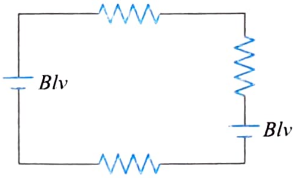


5. Induced emf = 0, because V is parallel to diameter.

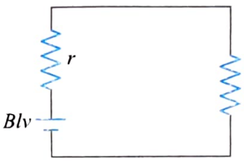
6. Induced emf $= 2Bav$

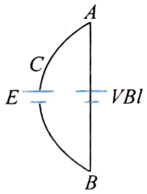


7. Net induced emf is zero.

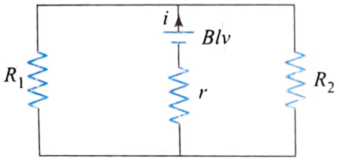


8.



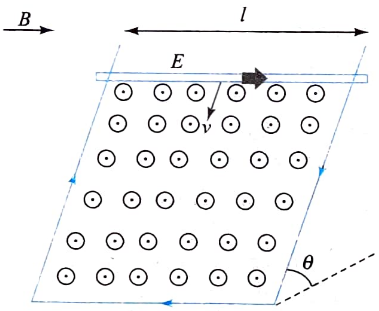


9. The given circuit is equivalent to the following diagram:

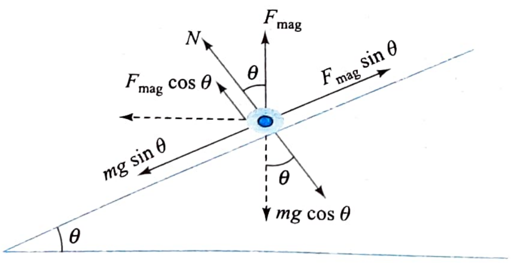


$$i = \frac{B\ell v}{r + \dfrac{R_1 R_2}{R_1 + R_2}}$$

10.



Conducting wire



The emf induced in the rod,

$$E = (B\sin\theta)lv$$
$$= Blv\sin\theta$$

The current in the circuit $I = \dfrac{E}{R} = \dfrac{Blv\sin\theta}{R}$

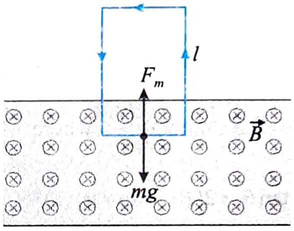
Magnetic force acting on the sliding rod, $F_{\text{mag}} = IlB$

$$F_{\text{mag}} = \left(\frac{Blv\sin\theta}{R}\right)lB$$
$$= \frac{B^2l^2v\sin\theta}{R}$$

If the rod is sliding with constant velocity,

$$F_{\text{mag}}\sin\theta = mg\sin\theta$$
$$\Rightarrow \frac{B^2l^2v\sin^2\theta}{R} = mg\sin\theta$$
$$\Rightarrow B = \sqrt{\frac{mgR}{l^2v\sin\theta}}$$

11. For the loop to move with constant velocity v in the magnetic field, the net force acting on it must be zero. Hence, magnetic force nullifies the effect of gravity.

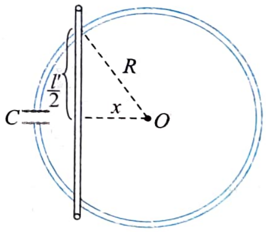


Then, $F_m = mg$; Substituting $F_m = \dfrac{B^2l^2v}{R}$, we obtain $v = \dfrac{mgR}{B^2l^2}$.

This is the terminal speed. This velocity is attained by the loop after falling through a height $h = \dfrac{v^2}{2g}$ in gravity.

Substitution of v yields $h = \dfrac{m^2gR^2}{2B^4l^4}$

12. (a) The length of the rod in the magnetic field , $l' = 2\sqrt{R^2 - x^2}$



Then, the induced emf, $\varepsilon = Bl'v = 2Bv\sqrt{R^2 - x^2}$

The change stored in the capacitor is $q = C\varepsilon = 2BCv\sqrt{R^2 - x^2}$

The current in the circuit is

$$i = \left|\frac{dq}{dt}\right| = \left|2BCv\frac{d}{dt}\left(\sqrt{R^2 - x^2}\right)\right| = \left|2BCv\left\{\frac{-2x\left(\frac{dx}{dt}\right)}{2\sqrt{R^2 - x^2}}\right\}\right|$$

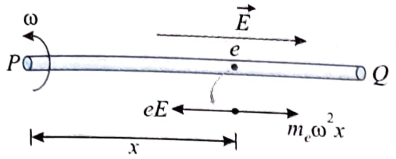
$$i\Big|_{x=\frac{R}{2}} = \frac{BCv^2x}{\sqrt{R^2 - x^2}}\Bigg|_{x=\frac{R}{2}} = \frac{2BCv^2}{\sqrt{3}R}$$

(b) $i = \dfrac{\varepsilon}{R_0} = \dfrac{2Bv}{R_0}\sqrt{R^2 - (R - vt)^2}$

xercise 4.3

1. Flux passing through the ring, $\phi = BA$ is constant here, therefore, emf induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.

2. The accumulation of free electrons will create an electric field which will finally balance the centrifugal forces and a steady state will be reached. In the steady state, $m_e\omega^2 x = eE$.


ω
$\vec{E}$
e
P
Q
eE
$m_e\omega^2 x$
x

$$V_P - V_Q = \int_{x=0}^{x=\ell} \vec{E}\cdot d\vec{x} = \int_0^{\ell} \frac{m_e\omega^2 x}{e}\,dx = \frac{m_e\omega^2\ell^2}{2e}$$

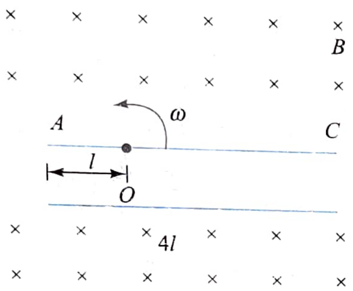
3. Rotational emf will be induced due to rotation of the rod.

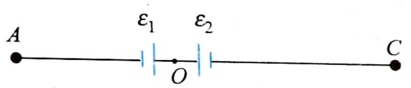
Emf of part OA is

$$\varepsilon_1 = \frac{1}{2}Bl^2\omega$$

Emf of part OC is $\varepsilon_2 = \frac{9}{2}Bl^2\omega$

The circuit can be separated as




ε_1
ε_2
A
O
C

$$V_A + \varepsilon_1 - \varepsilon_2 = V_C$$

$$V_A - V_C = \varepsilon_2 - \varepsilon_1 = \frac{9}{2}Bl^2\omega - \frac{1}{2}Bl^2\omega = 4\,Bl^2\omega$$

$$V_A - V_C = 4\,B\omega l^2$$

4. At any time t, $\phi = BA\cos\theta = BA\cos\omega t$

Now, induced emf in the loop

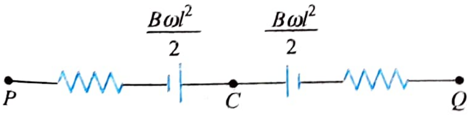
$$e = -\frac{d\phi}{dt} = BA\,\omega\sin\omega t$$

5. $E_{PQ} = \frac{1}{2}B\omega(2\ell)^2 = 100\text{ V} \Rightarrow B\omega\ell^2 = 50\text{ V}$

$$E_{PM} = \frac{1}{2}B\omega(\ell)^2 = 25\text{ V}$$

$$E_{MQ} = E_{PQ} - E_{PM} = 100 - 25 = 75\text{ V}$$

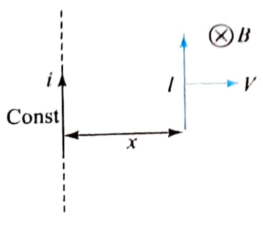
6. $\text{emf}_{PQ} = 0$; $\text{emf}_{PC} = \dfrac{B\omega\ell^2}{2}$


$\frac{B\omega l^2}{2}$
$\frac{B\omega l^2}{2}$
P
C
Q

7. $E = Blv = \dfrac{\mu_0 ilv}{2\pi x}$

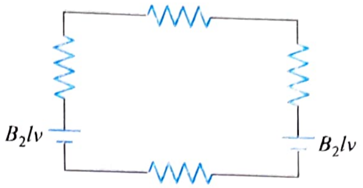
Alternatively:

emf is equal to the rate with which magnetic field lines are cut. In dt time the area swept by the rod is lv dt. The magnetic field lines cut in dt time $= Blv\,dt = \dfrac{\mu_0 i l v\,dt}{2\pi x}$.


⊗B
i
l
V
Const
x

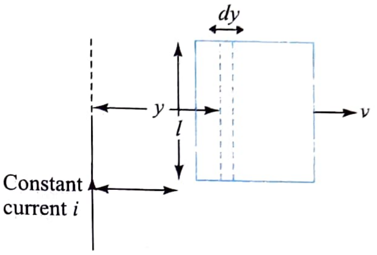
∴ The rate at which magnetic field lines are cut $= \dfrac{\mu_0 i l v}{2\pi x}$ and this will be induced e.m.f.

8. $E = B_1 lv - B_2 lv = \dfrac{\mu_0 i}{2\pi x}lv - \dfrac{\mu_0 i}{2\pi(x+b)}lv = \dfrac{\mu_0 ilbv}{2\pi x(x+b)}$



Alternatively:

Consider a small segment of width dy at a distance y from the wire. Let flux through the segment be $d\phi$.


dy
y
l
v
Constant current i

$$\therefore\quad d\phi = \frac{\mu_0 i}{2\pi y}l\,dy$$

$$\therefore\quad \phi = \frac{\mu_0 il}{2\pi}\int_x^{x+b}\frac{dy}{y} = \frac{\mu_0 il}{2\pi}\left(\ln(x+b) - \ln x\right)$$

Now,

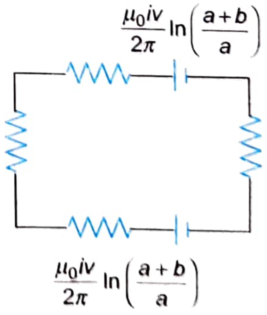
$$\frac{d\phi}{dt} = \frac{\mu_0 il}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right]$$

$$= \frac{\mu_0 il}{2\pi}\left[\frac{(-b)}{x(x+b)}\right]v = \frac{-\mu_0 iblv}{2\pi x(x+b)}$$

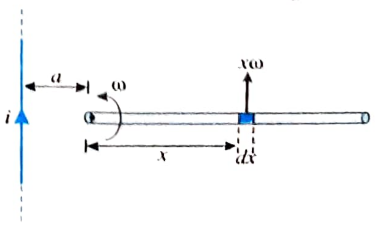
$$\therefore\quad \text{Induced emf} = \frac{\mu_0 iblv}{2\pi x(x+b)}$$

9. Net emf = 0

The equivalent circuit is


$\frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+b}{a}\right)$
$\frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+b}{a}\right)$

10. Consider a small segment of rod of length dx, at a distance x from one end of the rod. emf induced in the segment,



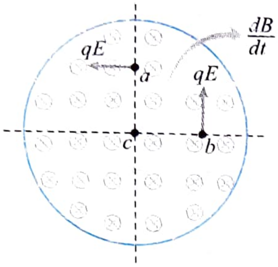
$$d\varepsilon = \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx$$

$$\therefore \quad \varepsilon = \int_0^{\ell} \frac{\mu_0 i}{2\pi(x+a)}(x\omega)\,dx = \frac{\mu_0 i\omega}{2\pi}\left[\ell - a\ln\left(\frac{\ell + a}{a}\right)\right]$$

Exercise 4.4

1. Since the field is increasing in inward direction, so electric field will be in anticlockwise direction. So, force on a and b are as shown in figure.

$$E2\pi r = \pi r^2\frac{dB}{dt} \Rightarrow E = \frac{r}{2}\frac{dB}{dt}$$



So force on a and b: $F = qE = \frac{qr}{2}\frac{dB}{dt}$

At centre, electric field will be zero, so no force on charge at c.

2. $\oint \vec{E}\cdot ds = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$

For loop a: $\oint \vec{E}\cdot d\vec{l} = -\frac{d(\vec{B}_1\cdot\vec{A}_1)}{dt} = -\frac{d(B_1A_1\cos 180°)}{dt}$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = A_1\frac{dB_1}{dt} = \pi r_1^2(-8.0 \text{ mT/s})$$

$$= -\pi(20.0\times10^{-2})^2(8.0\times10^{-3})$$

$$= -3.2\pi\times10^{-4} \text{ V}$$

For loop b: $\oint \vec{E}\cdot d\vec{l} = -\frac{d(\vec{B}_2\cdot\vec{A}_2)}{dt} = -A_2\frac{dB_2}{dt}$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = -\pi\left(\frac{25}{100}\right)^2\times\left(-8.0\times10^{-3}\right) = +5.0\pi\times10^{-4} \text{ V}$$

For loop c: $\oint \vec{E}\cdot d\vec{l} = -\frac{d(\vec{B}\cdot\vec{A})}{dt} = -\frac{d(\vec{B}_1\cdot\vec{A}_1 + \vec{B}_2\cdot\vec{A}_2)}{dt}$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = -\left[(-3.2\pi\times10^{-4}) + (5.0\pi\times10^{-4})\right]$$

$$= -1.8\pi\times10^{-4} \text{ V}$$

3. $\oint_1 \vec{E}\cdot d\vec{l} = mag$

$$\oint_3 \vec{E}\cdot d\vec{l} = 3(mag)$$

It means field in b and c should be out of the page.

$$\oint_4 \vec{E}\cdot d\vec{l} = 0$$

It means field in e will be opposite to that in c, i.e., into the page. Similarly field into d is also into the page.

4. (a) At point P_2 the induced electric field strength is given as

$$E = \frac{R^2}{2r_2}\left(\frac{dB}{dt}\right)$$

$$F = qE = \frac{eR^2}{2r_2}(6t^2 - 8t)$$

Substituting the values we get

$$F = \frac{(1.6\times10^{-19})(2.5\times10^{-2})}{2\times5\times10^{-2}}[6(2)^2 - 8(2)]$$

$$\Rightarrow \quad F = 8.0\times10^{-21} \text{ N}$$

(b) At P_1 being a point inside of the magnetic field, the induced electric field at this point is given as

$$E = \frac{r_1}{2}\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \frac{r_1}{2}[6t^2 - 8t]$$

$$\Rightarrow \quad E = \frac{0.02}{2}[6(3)^2 - 8(3)]$$

$$\Rightarrow \quad E = 0.3 \text{ V/m}$$

5. (a) As the loop is lying outside the magnetic field region. There is no flux linked to the loop. It means no emf is induced.

(b) Electric field at a distance r from center O outside the magnetic fieldis given by

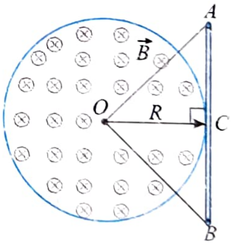
$$E = \frac{R^2}{2r}\frac{dB}{dt} \text{ (in tangential direction)}$$

Emf induced in arc AB $\varepsilon_{AB} = E.(b\theta) = \left(\frac{R^2}{2b}\frac{dB}{dt}\right)(b\theta) = \frac{R^2\theta\beta}{2}$

(c) Electric field lines are normal to arms BC. There is no emf in this arm.

6. (a) As emf induced in time varying magnetic field region is given by $e = \oint \vec{E}\cdot\vec{dl}$. Here the electric field lines are perpendicular to the rod at all points in Fig. (a), hence no emf will be induced.

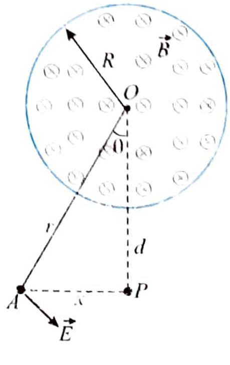
(b) Emf induced in AB = rate of change of flux through the triangular region.



$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\pi R^2}{4}\cdot\alpha$$

7. (a) Point P is situated outside the time varying magnetic field region. At this point the induced electric field is perpendicular to OP at all points in this path. The force and displacement both are perpendicular to each other. Hence the work done in moving it to infinity is zero.

(b) Induced electric field at a point A shown in the figure is given by



$$E = \frac{R^2}{2r}\left(\frac{dB}{dt}\right) = \frac{R^2\beta}{2r},$$

which is constant hence the electric force on the charge along AP is

$$F_e = qE\cos\theta = \frac{qR^2\beta \cdot d}{2r^2}$$

It means the external agent must apply equal and opposite force to keep the charge moving without gaining any kinetic energy. Work done by the external agent in small displacement dx will be

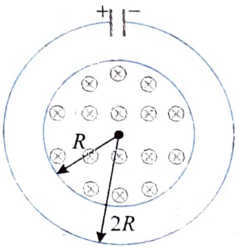
$$dW = \frac{qR^2\beta d}{2r^2}dx$$

But $x = d\tan\theta \Rightarrow dx = d\sec^2\theta\, d\theta$ and $r = d\sec\theta$

$$dW = \frac{qR^2\beta d}{2(d\sec\theta)^2}\cdot d\sec^2\theta d\theta = \frac{qR^2\beta}{2}d\theta$$

$$\therefore \quad W = \frac{qR^2\beta}{2}\int_0^{\pi/2} d\theta = \frac{\pi qR^2\beta}{4}$$

8. (a) The flux passing through the loop is



$$\phi = \pi R^2 B = \pi R^2 B$$

Where $B = \mu_0 ni$

Then $\varepsilon = \left|\frac{d\phi}{dt}\right| = \pi R^2\frac{dB}{dt} = \frac{\pi R^2 \mu_0 n i_0}{T}$

The charge in the capacitor is $q = CV = \frac{\varepsilon_0 A}{d}\varepsilon$

$$q = \frac{\pi R^2 \mu_0 n i_0 \varepsilon_0 A}{dT}$$

(b) The static electric field in the capacitor is

$$E_{\text{static}} = \frac{q}{A\varepsilon_0} = \frac{\mu_0 \pi R^2 n i_0}{dT}$$

(c) The induced electric field inside the capacitor ,

$$E_{\text{induced}} = \frac{R^2}{2r}\left(\frac{dB}{dt}\right) = \frac{R^2}{2.2R}\left(\frac{\mu_0 n i_0}{T}\right)$$

$$\Rightarrow \quad E_{\text{induced}} = \frac{R\mu_0 n i_0}{4T}$$

(d) $E_{\text{net}} = \left|\vec{E}_S + \vec{E}_i\right| = \left|\frac{\mu_0\pi R^2 n i_0}{dT} - \frac{R\mu_0 ni}{4T}\right|$

$$E_{\text{net}} = \frac{\mu_0 niR}{T}\left[\frac{\pi R}{d} - 1\right]$$

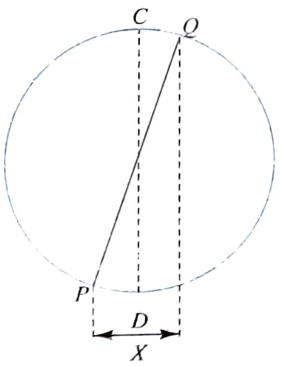
Exercises

Single Correct Answer Type

1. **(1)** Consider the force on an electron in PQ. This electron experiences a force towards Q. Free electrons in PQ tend to move towards N. So M will be positively charged.

2. **(2)** Apply Lenz's law.

3. **(2)** When the ring falls vertically, there will be an induced emf across A and B ($e = Bv(2r)$).



Note that there will be a potential difference across any two points on the ring, and the line joining these has a projected length in the horizontal plane. For example, between points P and Q there is a projected length x in the horizontal plane.

$\therefore$ P.D. across P and Q is

$V = B\,vx$

But for points C and D, $x = 0$

Therefore, P.D. = 0

Hence (2)

4. **(3)** Because A and C are at equal distance from B, and their flux across B is in opposite direction, so at any time flux in B will be zero. Hence no emf is induced.

5. **(2)** In the r–t graph, it is clear that from a to b there is no change in radius and hence no change in area and magnetic flux. Same is the situation from c to d.

Now, $|e| = \frac{d}{dt}(\phi)$

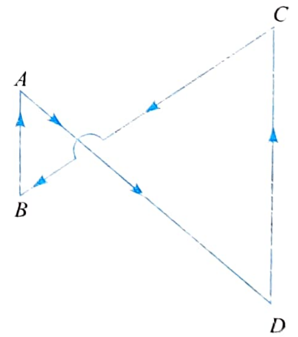
$$|e| = B\frac{d}{dt}(\pi r^2) = B\pi 2r\frac{dr}{dt}$$

Since $r \propto t$, $\therefore \frac{dr}{dt}$ = constant

$\therefore \quad |e| \propto r$

6. **(1)** Magnetic field in ⊗ direction is increasing. Therefore, induced current will produce magnetic field in ⊙ direction. Thus, current in both the loops should be anticlockwise. But as the area of the loop on the right side is more, induced emf in this side will be more compared to the left side loop.

Therefore, net current in the complete loop will be in a direction shown below:



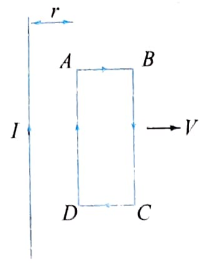
7. **(4)** When the coil is within the field, there is no change in the magnetic flux passing through it. Thus, no current will be induced and the acceleration will be g. But according to Lenz's law, the induced current will oppose its motion when it enters or leaves the field. Therefore, acceleration will be less than g.

8. **(4)** Magnetic flux in ⊗ direction through the coil is increasing. Therefore, induced current will produce magnetic field in ⊙ direction. Thus, the current in the loop is anticlockwise. Magnitude of induced current at any instant of time is

$$i = \frac{e}{R} = \frac{Bv(FG)}{\rho(FG + GD + DF)}$$

When the wire AH moves downwards FG, GD and DF all increase in the same ratio. Therefore, i is constant.

9. (4)



As the flux decreases, to maintain flux, current in the loop is clockwise. Force on DA due to the long wire is towards left while on BC is towards right.

10. (4) Induced emf depends upon vertical edge.

11. (4) $Bl\,Vt$ = constant

$$B = \frac{C}{\ell Vt}$$

12. (3) When the electron is closest, flux due to magnetic field of electron's motion is maximum through the loop. So slope of flux–time graph will be zero or induced emf will be zero.

13. (2) As anticlockwise direction is positive, so area vector outwards is positive. So net flux thrugh the given loop is

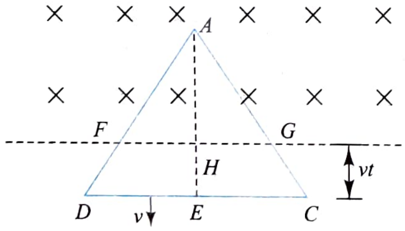
$$\phi = -BA - BA + BA = -BA$$

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\phi}{dt} = -\frac{d(-BA)}{dt} = A\frac{dB}{dt} = A(-\alpha)$$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{r} = -\alpha A$$

14. (1) From Lenz's law if one rod is moved away from the second rod then the second rod will be attracted towards the first rod so as to oppose the change in flux.

15. (2) Let $2a$ be the side of the triangle and b be the length AE.



$$\frac{AH}{AE} = \frac{GH}{EC} \Rightarrow GH = \left(\frac{AH}{AE}\right)EC$$

$$\text{or} \quad GH = \frac{(b-vt)}{b}.a = a - \left(\frac{a}{b}vt\right)$$

$$\therefore \quad FG = 2GH = 2\left[a - \frac{a}{b}vt\right]$$

$\therefore$ Induced emf, $e = Bv\,(FG) = 2Bv\left(a - \frac{a}{b}vt\right)$

$\therefore$ Induced current, $i = \frac{e}{R} = \frac{2Bv}{R}\left[a - \frac{a}{b}vt\right]$

or $\quad i = k_1 - k_2 t$

Thus i–t graph is a straight line with negative slope and positive intercept.

16. (3) The change in magnetic flux is zero, hence the current in the ring will be zero.

17. (2) $P = Fv = BI\ell v = 1.25 \times 10^{-3} \times 50 \times 0.1 \times 1$ W

$= 6.25 \times 10^{-3}$ W $= 6.25$ mW

18. (1) $|\vec{E}|$ = Magnitude of induced emf

$$= \frac{B\ell^2}{2}\omega, \ \ell = \sqrt{2}\,r$$

19. (3) $BI\ell = mg$ or $B\frac{Bv\ell}{R}\ell = mg$ or $v = \frac{mg\,R}{B^2\ell^2}$

20. (1) Speed of the loop should be

$$v = \frac{\ell}{t} = \frac{0.5}{2} = 0.25 \text{ m s}^{-1}$$

Induced emf, $e = Bv\ell = (1.0)\,(0.25)\,(0.5) = 0.125$ V

$\therefore$ Current in the loop, $i = \frac{e}{R} = \frac{0.125}{10}$

$= 1.25 \times 10^{-2}$ A

The magnetic force on the left arm due to the magnetic field is

$F_m = i\ell B = (1.25 \times 10^{-2})\,(0.5)\,(1.0)$

$= 6.25 \times 10^{-3}$ N

To pull the loop uniformly an external force of 6.25×10^{-3} N towards right must be applied.

$\therefore \quad W = (6.25 \times 10^{-3}\text{N})\,(0.5\text{m}) = 3.125 \times 10^{-3}$ J

21. (3) Use Lenz's law. Induced emf of the current opposes the change in flux through it.

22. (3) Magnetic field due to larger loop

$$= \frac{\mu_0 1}{2r} = \frac{4\pi \times 10^{-7} \times 1}{2 \times 0.1}\text{T} = 2\pi \times 10^{-6}\text{ T}$$

Now, magnetic flux linked with the smaller loop, ϕ

$= NBA \cos \omega t = 1 \times 2\pi \times 10^{-6} \times 5 \times 10^{-4} \cos \omega t$

$= \pi \times 10^{-9} \cos \omega t$ weber

23. (1) $ma_0 = eE \Rightarrow E = \frac{ma_0}{e}$

24. (1) $I = \frac{B\ell v}{R}$

$$I = \frac{5 \times 10^{-2} \times 0.3 \times 0.2}{5}\text{A} = 0.6\text{ mA.}$$

Area and flux are decreasing. So, current flows to increase the flux. Clearly, current should be clockwise. So, it flows from B to C through 5 Ω.

25. (2) ϕ (flux linked) $= a^2B \cos 0° - b^2B \cos 180°$

$= (a^2 - b^2)B$

$$E = -\frac{d\phi}{dt} = -(a^2 - b^2)\frac{dB}{dt}$$

$= (a^2 - b^2)\,B_0\,\omega \cos \omega t$

where $B = B_0$, $\sin \omega t$, $B_0 = 10^{-3}$ T, $\omega = 100$

$$\therefore \quad I_{\max} = (a^2 - b^2)\frac{B_0\omega}{R}$$

and $\quad R = (4a + 4b)\,r = 4\,(a + b)\,r$

$$\therefore \quad I_{\max} = \frac{(a-b)\,B_0\omega}{4r} = \frac{(1-0.4) \times 10^{-3} \times 100}{4 \times 5 \times 10^{-3}} = 3\text{A}$$

26. (1) $\vec{\ell}$, $\vec{v}$ and $\vec{B}$ are coplanar.

27. (1) Component of weight along the inclined plane $= mg \sin \theta$

Again, $F = BI\ell = B\frac{B\ell v}{R}\ell = \frac{B^2\ell^2 v}{R}$

Now, $\frac{B^2\ell^2 v}{R} = mg \sin \theta$ or $v = \frac{mgR\sin\theta}{B^2\ell^2}$

28. **(2)** $e=\left(\vec{v}\times\vec{B}\right).\vec{\ell}$

$$e=\left[\hat{i}\times\left(3\hat{i}+4\hat{j}+5\hat{k}\right)\right].5\hat{j}$$

$$\Rightarrow \quad e=-25\text{ V}$$

29. **(1)** Let v be the velocity of conductor at any time, then induced emf:

$$e=Blv \qquad \ldots\text{(i)}$$

Charge on capacitor: $q=Ce=CBlv$

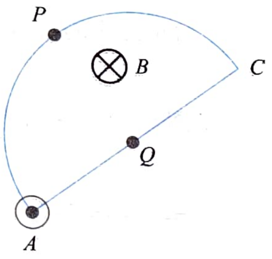
Current in circuit: $I=\dfrac{dq}{dt}=CBl\dfrac{dv}{dt}$

for conductor: $mg-IBl=\dfrac{mdv}{dt}$

$$\Rightarrow \quad mg-CB^2l^2\frac{dv}{dt}=\frac{mdv}{dt} \quad\Rightarrow\quad \frac{dv}{dt}=\frac{mg}{m+CB^2l^2}$$

This is the acceleration of conductor which is constant.

30. **(2)** We connect a conducting wire from A to C and complete the semicircular loop.



The emf in the semicircular loop is zero because its magnetic flux does not change.

$\therefore$ emf of section APC + emf of section $CQA=0$

$\therefore$ emf of section APC = emf of section $AQC=2BR^2\omega$

31. **(1)** Induced electric field at point P:

$$E=\frac{R}{2}\frac{dB}{dt} \text{ towards right}$$

Acceleration of electron: $a=\dfrac{eE}{m}=\dfrac{eR}{2m}\dfrac{dB}{dt}$ towards left

32. **(4)** Given that $\phi=at\,(T-t)$

Induced emf, $E=\dfrac{d\phi}{dt}=\dfrac{d}{dt}[at(T-t)]$

$$=at\,(0-1)+a\,(T-t)$$
$$=a\,(T-2t)$$

So, induced emf is also a function of time.

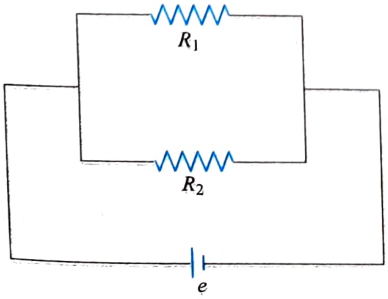
$\therefore$ Heat generated in time T is

$$H=\int_0^T\frac{E^2}{R}\,dt=\frac{a^2}{R}\int_0^T(T-2t)^2\,dt=\frac{a^2T^3}{3R}$$

33. **(4)** $W=F\ell(NIB\ell)\ell=NB\ell^2\left(\dfrac{BVN\ell}{R}\right)$

$$=\frac{N^2B^2\ell^3}{R}V=\frac{N^2B^2\ell^3}{R}\frac{\ell}{t}=\frac{N^2B^2\ell^4}{Rt}=0.1\text{ mJ}$$

34. **(1)** The equivalent diagram is



The induced emf across the centre and any point on the circumference is

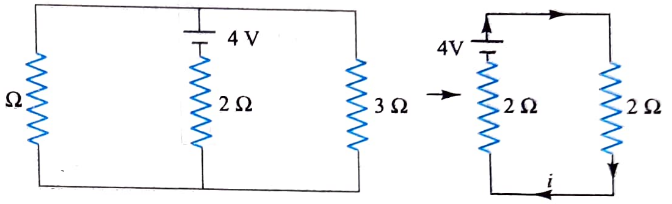
$$|\vec{e}|=\frac{1}{2}B\omega\ell^2=\frac{B\omega r^2}{2}$$

$\therefore$ Current through $R_1=\dfrac{B\omega r^2}{2R_1}$

35. **(3)** Motional emf

$$e=Bvl$$
$$e=(2)\,(2)\,(1)=4\text{ V}$$

This acts as a cell of emf $E=4$ V and internal resistance $r=2\ \Omega$. The simple circuit can be drawn as follows:



Therefore, current through the connector

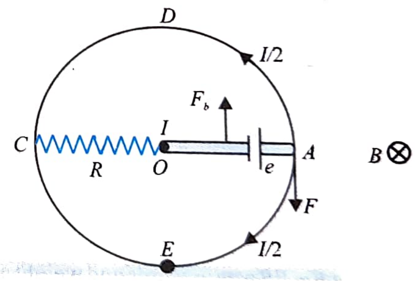
$$i=\frac{4}{2+2}=1\text{ A}$$

Magnetic force on connector

$F_m=ilB=(1)\,(1)\,(2)=2$ N (towards left)

Therefore, to keep the connector moving with a constant velocity, a force of 2 N will have to be applied towards right.

36. **(4)** Induced emf in the spoke is shown in figure below.



$$e=\frac{1}{2}B\omega r^2,\ I=\frac{e}{R}=\frac{B\omega r^2}{2R}$$

There will be no induced emf separately in parts ADC or AEC.

$$F_b=IrB=\frac{B^2\omega r^3}{2R}$$

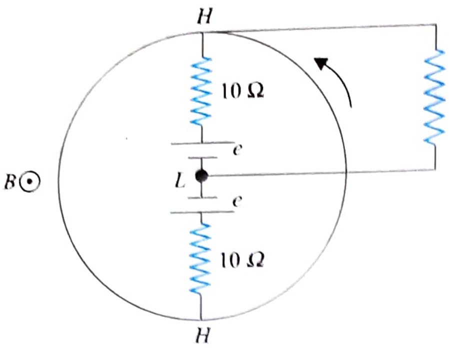
balancing torque about E: $Fr=F_b r/2$

$$\Rightarrow \quad F=\frac{F_b}{2}=\frac{B^2\omega r^3}{4R}$$

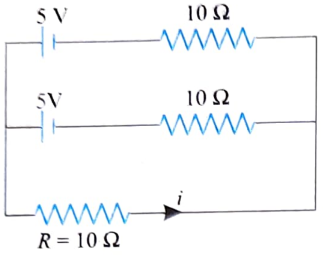
Note: Force due to currents $I/2$ will act on circular parts also, but their torque about E will be zero.

37. **(3)** Emf induced between centre of the ring and the rim is

$$e=\frac{1}{2}B\omega R^2=\frac{1}{2}(50(20)(0.1)^2=5\text{ V}$$



Now the circuit can be drawn as follows:



$$\therefore \quad i = \frac{5}{10+5} = \frac{1}{3}\ \text{A}$$

38. **(1)** Force on the wire = ilB,

$$\therefore \quad \text{Acceleration} = \frac{ilB}{m}$$

$$\therefore \quad \text{Velocity} = \frac{ilBt}{m}$$

39. **(2)** Let E be the electric field at a distance r from the centre of the disc. Then

$$eE = m\omega^2 r$$

$$\text{or} \quad E = \frac{m\omega^2 r}{e}$$

$$\therefore \quad \text{P.D.} = \int_{r=0}^{r=a} E dr$$

$$= \int_0^a \frac{m\omega^2 r}{e} dr = \frac{m\omega^2 a^2}{2e}$$

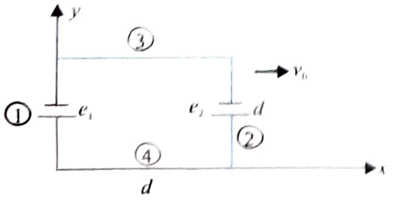
40 **(4)** Magnetic force $F = IlB$ acts upwards and weight acts downwards.

$$a = \frac{mg - F}{m}, \text{ depends upon } mg \text{ and } F.$$

41. **(4)** According to Lenz's law, emf of same magnitude in clockwise direction is induced in the two loops into which the figure is divided. So, current is induced in the clockwise direction in the outer boundary but no current is there in wire AB.

42. **(1)** Magnetic field at side (1): $B_1 = B_0$

Induced emf in (1): $e_1 = B_0 v_0 d$



Magnetic field at side (2): $B_2 = B_0\left[1 + \frac{d}{a}\right]$

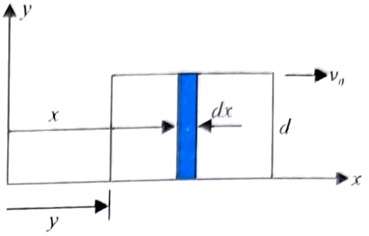
Induced emf in (2): $e_2 = B_0\left(1 + \frac{d}{a}\right) v_0 d$

Induced emf in 3 and 4 will be zero.

Net emf: $e = e_2 - e_1 = \frac{B_0 v_0 d^2}{a}$

Alternate method:

Let at any instant, the loop is at a distance y as shown below. Let us take anticlockwise direction to be positive.



Flux through a strip of width dx: $d\phi = B_0\left[1 + \frac{x}{a}\right](d)(dx)$

Net flux: $\phi = B_0 d \int_y^{y+d} \left(1 + \frac{x}{a}\right) dx$

$$\Rightarrow \quad \phi = B_0 d\left[x + \frac{x^2}{2a}\right]_y^{y+d}$$

$$\Rightarrow \quad \phi = B_0 d\left[d + \frac{1}{2a}\left[(y+d)^2 - y^2\right]\right]$$

$$\Rightarrow \quad \phi = B_0 d\left[d + \frac{1}{2a} d(2y + d)\right]$$

$$e = -\frac{d\phi}{dt} = -B_0 d\left[\frac{1}{2a} d \frac{2dy}{dt}\right]$$

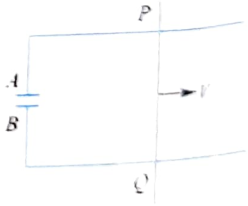
$$\Rightarrow \quad e = -\frac{B_0 d^2}{a}\frac{dy}{dt} = -\frac{B_0 d^2}{a} v_0$$

Negative sign indicates that emf induced is clockwise.

43. **(1)** $q = CV = C(Bvl)$

$= (10 \times 10^{-6})(4)(2)(1)$

$= 80\ \mu\text{C} = \text{constant}$



Magnetic force on the electron in the conducting rod PQ is towards Q. Therefore, A is positively charged and B is negatively charged.

44. **(1)** Emf induced in coil, $\varepsilon = -\frac{\Delta\phi}{\Delta t} = -\frac{(\phi_2 - \phi_1)}{t_2 - t_1}$

$$\phi_2 = BA_2 - B = \frac{4a}{2 \times 3} \times \frac{4a}{3} \times \sin 60^\circ = \frac{4\sqrt{3}}{9} Ba^2$$

$$\phi_1 = Ba^2$$

Work done, $W = \varepsilon q = \varepsilon I \Delta t = Ba^2\left(1 - \frac{4\sqrt{3}}{9}\right) i$

45. **(4)** Potential difference across capacitor

$V = Bvl = \text{constant}$

Therefore, charge stored in the capacitor is also constant. Thus, current through the capacitor is zero.

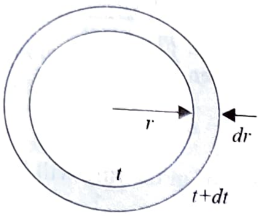
46. **(4)** Let radius of the loop is r at any time t and in further time dt, radius increases by dr.

Then change in flux: $d\phi = (2\pi r dr)B$

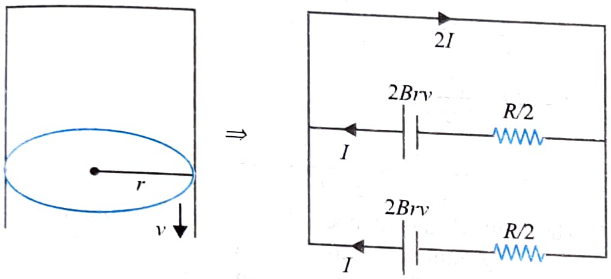
$$\Rightarrow \quad e = \frac{d\phi}{dt} = 2\pi r\left(\frac{dr}{dt}\right)\frac{k}{r}$$

$$\Rightarrow \quad e = 2\pi ck \text{ (constant)} \qquad \left[\because \frac{dr}{dt} = c,\ B = \frac{k}{r}\right]$$

Then change in flux: $d\phi = (2\pi r dr)B$



47. **(4)** $I = \dfrac{2Brv}{R/2} = \dfrac{4Brv}{R}$



Current in the top horizontal $= 2I = \dfrac{8Brv}{R}$.

48. **(4)** Volume of the balloon at any instant, when radius is r,

$$V = \frac{4}{3}\pi r^3$$

Time rate of change of volume,

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

Time rate of change of radius of balloon,

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

Flux through rubber band at the given instant,

$$\phi = B\,(\pi r^2)$$

$$\text{Induced emf} = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\pi r^2) = -2\pi rB\frac{dr}{dt}$$

$$= -2\pi rB\left(\frac{1}{4\pi r^2}\frac{dV}{dt}\right) = -\frac{B}{2r}\frac{dV}{dt}$$

As volume of the balloon is decreasing, $\dfrac{dV}{dt}$ is negative.

$$E_{\text{induced}} = -\frac{(0.04)}{2\times10\times10^{-2}}\times(-100\times10^{-6}) = 20\ \mu\text{V}$$

49. **(2)** When the copper rod is rotated, flux linked with the circuit varies with time.

Therefore, an emf is induced in the circuit.

At time t, plane of semi-circle makes angle ωt with the plane of rectangular part of the circuit. Hence, component of the magnetic induction normal to plane of semi-circle is equal to $B\cos\omega t$.

Flux linked with semicircular part is

$$\phi_1 = \frac{1}{2}\pi a^2 B\cos\omega t$$

Let area of rectangular part of the circuit be A.

$\therefore$ Flux linked with this part is

$$\phi_2 = BA$$

$\therefore$ Total flux linked with the circuit is

$$\phi = \frac{1}{2}\pi a^2 B\cos(\omega t) + BA$$

$\therefore$ Induced emf in the circuit,

$$e = -\frac{d\phi}{dt} = \frac{1}{2}\pi\omega a^2 B\sin(\omega t)$$

Since resistance of the circuit is negligible, therefore, potential difference across the capacitor is equal to induced emf in the circuit.

$\therefore$ Charge on the capacitor at time t is $q = Ce$

$$= \frac{1}{2}\pi\omega a^2 CB\sin(\omega t)$$

But current $I = \dfrac{dq}{dt} = \dfrac{1}{2}\pi\omega^2 a^2 CB\cos(\omega t)$

50. **(2)** Initial flux: $\phi_i = -\displaystyle\int_a^{2a}\frac{\mu_0 Ia}{2\pi}\frac{dx}{x} = -\frac{\mu_0 Ia}{2\pi}\ln 2$

Final flux: $\phi_f = \dfrac{\mu_0 Ia}{2\pi}\ln 2$

Charge flown: $q = \dfrac{\Delta\phi}{r} = \dfrac{\phi_f - \phi_i}{r} = \dfrac{\mu_0 Ia}{\pi r}\ln 2$

51. **(3)** $e = \displaystyle\int_{2l}^{3l} B\omega x\,dx = \frac{5B\omega l^2}{2}$

52. **(3)** $\phi_A = \dfrac{\mu_0 i\pi R^2}{2\pi(R^2+x^2)^{3/2}}\pi r^2$

$$\Rightarrow \quad E_A = -\frac{d\phi}{dt} = \frac{-\mu_0 i\pi}{2}R^2r^2(-3/2)\,(R^2+x^2)^{-5/2}2x$$

E_A is maximum when $\dfrac{dE_A}{dx} = 0$

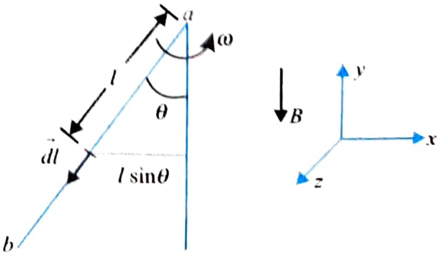
$$\Rightarrow \quad \frac{d}{dx}\frac{x}{(R^2+x^2)^{5/2}} = 0$$

$$\text{or} \quad (R^2+x^2)^{5/2} - \frac{5x}{2}(R^2+x^2)^{3/2}2x = 0$$

$$\text{or,} \quad R^2 + x^2 - 5x^2 = 0$$

$$\text{or,} \quad x = \frac{R}{2}$$

53. **(4)** $\varepsilon = \displaystyle\int(\vec{v}\times\vec{B}).\vec{dl}$



$$\Rightarrow \quad \varepsilon = \int\left[(l\sin\theta)\omega\hat{k})\times B(-\hat{j})\right]\cdot\left[dl\sin\theta(-\hat{i}) + dl\cos\theta(-\hat{j})\right]$$

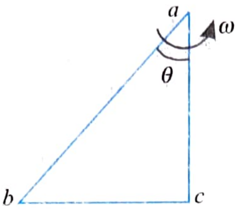
$$\Rightarrow \quad \varepsilon = \int[B\omega l\sin\theta\hat{i}]\cdot[dl\sin\theta(-\hat{i}) + dl\cos\theta(-\hat{j})]$$

$$= -B\omega\sin^2\theta\int_0^L l\,dl = -\frac{1}{2}B\omega\sin^2\theta L^2$$

$$\Rightarrow \quad \varepsilon = V_b - V_a = -\frac{1}{2}B\omega L^2\sin^2\theta$$

Negative sign indicates that end b will be negative w.r.t. a.

Alternate Method:



Consider two more conductors ac and bc. This completes a closed loop. The net emf induced in this closed loop should be zero, as net flux through this loop always remains zero.

$$e_{ab} + e_{bc} + e_{ca} = 0$$

but $e_{bc} = \frac{1}{2}B\omega(L\sin\theta)^2$, $e_{ca} = 0$ putting the values, we get

$$e_{ab} = -\frac{1}{2}B\omega(L\sin\theta)^2$$

54. (4) $e = \frac{d\phi}{dt} \Rightarrow i = \frac{e}{R} = \frac{1}{R}\frac{d}{dt}(BA) = \frac{A}{R}\frac{dB}{dt}$

where πr^2 = area of the loop of radius r and R = resistance of the loop of length $(2\pi r)$ and area of cross section πa^2.

$$R = \frac{\rho\ell}{\pi a^2} = \frac{\rho(2\pi r)}{\pi a^2}$$

Further mass of wire is $m = (\pi a^2)(2\pi r)(d)$

$$i = \frac{(\pi a^2)(\pi r^2)}{\rho(2\pi r)}\frac{dB}{dt}$$

$$i = \frac{(\pi a^2)(2\pi r)}{4\pi\rho}\frac{dB}{dt} \Rightarrow i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$$

55. (4) Induced electric field (in clockwise sense):

$$E = \frac{a^2}{2R}\frac{dB}{dt}$$

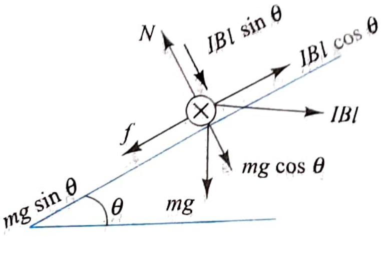
$$I\omega = \int\tau dt$$

$$\Rightarrow \quad mR^2\omega = \int qERdt = \int\frac{qa^2}{2R}\frac{dB}{dt}Rdt = \frac{\lambda 2\pi R}{2R}a^2R\int dB$$

$$\Rightarrow \quad \omega = \frac{B_0\pi a^2\lambda}{mR}$$

$$\Rightarrow \quad \vec{\omega} = -\frac{B_0\pi a^2\lambda}{mR}\hat{k} \text{ (because clockwise sense)}$$

56. (4) The front view of the arrangement is shown in figure.



From initial condition, $mg\sin\theta = \mu mg\cos\theta$

$$\Rightarrow \quad \mu = \tan\theta$$

$$ma = IB\ell\cos\theta - mg\sin\theta - \mu N$$

$$N = mg\cos\theta + IB\ell\sin\theta$$

$$\Rightarrow \quad a = \frac{IB\ell}{m}\cos\theta - 2g\sin\theta - \frac{IB\ell}{m}\frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{IB\ell}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta$$

Now, $s = \frac{1}{2}at^2 = \frac{1}{2}\left[\frac{IB\ell}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta\right]t^2$

57. (4) Induced emf between O and Q is

$$\frac{1}{2}B\omega(2r)^2 = B\omega 2r^2 = 2Bvr.$$

But net induced emf in the ring will be zero. Hence no current flows in the ring.

58. (3) $\oint\vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt}$ where $\phi = \int B2\pi r dr = \frac{2\pi}{3}B_0 t r^3$

$$E2\pi r = -\frac{2\pi B_0 r^3}{3} \Rightarrow E = -\frac{B_0 r^2}{3}$$

59. (3) $\phi = BA = \frac{\mu_0 i}{2\pi vt}a^2$

$$i = -\frac{1}{R}\frac{d\phi}{dt} = \frac{\mu_0 ia^2}{2\pi vt^2R} = \frac{\mu_0 ia^2v}{2\pi r^2R}$$

60. (1) The mutual inductance M between solenoid P and Q is given by emf induced in Q due to changing current in $P = M \times$ (rate of change of current I_p in P)

$$2 \times 10^{-3} = M \times (5)$$

$$\therefore \quad M = 4.0 \times 10^{-4}\text{ H}$$

Simiarly, emf induced in P due to changing current in $Q = M \times$ (rate of change of current I_Q in Q)

$\Rightarrow$ induced emf in $P = (4.0 \times 10^{-4}) \times (2) = 8.0 \times 10^{-4}$ V

61. (4) In first and third part, radius remains same, so there is no change in flux, hence no emf is induced. For second part:

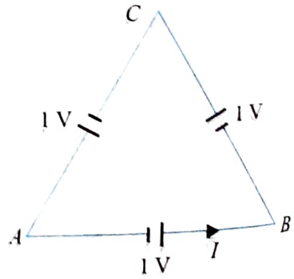
$$e = B\frac{dA}{dt} = B\frac{d(\pi r^2)}{dt} = B2\pi r\frac{dr}{dt}$$

dr/dt is constant, but r increases linearly, hence induced emf also increases linearly.

62. (1) $e = \frac{d\phi}{dt} = A\frac{dA}{dt} = \frac{1}{2}2 \times 2\sin 60°\sqrt{3} = 3$ V

Induced emf in each side = $e/3$

= 1 V



$$I = \frac{e}{R} = \frac{3}{1+2+2} = \frac{3}{5}\text{ A}$$

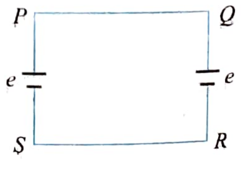
$$V_A + 1 - I \times 1 = V_B$$

$$\Rightarrow \quad V_A - V_B = I - 1 = \frac{-2}{5}$$

$$= -0.4\text{ V}$$

63. (4) Since entire loop is in magnetic field, so no net emf will be induced in the loop, hence no current will flow in the loop.

So $V_P = V_Q$, $V_S = V_R$, $V_P > V_S$, $V_P > V_R$, $V_Q > V_R$



64. (3) Induced emf's in the two loops will oppose each other. So net induced emf:

$$e = \frac{d}{dt}[(b^2 - a^2)B] = (b^2 - a^2)\frac{dB}{dt}$$

$$= (0.20^2 - 0.10^2)\,\omega B_0 \cos \omega t$$

$$= (0.04 - 0.01) \times 100 \times 10 \times 10^{-3} \cos \omega t$$

$$= 30 \times 10^{-3} \cos \omega t$$

Resistance: $R = 50 \times 10^{-3}\,[4 \times 20 + 4 \times 10] = 60 \times 10^{-3}\,\Omega$

$$I = \frac{e}{R} = 0.5 \cos \omega t$$

So amplitude of current = 0.5 A

65. (3) Given, $R = R_0 + t$

$$A = \pi R^2 = \pi(R_o + t)^2;\ \frac{dA}{dt} = 2\pi(R_o + t)$$

Therefore, emf induced in the loop is

$$e = B\frac{dA}{dt} = 2\pi(R_o + t)B$$

Since area is increasing, induced emf is in anticlockwise sense to oppose the change in flux.

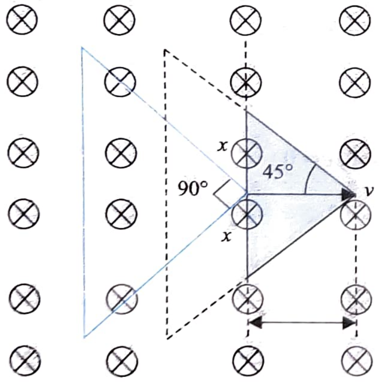
66. (3) For $0 < t < t_1$, $\frac{dB}{dt}$ is positive and hence, $e = -A\frac{dB}{dt}$ is constant and negative.

For $t_1 < t < t_2$, $\frac{dB}{dt}$ is zero and hence, $e = -A\frac{dB}{dt}$ is also zero.

For $t_2 < t < t_3$, $\frac{dB}{dt}$ is negative and hence, $e = -A\frac{dB}{dt}$ is constant and positive.

67. (3) Emf across the capacitor is constant and equal to Blv. In steady state, no current flows through the circuit

68. (4) $\phi = BA = B\left[\frac{1}{2}(2x)(x)\right]$



$$\phi = Bx^2$$

$$|\varepsilon| = \frac{d\phi}{dt} = 2Bx \cdot v$$

$$I = \frac{\omega}{(R)} = \left(\frac{2Bv}{R}\right)x$$

69. (2) $Q = EI = (Blv)I = Fv \Rightarrow F = \frac{Q}{v}$

70. (4) As rod is moving with constant velocity, the potential difference across capacitor will remain constant. Hence no current will flow through capacitor branch.

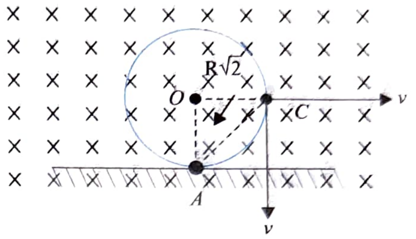
71. (2) $e = Bv\ell = \frac{B \times a \times \omega \times a}{2}$

$$i = \frac{Ba^2\omega}{2R}$$

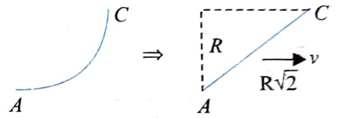
$$F = i\ell B = \frac{Ba^2\omega}{2R} \times a \times B = \frac{B^2a^3\omega}{2R}$$

towards right of OA

72. (4) Point C has two components of velocity: one is along the velocity of ring and other is along the tangent to the ring at that point. That component which is tangent to ring will not create any emf, because it is parallel to the length.



Since magnetic field is constant, so circular part of the ring between AC can be replaced by rod AC as shown.



So emf across $AC = BvR$

73. (1) As the loop is very small the distance of every point in its plane can be taken to be equal to $d = 1$ m

$$I = 3.36\,(1 + 2t) \times 10^{-2} \text{ ampere}$$

$$\frac{dI}{dt} = 2 \times 3.36 \times 10^{-2} \text{ m/sec}$$

Magnetic induction at every point on the loop, $B = \frac{\mu_0 I}{2\pi d}$

Magnetic flux linked with loop at any instant,

$$\phi = BA = \frac{\mu_0 I}{2\pi d} \cdot \pi r^2$$

Induced emf, $e = \frac{d\phi}{dt} = \frac{\mu_0 r^2}{2d}\left(\frac{dI}{dt}\right)$

Induced current, $I = \frac{e}{R} = \frac{\mu_0 r^2}{R \times 2d}\left(\frac{dI}{dt}\right)$

$$= \frac{4\pi \times 10^{-7} \times (10^{-3})^2 \times 2 \times 3.36 \times 10^{-2}}{8.4 \times 10^{-4} \times 2 \times 1}$$

$$= 5.024 \times 10^{-11} \text{ amp}$$

74. (3) Consider an element of width dy at a distance y from origin.

Induced emf across its ends is Bv_0dy, where $B = \frac{B_0}{L}y$.

Integrating from 0 to L, $E = \frac{B_0v_0l}{2}$

75. (4) $V_R - V_P = \int(\vec{v} \times \vec{B}) \cdot d\vec{\ell}$

$$= \int(v\hat{i} + v\hat{j}) \times (-3x\hat{k}) \cdot dx\hat{i}$$

$$= -3v\int x(-\hat{j} + \hat{i})\,.\,dx\hat{i}$$

$$= -3v\int_0^{1.5L} x\,dx = \frac{-27}{8}vL^2$$

$$V_Q - V_R = \int (v\hat{i} + v\hat{j}) \times (-3(1.5L)\hat{k}) . dy\hat{j}$$

$$= -4.5vL \int (-\hat{j} + \hat{i}) . dy\hat{j}$$

$$= 4.5vL \int_0^{1.5L} dy = \frac{27}{4} vL^2$$

From above:

$$V_P - V_Q = \frac{27}{8} vL^2 - \frac{27}{4} vL^2 = \frac{-27}{8} vL^2$$

76. **(3)** In steady state, velocity of both wires will become same.

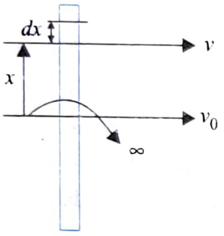
$$2\,mv = mv_0 \quad \Rightarrow \quad v = v_0/2$$

$$\text{Loss in } KE = \frac{1}{2} mv_0^2 - 2\left[\frac{1}{2} mv^2\right] = \frac{1}{4} mv_0^2$$

77. **(1)** $v = v_0 + wx = v_0 + \frac{v_0}{l} x$ due to mutual induction

$$\int de = \int B\,vdx$$

$$e = \int_0^{l/2} B\left(v_0 + \frac{v_0 x}{l}\right) dx = \frac{5}{8} B_0 v_0 l$$



78. **(1)** Flux through the ring increases in upward direction. To oppose the same, it is thrown up. As the ring moves up, the intensity of magnetic field due to electromagnet at upper points is less since $B \propto \left(\frac{1}{r^3}\right)$

79. **(2)** $F = Bil \quad \Rightarrow \quad i = \frac{F}{Bl} = 2.5$ A

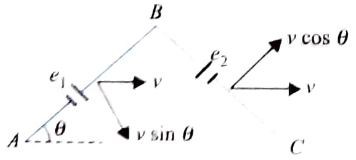
80. **(3)** We know that the moment of inertia of a symmetrical planar body about any axis lying in its plane and passing through its centre is same. Further, angular velocity is also same in both cases. Thus, induced emf is also same.

81. **(2)** At mean position, velocity is maximum and hence, rate of change of flux is also maximum.

82. **(2)** Let $AB = BC = l$

Velocity = v

Magnetic field = B



$E_1 = Bv\,l \sin \theta$

$E_2 = Bv\,l \cos \theta$

For induced emf at $A >$ induced emf at B.

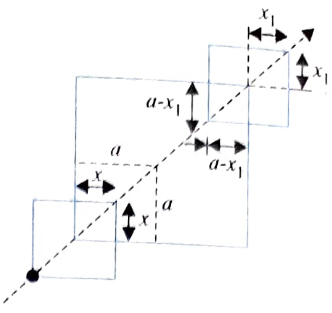
$E_2 > E_1$

$Bv\,l\,(\cos \theta - \sin \theta) > 0$

$\cos \theta > \sin \theta \quad \Rightarrow \quad \tan \theta < 1$

$\theta < 45°$

83. **(4)** The graph is to be drawn qualitatively



As shown, for $0 \le x \le a$

$$\phi = Bx^2$$

$$e = \frac{-d\phi}{dt} = -2Bx \cdot \frac{dx}{dt} = -2Bxv$$

At $x = 0, e = 0$

At $x = a/2, e = -Bav$

As, $x \to a$ but $x \ne a, e \approx -2\,Bav$

As, $x = a, e = 0$ as $\phi =$ constant

Distance along diagonal $x_0 = x\sqrt{2}$, thus, $x_0 = f(x)$.

When the loop reaches completely inside the field, it starts coming out of the field after some moment. When the loop is coming out, x_1 should be selected carefully, for which

$$\phi = B(a - x_1)^2$$

$$e = \frac{-d\phi}{dt} = -2B(a - x_1) \cdot (v)$$

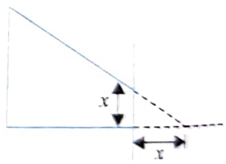
$$= 2B\,(a - x_1)\,v$$

At $x_1 = 0, e = 2\,Bav$

$x_1 = a, e = 0$

Thus, before the loop enters the field completely, $e = -2Bav$. When the loop is completely inside the field, $e = 0$ (ϕ = constant). As the loop starts coming out of the field ($e = 2Bav$). Option (4) is correct.

84. **(3)** For $0 \le x \le a,\ \phi = \frac{1}{2} Bx^2$



$$e = -\frac{d\phi}{dt} = -Bav$$

At $x = 0, e = 0$

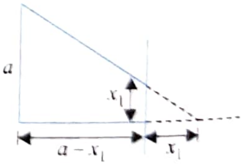
At $x = a/2, e = -\frac{Bav}{2}$

At $x = x \to a, e \approx -Bav$

As $x = a, e = 0$

(ϕ = constant)

For $a \le x \le 2a$



$$\phi = \frac{1}{2} Ba^2 = \text{constant}$$

For $2s \le x \le 3a,\ \phi = B\left[\frac{a^2}{2} - \frac{x_1^2}{2}\right]$

$$e = \frac{(-)d\phi}{dt} = -\frac{d}{dt}\left[\frac{B}{2}(a^2 - x_1^2)\right] = Bx_1 v$$

At $x_1 = 0, e = 0$

At $x_1 = a/2,\ e = \frac{B}{2} av$

At $x_1 = a, e = 0$, loop is outside the field.

85. **(2)** $$\frac{B^2 v_1 l_1^2}{R} - mg = ma_1 \quad \ldots(i)$$

$$\frac{B^2 v_1 l_2^2}{R} - mg = ma_2 \quad \ldots(ii)$$

$$a_1 - a_2 = \frac{B^2}{mR}(v_1 l_1^2 - v_2 l_2^2)$$

For $a_1 > a_2$, $v_1 l_1^2 > v_2 l_2^2$

$$\frac{l_1}{l_2} > \left(\frac{v_2}{v_1}\right)^{1/2}$$

86. **(1)** $$\frac{mgh - \frac{1}{2}mv_1^2}{mgv_1} = \frac{mgh - \frac{1}{2}mv_2^2}{mgv_2}$$

$$ghv_2 - \frac{1}{2}v_1^2 v_2 = ghv_1 - \frac{1}{2}v_2^2 v_1$$

$$gh(v_1 - v_2) = \frac{1}{2}v_1 v_2 (v_1 - v_2)$$

$$gh + \frac{1}{2}v_1 v_2 = 0$$

Multiple Correct Answers Type

1. **(1), (2)**

As $\vec{B} \perp \vec{A}$, hence $\phi = 0$ and $e = 0$.

2. **(1), (2), (3), (4)**

$\phi = BA\cos\theta$, $\theta = \omega t$

$$e = \frac{-d\phi}{dt} = BA\omega \sin\theta$$

at $\theta = \frac{\pi}{2}$, $\phi = 0$ and e is maximum. At $\theta = 0$, ϕ is maximum and e is zero. Emf is maximum when $\theta = \frac{\pi}{2}$ and for this plane of loop is parallel to magnetic field. Clearly phase difference between flux and emf is $\pi/2$.

3. **(2), (3)**

Thought based.

4. **(1), (3)**

Current induced in both A and B will be in same direction. So they will attract each other.

A is closer to magnet, so rate of change of flux in A will be more. So more current is induced in A.

5. **(1), (3)**

Use Lenz's law. The motion of ring will be opposed.

6. **(1), (3), (4)**

No emf is induced because flux through the ring does not change. Hence no current flows in the ring. Emf is induced between A and D because length AD is perpendicular to v. But no emf is induced across CE because length CE is parallel to v.

7. **(1), (2), (3), (4)**

Due to rotation, emf $= \dfrac{Br^2\omega}{2}$

Due to translation indeed emf $= Bvr$

where r is the separation.

8. **(2), (3)**

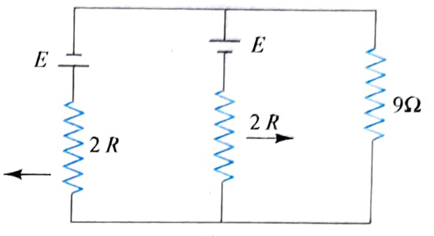
Each wire can be replaced by a battery whose emf is equal to

$$Blv = 1 \times 4 \times 10^{-2} \times 5 \times 10^{-2}$$
$$= 20 \times 10^{-4}\ \text{V}$$

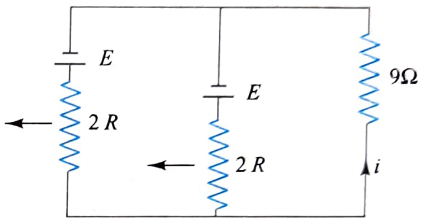
The polarity of the battery can be given by Fleming's right hand rule. When both wire move in opposite direction, the circuit diagram looks like as shown in Fig. (a).

The effective emf of the two batteries shown in the diagram is zero. So, choice (2) is correct and choice (4) is wrong.

When both wires move towards left, the circuit diagram looks like as shown in Fig. (b).



(a)



(b)

Effective emf of two batteries shown is E ($= 20 \times 10^{-4}$ V) and internal resistance is 1 Ω.

Hence, current in the circuit is

$$i = \frac{20 \times 10^{-4}}{10} = 0.2\ \text{mA}$$

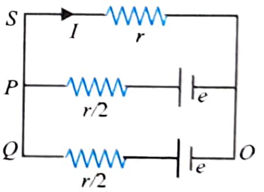
Hence, choice (3) is correct and choice (1) is wrong.

9. **(1), (2), (3), (4)**

Use concept of motional emf

10. **(2), (4)**

Equivalent circuit:



Current is from S to O, i.e., from circumference to centre

$$I = \frac{e}{r + r/4} = \frac{4}{5r}\left(\frac{1}{2}B\omega a^2\right) \Rightarrow I = \frac{2B\omega a^2}{5r}$$

11. **(1), (2), (3)**

$$\phi = 4t^n + 6$$

$$\frac{d\phi}{dt} = 4nt^{n-1}$$

$$|e| = 4nt^{n-1},\ |e| = \frac{4n}{t^{1-n}}$$

12. **(2), (4)**

Flux remains constant here, so emf induced is zero.

13. **(2), (4)**

$$i = \frac{dq}{dt} = \frac{d}{dt}(CvBl) = CBl\frac{dv}{dt} = CBla$$

$\therefore$ $F - CB^2l^2 a = ma$

$$\Rightarrow \quad a = \frac{F}{m + B^2l^2C}$$

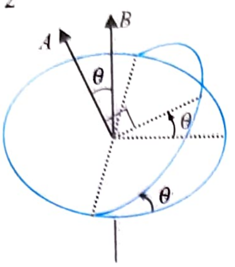
$\Rightarrow$ emf increases

$\Rightarrow$ charge increases.

14. (1), (4)

$\theta = \omega t$. Only half circular part will be involved in inducing emf, so

effective area $A = \dfrac{\pi a^2}{2}$



$\phi = BA\cos\theta$

$$e = -\frac{d\phi}{dt} = +BA\sin\theta\left(\frac{d\theta}{dt}\right) \Rightarrow e = \frac{B\pi a^2}{2}\omega\sin\theta$$

$$I = \frac{e}{R} = \frac{B\pi a^2\omega}{2R}\sin\theta$$

Clearly $I = 0$, when $\theta = 0°$ and when $\theta = \dfrac{\pi}{2}$, $I = \dfrac{B\pi a^2\omega}{2R}$

15. (1), (4)

Apply Lenz's law

16. (2), (3)

$$|\varepsilon_i| = \left|-\frac{d\phi}{dt}\right| = |\varepsilon_i| \propto \left|-\frac{d\phi}{dt}\right|$$

and also area under the curve gives

Change in flux $= \phi_f - \phi_i = \displaystyle\int d\phi = \int \varepsilon_i dt$

Total charge that will flow $= \Delta\phi/R$

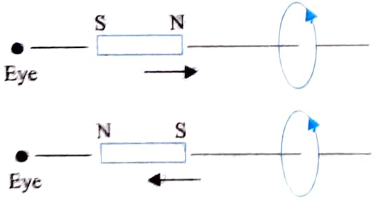
17. (1), (3), (4)

Apply Lenz's law

18. (1), (2), (3), (4)

Since line joining the ends of the ring is parallel to B, hence no emf will be induced in any case.

19. (2), (3)



20. (1), (4)

If $\dfrac{di}{dt} = 0$, it means current is constant, its flux through loop will remain constant. Hence no emf is induced.

If $\dfrac{di}{dt} > 0$, current increases so flux increates in loop in inward direction, so induced current should be anticlockwise which produces flux in outward direction.

21. (1), (2), (4)

As long as the rod moves at a constant speed, all the work done by external person pulling the rod is dissipated as heat in the resistor. As per work–energy theorem, there is no change in kinetic energy of rod.

Here, force applied by external agent is $F = BIl$.

However, if applied external force is doubled then a part of external power increases the velocity of rod and the rest is dissipated as heat. Obviously, Lenz's law is not satisfied under such circumstances. Further, $P = E^2/R = B^2l^2v^2/R$. Thus, if resistance R is doubled then power required to maintain the constant velocity v_0 becomes half.

22. (1), (2), (3)

$E = NAB\omega\cos\theta$, where θ is the angle between plane of coil and magnetic field. Substituting values, options (1), (2) and (3) are correct.

23. (1), (2), (3), (4)

While the frame enters the magnetic field, magnetic flux through the frame is increasing. Hence from Lenz's law, induced current will be anticlockwise.

So, choice (1) is correct.

After 1 sec, Distance moved by frame

$$S = \frac{1}{2}at^2$$

$$S = \frac{1}{2} \times 1 \times 1^2 = 0.5 \text{ m}$$

and speed $v = at = 1 \times 1 = 1$ m/s

$$\text{Induced emf} = Bv\frac{L}{5}$$

$$= 5 \times 1 \times 1 = 5 \text{ V}$$

Hence induced current

$$i = \frac{\varepsilon}{R} = \frac{5}{4} = 1.25 \text{ A}$$

(resistance of loop = 4 Ω)

So, choice (2) is correct.

$$\text{Force } F = iB\frac{L}{5} = 1.25 \times 5 \times 1 = 6.25 \text{ N}$$

Since magnetic moment and magnetic field are anti-parallel so torque will be zero. So, choice (4) is correct.

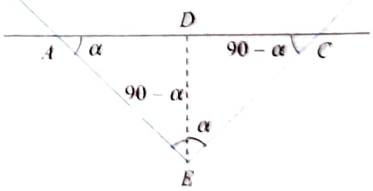
Hence choices (1); (2), (3) and (4) are correct.

24. (1), (4)

PQ is parallel to magnetic field, hence no emf is induced in it. QR is ⊥ perpendicular to magnetic field. Induced emf $= Bvl = B_0\omega a a = B_0\omega a^2$

25. (2), (4)

Let $\angle CAE = \alpha = \text{constant}$



At time t, $DE = vt$

$AD = DE\cot\alpha$

and $DC = DE\tan\alpha$

$AC = DE(\tan\alpha + \cot\alpha)$

$= vt(\tan\alpha + \cot\alpha) = l$ (say)

Induced emf $e = Bvl$

$e = Bv^2t(\tan\alpha + \cot\alpha)$

$e \propto v^2$ and $e \propto t$

26. (1), (3)

According to Lenz's law, current I is in clockwise direction

$$A'B' = 2A'O = \frac{2}{\tan 60°}\left(\frac{r\sqrt{3}}{4}\right) = \frac{r}{2}$$

For equilibrium $mg = I\left(\frac{r}{2}\right)B$

$$I = \frac{2mg}{rB}$$

If loop is displaced by x,

F = Restoring force

$$= -I\left[\left(\frac{2}{\tan 60^\circ}\right)\left(\frac{r\sqrt{3}}{4} + x\right)\right]B + mg$$

$$= -\frac{IrB}{2} + mg - \frac{2IB}{\sqrt{3}}x$$

$$= -\frac{2IB}{\sqrt{3}}\cdot x$$

$F \propto (-x)\, x$, motion is SHM.

$$a = -\frac{2IB}{m\sqrt{3}}\cdot x$$

$$T = 2\pi\left[\frac{m\sqrt{3}}{2IB}\right]^{1/2} = \pi\left[\frac{r\sqrt{3}}{g}\right]^{1/2}$$

Linked Comprehension Type

For Problems 1–2

1. (1) **2. (4)**

Current should enter the bar from P so that magnetic force is upwards.

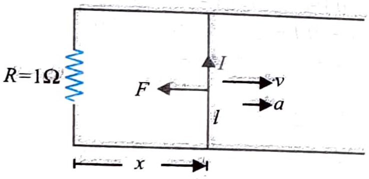
$$ilB = mg \quad \text{or} \quad \frac{V}{5}lB = mg$$

or $$m = \frac{150 \times 0.6 \times 1.5}{5 \times 10} = 2.7 \text{ kg}$$

For Problems 3–5

3. (1) **4. (4)** **5. (1)**

Let anticlockwise direction is positive



$$\phi = BA\cos 180^\circ = cxlx\cos 180^\circ = -clx^2$$

$$e = -\frac{d\phi}{dt} = cl2x\frac{dx}{dt}, \quad I = \frac{e}{R} = \frac{2clxv}{1} = 2clxv$$

This is positive so current is anticlockwise.

$$F = m(-a) \Rightarrow IBl = -mv\frac{dv}{dx} \Rightarrow 2c^2l^2x^2v = -mv\frac{dv}{dx}$$

$$\Rightarrow \quad 2c^2l^2\int_0^x x^2 dx = -\int_{10}^{5} m dv \Rightarrow 2c^2l^2\frac{x^3}{3} = m(5)$$

$$\Rightarrow \quad x^3 = \frac{15m}{2c^2l^2} \Rightarrow x = \left(\frac{15}{2}\right)^{\frac{1}{3}}$$

Heat produced = loss in KE of rod $= \frac{1}{2} \times 1[10^2 - 5^2] = \frac{75}{2} = 37.5$ J

Magnetic force is not doing any work.

For Problems 6–7

6. (2) **7. (3)**

$$\varepsilon_{av} = -\frac{\Delta\Phi_B}{\Delta t} = -B\frac{\Delta A}{\Delta t} = -B\frac{(-\pi r^2)}{\Delta t}$$

$$= \frac{1 \times \pi(0.10)^2}{0.314} = 0.1 \text{ V}$$

Since the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore, the current flows in clockwise direction

$$I_{av} = \frac{\varepsilon_{av}}{R} = \frac{0.1}{0.01} = 10 \text{ A}$$

For Problems 8–9

8. (2) **9. (3)**

Magnetic field on the axis of a circular coil is given by

$$B = \frac{\mu_0 iR^2}{2(R^2 + x^2)^2}$$

Since $x >> R$, therefore, magnetic field at the centre of the smaller loop is

$$B \approx \frac{\mu_0 iR^2}{2x^3}$$

Flux linked with coil is

$$\phi = B(\pi r^2) = \frac{\mu_0 \pi iR^2 r^2}{2x^3}$$

From Faraday's law we have

$$E = -\frac{d\phi}{dt} = \frac{3}{2}\frac{\mu_0 \pi iR^2 r^2}{x^4}v$$

For Problems 10–12

10. (4) **11. (1)** **12. (2)**

The large circuit is a time constant of

$$\tau = RC = (10\ \Omega)(20 \times 10^{-6}\text{ F}) = 200\ \mu\text{s}.$$

Thus, the current as a function of time is

$$i = \left(\frac{100\text{ V}}{10\ \Omega}\right)e^{-\frac{t}{200\mu\text{s}}}$$

At $t = 200$ ms, we obtain

$$i = (10\text{ A})(e^{-1}) = 3.7\text{ A}.$$

Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop,

$$\Phi_B = \int_c^{c+a}\frac{\mu_0 ib}{2\pi r}dr = \frac{\mu_0 ib}{2\pi}\ln\left(1 + \frac{a}{c}\right)$$

So the emf induced in the small loop at $t = 200$ ms

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{\mu_0 b}{2\pi}\ln\left(1 + \frac{a}{c}\right)\frac{di}{dt}$$

$$= -\frac{\left(4\pi \times 10^{-7}\frac{Wb}{A \times m^2}\right)(0.200\text{ m})}{2\pi} \times \ln(3.0)\left(-\frac{3.7\text{ A}}{200 \times 10^{-6}\text{s}}\right)$$

Thus, the induced current in the small loop is

$$i' = \frac{\varepsilon}{R} = \frac{0.81\text{ mV}}{25(0.600\text{ m})(1.0\ \Omega/\text{m})} = 54\ \mu\text{A}.$$

Initially current in large loop is maximum and afterwards decreases. Hence flux through the smaller loop decreases with time. The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

For Problems 13–14

13. (2) **14. (1)**

Let us take anticlockwise direction as positive, then area vector in upward direction will be positive

$$\phi = BA = Bl\,x$$

$$e = -\frac{d\phi}{dt} = -\ell\left[B\frac{dx}{dt} + x\frac{dB}{dt}\right]$$

$$= -5\times10^{-2}\,[0.1\times(-5\times10^{-2}) + 5\times10^{-2}\times0.2]$$

$$= -250\times10^{-6}\text{ V} = -250\ \mu\text{V}$$

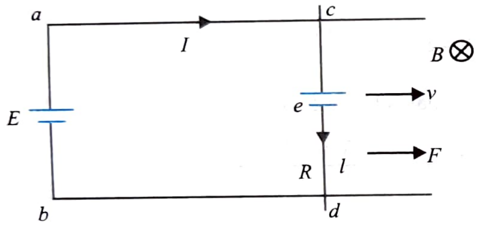
Emf is coming out to be negative, so it should be clockwise.

$$\text{Current: } I = \frac{e}{R} = \frac{250\times10^{-6}}{10^{-4}} = 2.5\text{ A}$$

For Problems 15–17

15. (1) **16. (3)** **17. (2)**

Due to applied emf, current will flow as shown. Due to this current and magnetic field B, force on the wire will act towards right and rod will start moving towards right. Let at any instant, its velocity is v. Due to this velocity, emf $e = Blv$ will be induced as shown.



$$\text{Net emf: } E - B\ell v,\ I = \frac{E - B\ell v}{R} \qquad \text{...(i)}$$

$$F = IB\ell = \left(\frac{E - B\ell v}{R}\right)B\ell \qquad \text{...(ii)}$$

$$\Rightarrow \quad m\frac{dv}{dt} = (E - B\ell v)\frac{B\ell}{R}$$

$$\frac{mR}{B\ell}\frac{dv}{dt} = E - B\ell v \Rightarrow \int_0^v \frac{dv}{E - B\ell v} = \int_0^t \frac{B\ell}{mR}dt$$

$$\Rightarrow \quad \left[\frac{\ln(E - B\ell v)}{-B\ell}\right]_0^v = \frac{B\ell}{mR}t \Rightarrow \ln\left(\frac{E - B\ell v}{E}\right) = \frac{-B^2\ell^2}{mR}t$$

$$\Rightarrow \quad \frac{E - B\ell v}{E} = e^{-t/\tau} \Rightarrow v = \frac{E}{B\ell}\left[1 - e^{-t/\tau}\right]$$

Velocity will increase upto $t = \infty$, and at $t = \infty$, velocity will be maximum and constant.

$$v_{t=\infty} = \frac{E}{B\ell} \quad \rightarrow \quad \text{This is the required terminal velocity.}$$

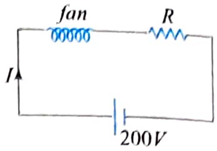
Also, when the velocity is constant, net force on the rod will become zero, so putting $F = 0$ in equation (ii), we get

$$v_{\text{terminal}} = \frac{E}{B\ell}$$

when the rod attains terminal velocity, then from equation (i), $I = 0$.

For Problems 18–20

18. (3) **19. (3)** **20. (2)**



The fan is running at 200 V, consuming 1000 W, then

$$I = \frac{1000}{200} = 5\text{ A}$$

But as coil resistance is 1 Ω, power dissipated by internal resistance as heat is $P_1 = I^2R = 25$ W.

If V is the net emf across the coil, then

$$\frac{V^2}{R} = 25\text{ W} \quad \text{or} \quad V = 5\text{ V}$$

Net emf = source emf – back emf

or $\quad V = V_S - e \quad \Rightarrow \quad e = 195$ V

The work done $P_2 = 1000 - 25 = 975$ W.

For Problems 21–23

21. (1) **22. (2)** **23. (3)**

$$\frac{dB}{dt} = 2\text{ T/s}$$

$$E = -\frac{AdB}{dt} = -800\times10^{-4}\text{ m}^2\times2 = -0.16\text{ V}$$

$$i = \frac{0.16}{1\,\Omega} = 0.16\text{ A, clockwise}$$

$$\text{At } t = 2\text{ s, } B = 4\text{ T, } \frac{dB}{dt} = 2\text{ T/s}$$

$$a = 20\times30\text{ cm}^2$$

$$= 600\times10^{-4}\text{ m}^2;\ \frac{dA}{dt} = -(5\times20)\text{ cm}^2/\text{s}$$

$$= -100\times10^{-4}\text{ m}^2/\text{s}$$

$$E = -\frac{d\phi}{dt} = -\left[\frac{d(BA)}{dt}\right] = -\left[\frac{BdA}{dt} + \frac{AdB}{dt}\right]$$

$$= -[4\times(-100\times10^{-4}) + 600\times10^{-4}\times2]$$

$$= -[-0.04 + 0.120] = -0.08\text{ V}$$

Alternative: $\phi = BA = 2t\times0.2\,(0.4 - vt)$

$$= 0.16\,t - 0.4\,vt^2$$

$$E = -\frac{d\phi}{dt} = 0.8\,vt - 0.16$$

At $\quad t = 2$s

$$E = 0.08\text{ V}$$

At $t = 2$ s, length of the wire = (2 × 30 cm) + 20 cm = 0.8 m

Resistance of the wire = 0.8 Ω

$$\text{Current through the rod} = \frac{0.08}{0.8} = \frac{1}{10}\text{ A}$$

Force on the wire is $= ilB$

$$= \frac{1}{10}\times(0.2)\times4 = 0.08\text{ N}$$

Same force is applied on the rod in opposite direction to make net force zero.

For Problems 24–26

24. (3)

25. (4) When $x = -\frac{9}{5}a : |x| > a + \frac{a}{2} = \frac{3}{2}a$, so the loop is outside the magnetic field therefore, both induced current and external force required are zero.

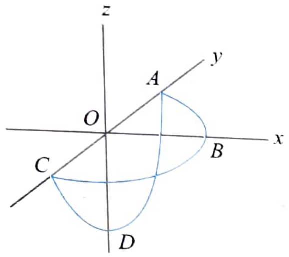
26. **(3)** At $x = -\frac{a}{4}$, loop is completely inside the field. Flux through the loop is $BA = Ba^2$. As $\frac{dB}{dt} = 0$, induced current is zero so is external force is required.

For Problems 27–29

27. **(4)** It is given $\vec{B} = \frac{(\vec{i} - \vec{k})}{\sqrt{2}}|B|$ and $\left|\frac{dB}{dt}\right| = 10^{-2}$ T/s

It is obvious that B is directed perpendicular to y-axis. Components B_x and B_z of the field B are $B_x = \frac{B}{\sqrt{2}}$ and $B_z = -\frac{B}{\sqrt{2}}$.

Then the flux (at instant t) through $ABC = \phi_1 = \left(\frac{\pi a^2}{2}\right)B_z$



Hence emf e_1 induced in loop ABC is

$$e_1 = -\frac{d\phi_1}{dt} = -\frac{\pi a^2}{2}\cdot\frac{dB_z}{dt} = +\frac{\pi a^2}{2\sqrt{2}}\left(\frac{dB}{dt}\right)$$

This emf will be directed clockwise (looking along negative z-axis) $\overline{ABC}$.

Similarly the flux (at instant t) through $ADC = \phi_2 = \frac{\pi a^2}{2}Bx$.

Hence emf e_2 induced in loop ADC is

$$e_2 = -\frac{d\phi_2}{dt} = -\frac{\pi a^2}{2}\frac{dB_x}{dt} = -\frac{\pi a^2}{2\sqrt{2}}\left(\frac{dB}{dt}\right)$$

This emf will be directed clockwise (looking along positive x-axis) $\overline{CDA}$.

It is easy to see that these two emfs along the two semicircles will add up.

Hence induced emf in the closed loop will be

$$e = |e_1| + |e_2| = \frac{\pi a^2}{2\sqrt{2}}\frac{dB}{dt} + \frac{\pi a^2}{2\sqrt{2}}\frac{dB}{dt}$$

It is given $\frac{dB}{dt} = 10^{-2}$ T/s and $a = 0.5$ m

Hence $e = \frac{\pi(0.5)^2}{\sqrt{2}}\cdot 10^{-2}\text{ V} = \frac{5\pi}{2\sqrt{2}}\times 10^{-3}\text{ V} = \frac{5\pi}{2\sqrt{2}}$ mV

This will be in clockwise sense, when looking along B.

28. **(3)** It is given that $\vec{B} = \frac{(\vec{i} - \vec{j})}{\sqrt{2}}|B|$

This direction is perpendicular to z-axis.

Hence there will be no flux in the semicircular section ABC.

The induced emf in the other semicircular section ADC will be

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\vec{B}\cdot\vec{A}\right]$$

$$= -\frac{d}{dt}\left[B\cdot\frac{\pi a^2}{2}\cdot\cos\frac{\pi}{4}\right]$$

since the field makes an angel of $\pi/4$ with the yz plane

$$e = \frac{\pi a^2}{2\sqrt{2}}\cdot\frac{dB}{dt} = \frac{\pi a^2}{2\sqrt{2}}\cdot 10^{-2}\text{ V}$$

Putting $a = 0.5$ m, $e = \frac{\pi(0.5)^2}{2\sqrt{2}}10^{-2}\text{ V} = \frac{5\pi}{4\sqrt{2}}$ mV

This will be in the clockwise sense looking along positive x-axis i.e., $\overline{CDA}$.

29. **(2)** Since the field is directed along the z-axis, there will be no flux through the semicircular section ADC.

The flux through section $ABC = \phi = B.A$ or $\phi = B\cdot\frac{\pi a^2}{2}$ direction along z-axis.

The induced emf $e = -\frac{d\phi}{dt} = -\frac{\pi a^2}{2}\left|\frac{dB}{dt}\right|$

Substituting values, $a = 0.5$, $\frac{dB}{dt} = +10^{-2}$ T/s

$$|e| = \frac{\pi a^2}{2}\left|\frac{dB}{dt}\right| = \frac{\pi(0.5)^2}{2}10^{-2}\text{ V} = \frac{5\pi}{4}\text{ mV}$$

When looking along field direction, (i.e.,) along z-axis, this will be counter clockwise sense, since the field increases with time.

For Problems 30–31

30. **(1)** 31. **(3)**

i. Velocity of wire frame when it starts entering the magnetic field.

$$V_1 = \sqrt{2gh} = \sqrt{2(10)(5)} = 10\text{ m/s}$$

and the time taken is, $t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(5)}{10}} = 1$ s

ii. When the frame has partially entered the field, the induced emf produced is $\varepsilon = Blv$

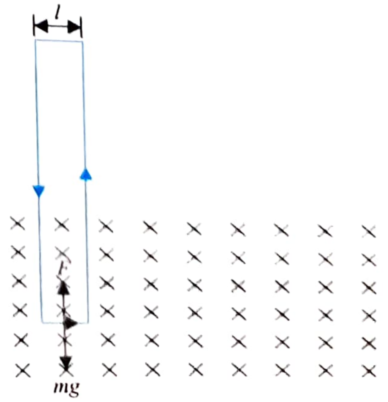
$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}\text{ (anticlockwise)}$$

Ampere's force, $F = \frac{B^2l^2v}{R}$ (upward)

Putting $m = 0.5$ kg, $B = 1$ T, $\ell = 0.25$ m, $v = 10$ m/s, $R = 1/8$ Ω

We get, $F = \frac{(1)^2(0.25)^2(10)}{1/8} = 5$ N

Since, $mg = (0.5)(10) = 5$ N



Therefore, using Newton's second law, the acceleration of the wire frame while entering the magnetic field is zero. Thus, time taken to completely enter into the field is $t_2 = \frac{2}{10} = 0.2$ s

iii. When the frame has completely entered the field, the current becomes zero and thus, the ampere's force also become zero. The frame accelerates under gravity only.

$$\therefore \quad 15 = 10t_3 + 5t_3{}^2$$

$$\text{or,} \quad t_3^2 + 2t_3 - 3 = 0 \quad \text{or,} \quad t_3 = 1 \text{ s}$$

The total time taken is

$$T = t_1 + t_2 + t_3 = 1 + 0.2 + 1 = 2.2 \text{ s}$$

For Problems 32–33

32. (1)

33. (2) Since magnetic field strength B varies with time, therefore an emf is induced in the loop and a current starts flowing through the loop.

Due to flow of current heat is generated.

At an instant t, flux linked with the loop

$$\phi = \pi a^2 B_0(tT - t^2)$$

Induced emf, $e = -\frac{d\phi}{dt} = -\pi a^2 B_0(T - 2t)$

Induced current, $i = \frac{e}{R} = \frac{\pi a^2 B_0}{R}(2t - T)$

Thermal power generated at this instant,

$$P = \frac{\pi^2 a^4 B_0^2}{R}(2t - T)^2$$

During an elementary time interval dt, heat generated

$$= Pdt = \frac{\pi^2 a^4 B_0^2}{R}(2t - T)^2 \cdot dt$$

Total heat generated from $t = 0$ to $t = T$.

$$Q = \int Pdt = \frac{\pi^2 a^4 B_0^2}{R}\int_0^T (2t - T)^2\, dt$$

$$= \frac{\pi^2 a^4 B_0^2 T^3}{3R}$$

The current reverses its sign when its magnitude reduces to zero. Let this happen at instant $t = t_0$. Substituting t by t_0 in equation (i).

$$\frac{\pi a^2 B_0}{R}(2t_0 - T) = 0$$

$$t_0 = \frac{T}{2}$$

Substituting I by dq/dt in equation (i),

$$dq = \frac{\pi a^2 B_0}{R}(2t - T)dt$$

$\therefore$ Charge that flows from $t = 0$ to $t = T/2$,

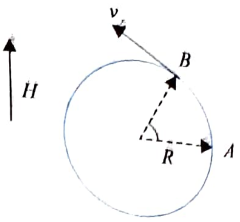
$$q = \frac{\pi a^2 B_0}{R}\int_0^{T/2} (2t - T)dt = -\frac{\pi a^2 B_0 T^2}{4R}$$

or magnitude of charge that flows $= \frac{\pi a^2 B_0 T^2}{4R}$

For Problems 34–35

34. (3)

35. (2) Path of the car is a circle of radius R as shown in figure.



Let at instant $t = 0$, the car be at A when the axis of ring was parallel to horizontal component H of earth's magnetism.

Since, car completes n revolutions per minute, therefore, angular velocity of car is $\omega = \left(\frac{2\pi n}{60}\right)$

Hence, at time t, its position B is given by,

$$\theta = \omega t = \frac{2\pi nt}{60} = \frac{\pi nt}{30}$$

Axis of the ring has also turned through the same angle θ. Therefore, component of H along ring axis is now equal to $H \cos \theta$.

Hence, flux linked with the ring at this instant is,

$$\phi = \pi a^2(H \cos \theta) = \pi a^2 H \cos\left(\frac{\pi nt}{30}\right)$$

Induced emf,

$$e = -\frac{d\phi}{dt} = \frac{\pi^2 a^2 nH}{30}\sin\left(\frac{\pi nt}{30}\right)$$

Resistance of the ring $= 2\ \pi a l$.

Due to induced emf, a current starts to flowing through the ring and due to this current heat is produced in the ring. At time t, thermal power,

$$P = \left(\frac{e^2}{\text{Resistance}}\right)$$

Period of one complete revolution of the car is,

$$T = \frac{60}{n} \text{ sec}$$

Average power,

$$P_{\text{avg}} = \frac{\int_0^T p\, dt}{T} = \frac{\pi^3 a^3 H^2 n^2}{3600\,\lambda} \text{ Js}^{-1}$$

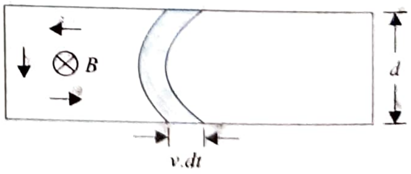
For Problems 36–37

36. (4)

37. (4) When ring moves to the right, emf is induced in each of the two semi-circles. During an elemental time interval dt, displacement of the ring is $v\ dt$.

Consider two semi-circular separately.

During this interval, left semi-circle cuts flux area shown as shaded part in figure. This area is $d\,(v\ dt)$. Therefore, flux cut by semi-circle during this interval is,

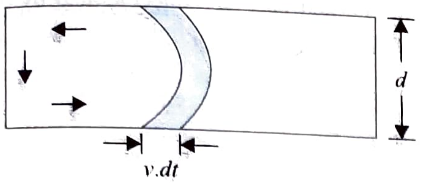


$$d\phi = Bd(vdt)$$

Hence, emf induced in it is $e = \frac{d\phi}{dt} = Bvd$

This emf tries to force an anticlockwise current in the circuit as shown in figure.

Similarly, emf induced in right semi- circle is also equal to $e = Bvd$ and it also tries to force current in the same direction as shown in figure.



Hence, these two semi-circles are two identical electrical sources connected in parallel with each other. EMF of each source is $e = Bvd$ and internal resistance is, $r = (\pi d/2)\lambda$

$\therefore$ Equivalent internal resistance of parallel combination of two sources is,

$$\frac{r}{2} = \frac{1}{4}\pi d\lambda$$

Since, rails have negligible resistance, therefore, equivalent resistance of the circuit is,

$$R = \frac{r}{2} = \frac{1}{4}\pi\lambda d$$

Hence, induced current through rails is

$$I = \frac{e}{R}$$

But current through each semi-circle is equal to $I/2$.

Force required to maintain velocity of ring at constant = Retarding force acting on ring due to induced current = 2 × Retarding force on each semi-circle.

$$= 2 \times B\frac{1}{2}d = Bld = \frac{4B^2vd}{\pi\lambda}$$

For Problems 38–41

38. (4) **39. (1)** **40. (3)** **41. (1)**

Given, $\vec{A} = \pi a^2\hat{j}$ and $B = (\hat{i} + t^2\hat{j})$

Associated flux is $\phi = \vec{B}.\vec{A} = \pi a^2 t^2$

Differentiating, $e = -2\pi a^2 t$

Thus, induced electric field is

$$E = \frac{e}{2\pi a} = at$$

If induced current is I, Torque due to its weight is πga

Torque due to applied field is $\vec{\tau} = I\vec{A} \times \vec{B} = -\pi a^2 I\vec{k}$

where, $I = e/R$

Ring just begins to topple when torque due to field becomes numerically equal to torque due to weight of ring.

$\pi a^2 \cdot \frac{2\pi a^2 t}{R} = \pi ga$. Putting $a = 1$ m and $R = 2\ \Omega$, we get $t = \frac{10}{\pi}$ s

Further, heat produced in time t is

$$H = \int \frac{e^2}{R}dt = \frac{4\pi^2 a^4}{R} \cdot \frac{t^3}{3}$$

Substituting the value of time,

$$H = \frac{2000}{3\pi}\ \text{J}$$

Induced electric field at the circumference of ring at the instant ring start toppling is $E = at = \frac{10}{\pi}$

Matrix Match Type

1. i. → b., d.; ii. → a., c.; iii. → a., c.; iv. → b., d.

i. If current is increased, flux in the loop will increase in inside direction, then due to Lenz's law induced emf in the loop will be in anticlockwise direction. Due to this current, the current in the nearer side of loop to the wire will be in opposite direction to that of wire. Hence, there will be repulsion.

ii. This situation is opposite to part (i)

iii. If loop is moved away, then flux decreases and this becomes similar to part (ii)

iv. Similar to part (i)

2. i. → b., d.; ii. → b., c.; iii. → a., d.; iv. → a., c.

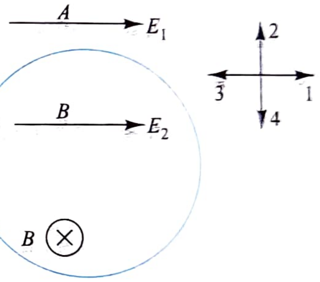
Since field is decreasing, so induced electric field at both points A and B will be in clockwise direction or towards 1. Hence, force on an electron will be along 3 at both points A and B.

For A: $E2\pi r = \pi a^2 \frac{dB}{dt}$

$E \propto \frac{1}{r}$

For B: $E2\pi r = \pi r^2 \frac{dB}{dt}$

$E \propto r$



3. i. → b., d.; ii. → a., d.; iii. → c.; iv. → c.

i. If loop is moved away, then flux through loop decreases in −ve z direction. To increase this decreasing flux, current induced should be in clockwise direction.

Now, induced current in AB will be parallel to I, so that there will be net attraction between wire and loop.

Hence **(i) → (b, d)**

ii. Explain in the similar way as in (i).

In cases (iii) and (iv), just after the rotation is started, velocity of each element of the loop will be parallel or antiparallel to the magnetic field produced by I. Hence, no emf is induced.

So **(iii) → (c), (iv) → (c)**

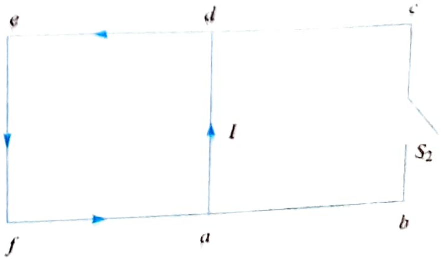
4. i. → b.; ii. → a.; iii. → d.; iv. → d.

i. $\frac{dB}{dt} = 10 \times 10^{-3}$ T/s

$A = 2^2 = 4\ \text{cm}^2 = 4 \times 10^{-4}\ \text{m}^2$

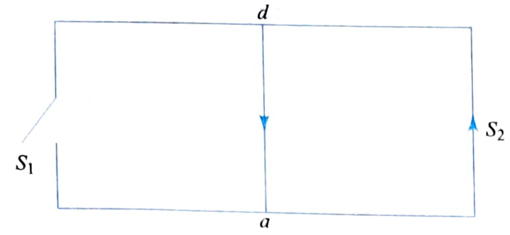
$e = \frac{d\phi}{dt} = A\frac{dB}{dt} = 4 \times 10^{-4} \times 10 \times 10^{-3} = 4 \times 10^{-6}$ V

$I = \frac{e}{R} = \frac{4 \times 10^{-6}}{2 \times 4} = 5 \times 10^{-7}$ A



The emf will be in anticlockwise direction, so current will be from a to d.

ii. Again, current will be in anticlockwise direction.



This makes the direction of current from d to a, magnitude same as that in part (i).

iii. If both are open, induced emf will develop, but no current will flow.

iv. If both are closed, then induced emf in the left part will tend to flow will from a to d and in right part, the current will tend to flow from d to a. So, from the principle of superposition, no current will flow in ad.

5. i. → c., d.; ii. → c., d.; iii. → b., d.; iv. → a., d.

i. $e_{OA} = \frac{1}{2} B\omega (OA)^2 = \frac{1}{2} B\omega (\sqrt{2}\, L)^2 = B\omega L^2$

ii. $e_{OD} = \frac{1}{2} B\omega (OD)^2 = \frac{1}{2} B\omega (\sqrt{2}\, L)^2 = B\omega L^2$

iii. $e_{OC} = \frac{1}{2} B\omega L^2$ or $e_{CD} = E_{OD} - E_{OC} = \frac{1}{2} B\omega L^2$

iv. $e_A - e_O = B\omega L^2$

$e_D - e_O = B\omega L^2$

$e_A - e_D = 0$

6. i. → b.; ii. → c.; iii. → a.; iv. → b.

i. At $t = 1$s, flux is increasing in the inward direction, hence induced emf will be in anticlockwise direction.

ii. At $t = 5$ s, there is no change in flux, so induced emf is zero.

iii. At $t = 9$ s, flux is increasing in upward direction, hence induced emf will be in clockwise direction.

iv. At $t = 15$ s, flux is decreasing in upward direction, so induced emf will be in anticlockwise direction.

7. i. → c.; ii. → a., b.; iii. → d.; iv. → c.

We know that $e = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$. If we take area vector in the upward direction, then anticlockwise direction will be positive. From 0 to t_1 and t_5 to t_6, dB/dt is +ve. Hence induced emf e is –ve. So, induced current will be in clockwise direction. From t_2 to t_4, dB/dt is –ve. Hence induced emf e is +ve. So, induced current will be in anticlockwise direction. From t_1 to t_2 and t_4 to t_5, dB/dt is zero. Hence, no emf is induced. Induced emf or current is maximum from 0 to t_1 and t_5 to t_6, because here magnitude of dB/dt is maximum.

8. i. → a., c.; ii. → a., c.; iii. → b., d.; iv. → a., b., c., d.

a. Speed of the charged particle cannot be changed by magnetic force because magnetic force does no work on charged particle. Only electric field in case (a) and induced electric field in case (c) can change speed of the charged particle.

b. Magnetic field cannot exert force on the charged particle at rest. Only electric field in case (a) and induced electric field in case (c) can exert force on charge initially at rest. In case (c) after the charged particle starts moving, the magnetic field can exert force on the charge.

c. A charged particle can move on a circle with a uniform speed due to uniform and constant magnetic field. Even within a region of non-uniform magnetic field, at all points on the circle, the field may be uniform, for example, on any circle concentric with a current-carying ring.

d. A moving charged particle is accelerated by electric field and also accelerated by magnetic field (provided v is not parallel to B).

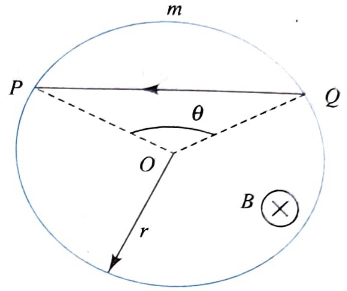
9. i. → b., c.; ii. → a., d.; iii. → a., c.; iv. → b., d.

i. Area $OPMQO = \frac{1}{2} r^2\theta$

Flux in this area, $\phi_1 = \frac{1}{2} r^2\theta B$

Induced emf in this area,

$$e_1 = \frac{d\phi_1}{dt} = \frac{1}{2} r^2\theta \frac{dB}{dt}$$



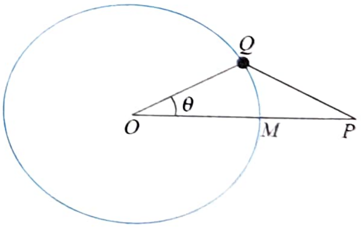
Area $OPQ = r\sin\left(\frac{\theta}{2}\right) r\cos\left(\frac{\theta}{2}\right) = \frac{1}{2} r^2 \sin\theta$

Induced emf in OPQ,

$$e_2 = \frac{d\phi_2}{dt} = \frac{1}{2} r^2 \sin\theta \frac{dB}{dt}$$

e_2 will be only in part PQ, because in OQ and OP induced emf will be zero. Clearly, $e_2 < e_1$, because area $OPQ <$ area $OPMQO$. Since B is increasing, so e.m.f. will be in anticlockwise direction. Hence end P will be positive w.r.t. Q.

ii. Here emf in OPQ will be due to flux changing in area OMQ. This area is $\frac{1}{2} r^2\theta$. The entire emf will be in part PQ. End Q will be positive.



iii. Induced emf $= \frac{1}{2} r^2\theta \frac{dB}{dt}$. End P will be positive.

iv. Area in which flux is changing is lesss than $\frac{1}{2} r^2\theta$. End Q will be positive.

10. i. → c.; ii. → d.; iii. → a.; iv. → b.

Magnetic field is along x-axis because when the cube is moved along x-axis, there is no motional emf as $\vec{v} \times \vec{B} = 0$. When the block is moved along y-axis, force on the electrons is in direction

$-(\hat{j} \times \hat{i}) = \hat{k}$

Therefore, electric field will be created along z-axis.

Now, $c\, v\, B = 24$ mV

$\Rightarrow c = 20$ cm

Similarly, $bvB = 36$ mV

$\Rightarrow b = 30$ cm

$\therefore a = 25$ cm

11. **i. → a., d.; ii. → a., d.; iii. → d.; iv. → d.**

i. Switch is closed, due to the heating effect, R increases as temperature increases, so current decreases and hence flux linked with B is decreasing with time, so, the induced current in R_1 will try to strengthen the original flux. Direction can be found by using right hand palm rules.

ii. As switch is opened, current in R decreases from finite to zero value, and hence flux also decreases (same as for a).

iii. Due to motion of A, flux linked with B is increasing and due to heating effect flux linked with B is decreasing. So, net effect on flux cannot be determined precisely, so current in R_1 can be in any direction and even it may be zero if effects have been all cancelled.

iv. The varying emf can be increasing or decreasing (explanation somewhat similar to r).

12. **i. → c.; ii. → b.; iii. → a.; iv. → b.**

Using Faraday's law, whenever there is a change in flux linked with coil, emf is induced the coil.

For $0 < x < b$

Flux linked with coil

$$e = -\frac{d\phi}{dt} = -\frac{dBIlx}{dt} - BIl\frac{dx}{dt}$$

$$e = -BIlv$$

When $b < x \le 2b$, there is no change in flux, hence no *emf* is induced

When $2b < x < 3b$ there is a decrease in flux hence $e = BIl\ v$

When $x > 3b$, again flux linked with coil is zero, hence no emf is induced.

13. **i. → a., b.; ii. → a., b.; iii. → c.; iv. → a., d.**

i. $$\frac{V - Blv}{R} = i\ ,\ F = \frac{V - Blv}{R} = Bl$$

Here magnetic force is to the right side. Actually rod will first accelerate towards right for some time and then finally will attain constant velocity. Constant velocity will be attained when F becomes zero and that constant velocity will be $v = V/Bl$.

If we apply another force to the right then rod cannot move with constant velocity rather its acceleration will be more.

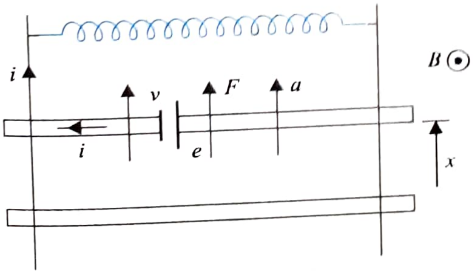
ii. Let at any time charge on the capacitor is Q and current in the circuit is i, then

$$F = ma \quad \Rightarrow \quad i\,l\,B = ma \quad \frac{lBdQ}{dt} = \frac{mdv}{dt}$$

$$\Rightarrow \quad \int_0^v mdv = -\int_{Q_0}^{Q} BldQ \Rightarrow v = \frac{Bl(Q_0 - Q)}{m}$$

After the whole charge of capacitor discharges, the rod achieves constant velocity.

iii. No energy dissipated due to no resistance. Let at on time current in the circuit is i.



$$F = Bil \Rightarrow ma \Rightarrow Bil \Rightarrow i = \frac{ma}{Bl}$$

$$e = -L\frac{di}{dt} \Rightarrow Blv = -L\frac{di}{dt}$$

$$\Rightarrow \quad Bl\frac{dx}{dt} = -L\frac{di}{dt} \Rightarrow Bl\int_0^x dx = -L\int_{i_0}^{i} di$$

$$\Rightarrow \quad Blx = -L(i - i_0)$$

Simplify to get $a = \dfrac{B^2 l}{ml}\left[x - \dfrac{Li_0}{Bl}\right]$

This represents SHM. So rod executes SHM.

iv. Induced emf will be developed which will oppose the motion of rod by applying a force to the left. And after some time, the rod will come to rest. But if a constant force to the right is applied immediately, then it can move with constant velocity provided the applied force is same as that of initial force developed due to motion.

Numerical Value Type

1. **(2)** $\Delta\phi = R(\Delta q) = R\int idt$

$$= R\ [\text{area under } i\text{–}t \text{ graph}]$$

$$= \frac{1}{2}(4)(0.1)(10) = 2 \text{ Wb}$$

2. **(5)** The total flux through N turns of the coil,

$$\phi_{\text{total}} = NBA \cos\theta$$

According to Faraday's law of electromagnetic induction,

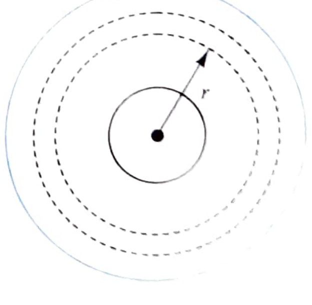
$$E_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\theta)$$

$$= -(NA\cos\theta)\frac{dB}{dt}$$

The current induced in the coil,

$$I_{\text{induced}} = \frac{E_{\text{induced}}}{R} = 5 \text{ A}$$

3. **(7)**



The magnetic field inside is only due to the current of the inner cylinder.

$$B = \frac{\mu_0 i}{2\pi r}$$

Magnetic field energy density is not uniform in the space between the cylinders. At a distance r from the centre,

$$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0 i^2}{8\pi^2 r^2}$$

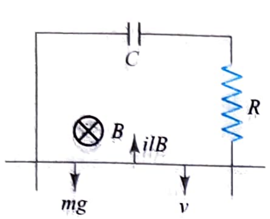
Energy in volume of element (length ℓ)

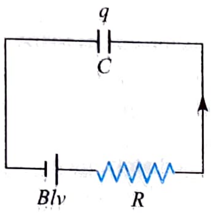
$$dU_B = u_B\,dV = \frac{\mu_0 i^2}{8\pi^2 r^2}(2\pi r\ell)dr = \frac{\mu_0 i^2\ell}{4\pi}\frac{dr}{r}$$

$$U_B = \frac{\mu_0 i^2\ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i^2\ell}{4\pi}\ell n\frac{b}{a}$$

Using values, we get $U = 7$ nJ

4. (5)





By Newton's law, $mg - ilB = m\frac{dv}{dt}$ ...(i)

Using KVL $Blv = iR + \frac{q}{C}$ ...(ii)

Differentiating equation (ii) w.r.t. time, we get

$$Bl\frac{dv}{dt} = R\frac{di}{dt} + \frac{i}{C} \quad \text{...(iii)}$$

Eliminating $\frac{dv}{dt}$ from equations (i) and (iii), we get

$$mg - ilB = \frac{m}{Bl}\left[R\frac{di}{dt} + \frac{i}{C}\right]$$

$$\Rightarrow \quad mg\,Bl - iB^2l^2 = m\left(R\frac{di}{dt} + \frac{mi}{C}\right) \quad \text{...(iv)}$$

I will be maximum when $\frac{di}{dt} = 0$. Use this in equation (iv)

$$\Rightarrow \quad mg\,B\ell C = i(B^2\ell^2 C + m)$$

$$\Rightarrow \quad i_{\max} = \frac{mgB\ell C}{m + B^2\ell^2 C}$$

5. (8) After long time, from conservation of momentum.

$$mv_0 = 2mv;\ v = \frac{v_0}{2} = 8\text{ m s}^{-1}.$$

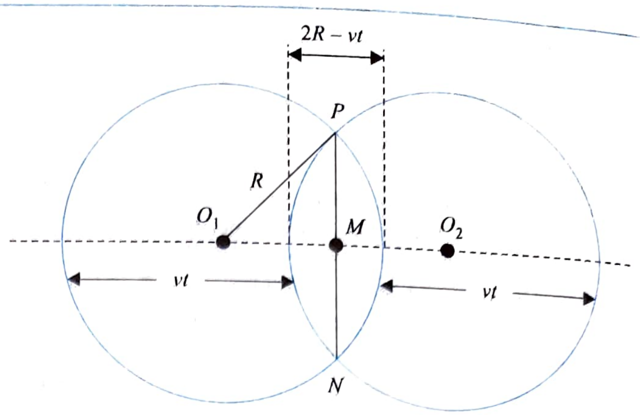
6. (7) Induced emf should be equal to 10 V

$$e = A\frac{dB}{dt} \Rightarrow 10 = \left(\frac{10}{100}\right)^2\frac{7}{\Delta t} \Rightarrow \Delta t = 7\text{ ms}$$

7. (1) The rate of electrical energy consumed in the bulb = rate of loss of gravitational PE of the mass = Mgv = 100 W. Hence

$$M = \frac{100}{10\times 10} = 1\text{ kg}.$$

8. (8) $O_1M = (vt - R) + \left(\frac{2R - vt}{2}\right) = \frac{vt}{2}$



$$\ell = PN = 2\sqrt{R^2 - (O_1M)^2} = 2\sqrt{R^2 - \left(\frac{vt}{2}\right)^2} = \sqrt{4R^2 - v^2t^2}$$

so $\quad E = Bvl = Bv\sqrt{4R^2 - v^2t^2}$

given $R = 5$ m, $v = 2$ m/s, $t = 3$ s

$$E = 0.5\times 2\sqrt{4\times 25 - 4\times 9} = 8\text{ V}$$

9. (5) $a = \frac{ilB}{m}$ and $v = \left(\frac{ilB}{m}\right)t = 5$ cm/s

10. (4) It is clear that the diameter AC traces out a circle as the given wire rotates about axis XY. The emf developed by induction between the ends A and C of the given semicircular wire is the same as if a straight wire AC were rotating perpendicular to field B.

Hence flux linked $= \phi_B = B\left(\frac{1}{2}(AC)^2\theta\right)$

where θ is the angle traced in time t.

By Faraday's law, the induced emf $e = \oint \vec{E}\cdot d\vec{l}$

But we also have, $\oint E\cdot dl = e = -\frac{d\phi_B}{dt}$

$$\oint E\cdot dr = -\frac{d}{dt}\left(\frac{1}{2}(d)^2\theta\right)B$$

$$= -\frac{1}{2}d^2B\left(\frac{d\theta}{dt}\right) = -\frac{B}{2}\omega d^2$$

But we desire only the $\int_C^A E\cdot dr$ which will be $\frac{1}{2}$ or $\oint E\cdot dr$.

Hence $\int_C^A E\cdot dr = -\frac{B}{4}\omega d^2$

11. (8) Dynamic emf

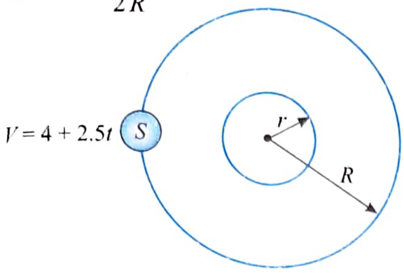
$$e_d = -\vec{l}.(\vec{v}\times\vec{B}) = \vec{l}.(\vec{B}\times\vec{v})$$

$$(\vec{B}\times\vec{v}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 4 & 0\\ 4 & 6 & 8\end{vmatrix} = \hat{i}(32-0) - \hat{j}(16-0) + \hat{k}(12-16)$$

$$= 32\hat{i} - 16\hat{j} - 4\hat{k}$$

$$e_d = (2\hat{k})\cdot(32\hat{i} - 16\hat{j} - 4\hat{k}) = -8\text{ volt}$$

12. (1.25) The magnetic field at the centre O due to the current in the larger loop is $B=\dfrac{\mu_0 I}{2R}$.



If r is the resistance per unit length, then

$$I=\frac{\text{potential difference}}{\text{resistance}}=\frac{4+2.5t}{2\pi R\cdot\rho}$$

$$\therefore \quad B=\frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}$$

$\therefore$ $r << R$, so the field B can be taken almost constant over the entire area of the smaller loop.

$\therefore$ the flux linked with the smaller loop is

$$\phi = B\times\pi r^2=\frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}\cdot\pi r^2$$

Induced emf $e=\dfrac{d\phi}{dt}=\dfrac{\mu_0 r^2}{4R^2\rho}\times 2.5$

The corresponding current in the smaller loop is I' then

$$I'=\frac{e}{R}=\frac{\mu_0 r^2}{4R^2\rho}\times 2.5\times\frac{1}{2\pi r\rho}$$

$$=\frac{2.5\,\mu_0 r}{8\pi R^2\rho^2}=\frac{2.5\times4\pi\times10^{-7}\times0.1}{8\pi\times(1)^2\times(10^{-4})^2}=1.25\,\text{A}$$

13. (1) The magnetic field is decreasing with time. Using Faraday's law(or Lenz's law) one can see that the emf induced in a closed path will be in clockwise sense. It means that the induced electric field is clockwise. Therefore, end A will reach point C. Magnitude of induced electric field is

$$E=\frac{l}{2}\left|\frac{dB}{dt}\right|=\frac{l}{2}\frac{B_0}{2}=\frac{B_0 l}{4}=\frac{B_0\times0.04}{4}=1\text{ V/m}$$

$$\tau = 2Eq\times l$$

$$\therefore \quad \alpha=\frac{2Eql}{I}=\frac{2Eql}{2\times ml^2}=\frac{Eq}{ml}$$

$$\therefore \quad \theta=\frac{1}{2}\alpha t^2$$

$$t^2=\frac{2\theta}{\alpha}=\frac{2\times\frac{\pi}{2}}{\alpha}=\frac{\pi ml}{Eq}$$

$$=\frac{\pi l}{E}\times\frac{1}{q/m}=\frac{\pi\times0.04}{1}\times\frac{1}{\frac{4\pi}{100}}$$

$\therefore \quad t^2 = 1 \qquad \therefore\ t = 1\text{s}$

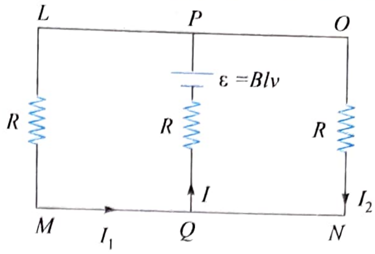
Archives

JEE Main

Single Correct Answer Type

1. (2) Emf induced across sliding connector, $\varepsilon = Blv$.

The equivalent circuit diagram can be drawn as shown in the figure.



At junction Q we can write

$$I = I_1 + I_2 \qquad \text{...(i)}$$

Applying Kirchhoff's loop law for the closed loop $PLMQP$, we get

$$-I_1R - IR + \varepsilon = 0$$

$$I_1R + IR = Blv \qquad \text{...(ii)}$$

Again, applying Kirchhoff's loop law for the closed loop $PONQP$, we get

$$-I_2R - IR + \varepsilon = 0$$

$$I_2R + IR = 2Blv \qquad \text{...(iii)}$$

Adding equations (ii) and (iii), we get

$$2IR + I_1R + I_1R = 2\,Blv$$

$$2IR + R(I_1 + I_2) = 2Blv$$

$$2IR + IR = 2Blv \qquad \text{(Using (i))}$$

$$3IR = 2Blv$$

$$I=\frac{2Blv}{3R} \qquad \text{...(iv)}$$

Substituting this value of I in equation (ii), we get $I_1=\dfrac{Blv}{3R}$

Substituting the value of I in equation (iii), we get

$$2IR+I_1R+I_2R=I_2=\frac{Blv}{3R}$$

Hence, $I_1=I_2=\dfrac{Blv}{3R}, I=\dfrac{2Blv}{3R}$

2. (4) Here earth magnetic field $B_H = 5.0\times10^{-5}\text{ N A}^{-1}\text{ m}^{-1}$

Length of vertical aerial $l = 2$ m

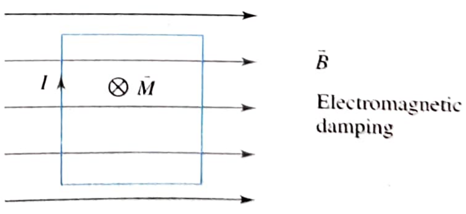
Velocity of this aerial, $v = 1.5\text{ ms}^{-1}$

This aerial moves in horizontal magnetic field hence motional e.m.f will be induced across the aerial

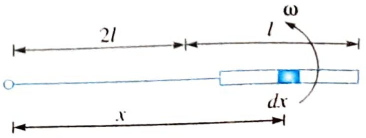
Induced e.m.f, $\varepsilon = B_H vl = 5\times10^{-5}\times1.50\times2$

$= 15\times10^{-5}\text{ V} = 0.15\text{ mV}$

3. (4)



4. (3) As rod is rotating in magnetic field thus cutting magnetic field lines, motional emf will develop across the rod. As each and every element of the rod is moving with different velocity. For calculating emf across rod we first consider an element of length dx at a distance x from the fixed end of the string.



e.m.f. induced in the element is

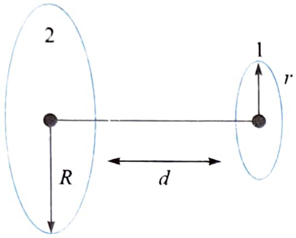
$$d\varepsilon = B(\omega x)dx$$

Hence, the e.m.f. induced across the ends of the rod is

$$\varepsilon = \int_{2l}^{3l} B\omega x dx = B\omega\left[\frac{x^2}{2}\right]_{2l}^{3l} = \frac{B\omega}{2}\left[(3l)^2 - (2l)^2\right]$$

$$= \frac{5B\omega l^2}{2}$$

5. **(4)** Let M_{12} be the coefficient of mutual induction between loops



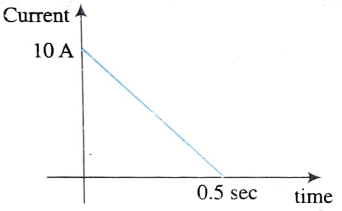
$$\phi_1 = M_{12}i_2$$

$$\Rightarrow \quad \frac{\mu_0 i_2 R^2}{2(d^2+R^2)^{3/2}}\pi r^2 = M_{12}i_2$$

$$\Rightarrow \quad M_{12} = \frac{\mu_0 R^2 \pi r^2}{2(d^2+R^2)^{3/2}}$$

$$\phi_2 = M_{12}i_1 \Rightarrow \phi_2 = 9.1\times10^{-11}\text{ weber}$$

6. **(1)**



Charge flowing in the coil, $q = \dfrac{\Delta\phi}{R}$

$\Delta\phi$ = change in flux and R = resistance of the coil

Also, $q = \int I\,dt$ = Area of current–time graph

$$\Rightarrow \quad q = \frac{1}{2}\times 10\times 0.5 = 2.5\text{ coulomb}$$

Change in flux $\Delta\phi = qR$

$\Rightarrow \quad \Delta\phi = 2.5 \times 100 = 250$ Wb

JEE Advanced

Single Correct Answer Type

1. **(4)** According to Lenz's law, current will be in anticlock-wise sense as magnetic field is increasing into the plane of paper.
2. **(3)** True for induced electric field and magnetic field.

Multiple Correct Answers Type

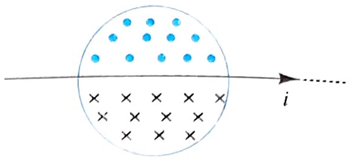
1. **(2), (4)**

   As $\dfrac{d\phi}{dt}$ = emf is the same, the current induced in the ring will depend upon the resistance of the ring. Larger the resistivity, smaller the current.

2. **(1), (3)**

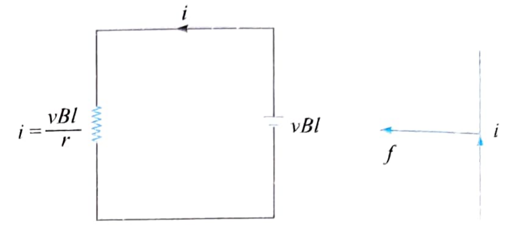
   $(\phi)_{\text{loop}} = 0$ for all cases

   So induced emf = 0



3. **(3), (4)**

   While entering i.e. $x < L$



$$f = i\ell B = \frac{B^2\ell^2 v}{R}$$

$$a_2 = \frac{f}{m} = \frac{B^2\ell^2 v}{mR} = kv = -\frac{vdv}{dx}\left[k = \frac{B^2\ell^2}{m}\right]$$

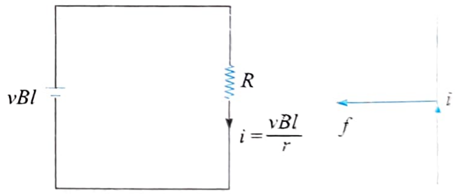
$$\int_{v_0}^{v} dv = -k\int_0^x dx \Rightarrow v = v_0 - kx$$

$$f = \frac{B^2\ell^2}{R}(V_0 - kx) = \alpha - \beta x$$

$$i = (v_0 - kx)\frac{B\ell}{R} = i_0 - \gamma x$$

For $3L > x > L$ $f = 0$ $i = 0$ v = constant.

For $4L > \times > 3L$



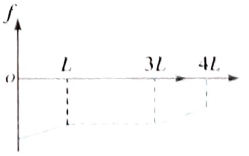
$$f = i\ell B = \frac{B^2\ell^2 v}{R}$$

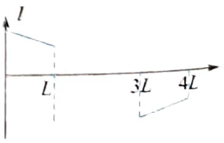
$$a = \frac{B^2\ell^2}{mR}v = kv = -\frac{vdv}{dx}$$

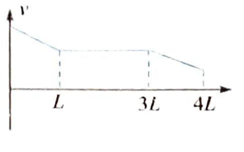
$$v = v'_0 - kx$$

$$f = \alpha' - \beta'.x$$

$$i = i'_0 - \gamma' x$$







4. (1), (2)

The angle that area vector makes with $\vec{B}$ at time t is $\theta = \omega t$.

$$|\phi_1| = BA\cos\omega t \Rightarrow |\varepsilon_1| = BA\omega\sin(\omega t)$$

$$|\phi_2| = 2BA\cos(\omega t) \Rightarrow |\varepsilon_2| = 2BA\omega\sin(\omega t)$$

Due to orientation of loops, the two EMFs will work against each other.

So, $|\varepsilon(\text{net})| = BA\,\omega\sin(\omega t)$

So, (1) is correct.

We can see that ε_{net} is maximum when $\theta = \dfrac{\pi}{2}$.

So, (2) is correct. Obviouly, (3) is incorrct. (4) is incorrect as the EMF is proportional to difference in areas.

Linked Comprehension Type

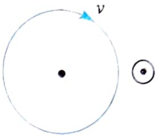
For Problems 1–2

1. (2) $E(2\pi R) = \pi R^2 \dfrac{dB}{dt}$

$$E = \frac{RB}{2}$$

2. (2) $\Delta L = \int \tau\, dt$

$$= Q\left(\frac{R}{2}B\right)R(1) = \frac{QR^2B}{2}\text{, in magnitude}$$



$$\Delta\mu = \gamma\Delta L = -\gamma\frac{BQR^2}{2} \quad \text{(taking in account the direction)}$$

Chapter 5

Concept Application Exercises

Exercise 5.1

1. $L = \dfrac{\varepsilon}{di/dt} = \dfrac{6}{600} = 10^{-2}\text{ H}$

2. $< V_L > = L\dfrac{\Delta i}{\Delta t} = 4\times10^{-3}\left(\dfrac{2-0.25}{0.5}\right) = 14\text{ mV}$

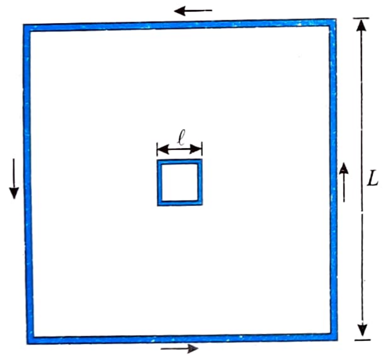
3. $M = \dfrac{\phi_2}{i_1} = \dfrac{\mu_0 i_1 l^2}{2Ri_1} = \dfrac{\mu_0 l^2}{2R} = \dfrac{(4\pi\times10^{-7})(10^{-4})}{(2)\left(\frac{1}{4}\right)} = 8\pi\times10^{-11}\text{ H}$

4. $M = \dfrac{\phi_2}{i_1} = \dfrac{\mu_0 i_1 A}{2\pi x} / i_1$

$$= \frac{\mu_0 A}{2\pi x} = \frac{4\pi\times10^{-7}\times10^{-8}}{2\pi\times0.5} = 4\times10^{-15}\text{ H}$$

5. Considering the larger loop to be made up of four rods, each of length L, the field at the center, i.e., at a distance $L/2$ from each rod, will be

$$B = \frac{\mu_0 I}{4\pi d}[\sin\alpha + \sin\beta] = \frac{\mu_0 I}{4\pi(L/2)}2\sin45^\circ = \frac{\mu_0 I}{\sqrt{2}\pi L}$$



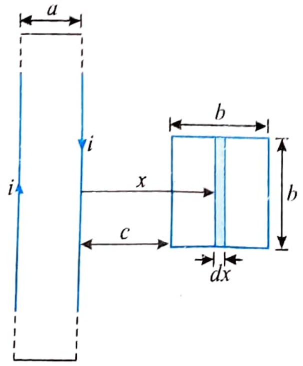
Net field: $B_1 = 4B_0 = 2\sqrt{2}\dfrac{\mu_0 I}{\pi L}$

So the flux linked with the smaller loop is

$$\phi = B_1 S_2 = \frac{2\sqrt{2}\mu_0 I}{\pi L} l^2$$

Hence, $M = \dfrac{\phi}{I} = 2\sqrt{2}\dfrac{\mu_0 l^2}{\pi L}$

6. Let us assume the rectangular loop of infinite size have current i. Consider a segment of width dx at a distance x as shown.



The flux through the segment, $d\phi = \left[\dfrac{\mu_0 i}{2\pi x} - \dfrac{\mu_0 i}{2\pi(x+a)}\right]b\,dx$.

$$\phi = \int_c^{c+b}\left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right]b\,dx$$

$$= \frac{\mu_0 ib}{2\pi}\left[\ln\frac{c+b}{c} - \ln\frac{a+b+c}{a+c}\right]$$

The mutual inductance between two rectangular loops is

$$M = \frac{\phi}{i} = \frac{\mu_0 b}{2\pi}\ln\frac{(c+b)(a+c)}{c(a+b+c)}$$

7. If a current I_1 flows around the first wire then a uniform axial magnetic field of strength $B_1 = \dfrac{\mu_0 N_1 I_1}{\ell}$ is generated in the core.

The magnetic field in the region outside the core is of negligible magnitude. The flux linked to a single turn of the second wire is $B_1 A$. Thus, the flux linked to all N_2 turns of the second wire is

Now, $\phi = N_2 B_1 A = \dfrac{\mu_0 N_1 N_2 A I_1}{\ell} = MI_1$

$\therefore \quad M = \dfrac{\mu_0 N_1 N_2 A}{\ell}$

As described previously, M is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

8. Let a current i flows in coil of radius a_2.

Magnetic field at the centre of the coil $= \dfrac{\mu_0 i}{2a_2}$

Flux in smaller coil $\phi = \dfrac{\mu_0 i}{2a_2}\pi a_1^2$

or $\quad Mi = \dfrac{\mu_0 i}{2a_2}\pi a_1^2 \quad$ or $\quad M = \dfrac{\mu_0 \pi a_1^2}{2a_2}$

9. Let a current i flows in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of the smaller coil and hence no flux will pass through it, so $M = 0$.

10. If current i flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of the larger coil.

Now, the area vector of the smaller coil which is perpendicular to the plane of the smaller coil will make an angle θ with the magnetic field.

Thus, flux $\vec{B}\cdot\vec{A} = \dfrac{\mu_0 i}{2a_2}\pi a_1^2\cos\theta$

or $\quad M = \dfrac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}$

11. (a) To find mutual inductance, it does not matter in which coil we consider current and in which coil flux is calculated (reciprocity theorem). Let current i be flowing in the larger coil. Magnetic field at the centre $= \dfrac{\mu_0 i}{2b}$.

Flux through the smaller coil $= \dfrac{\mu_0 i}{2b}\pi a^2$

$\therefore \quad M = \dfrac{\mu_0}{2b}\pi a^2$

(b) $|\text{emf induced in larger coil}| = \left[\left(\frac{di}{dt}\right)\text{in smaller coil}\right]$

$$= \frac{\mu_0}{2b}\pi a^2(2) = \frac{\mu_0 \pi a^2}{b}$$

(c) current in the larger coil $= \frac{\mu_0 \pi a^2}{bR}$.

Exercise 5.2

1. (i) (a) The bulb glows brightly right away, and then more and more faintly as the capacitor charges up. (b) The bulb gradually gets brighter and brighter, changing rapidly at first and then more and more slowly. (c) The bulb immediately becomes bright. (d) The bulb glows brightly right away, and then more and more faintly as the inductor starts carrying more and more current (the inductor eventually acts as a short).

   (ii) (a) The bulb goes out immediately because current stops immediately (charge ceases to flow). (b) The bulb glows for a moment as a spark jumps across the switch. (c) The bulb stays lit for a while, gradually getting fainter and fainter as the capacitor discharges through the bulb. (d) The bulb suddenly glows brightly. Then its brightness decreases to zero, changing rapidly at first and then more and more slowly.

2. The equivalent resistance is

$$L_{eq} = L_1 + L_4 + L_{23} = L_1 + L_4 + \frac{L_2 L_3}{L_2 + L_3}$$

$$= 30.0 + 12.0 + \frac{(40.0)(10.0)}{(40.0 + 10.0)} = 50.0 \text{ mH}.$$

3. (a) In given situation the coil oppose the changes, so if it pushes current rightward (when the current is already going rightward), then i must be in the process of decreasing.

   (b) From Eq. (in absolute value) we get,

$$L = \left|\frac{\varepsilon}{di/dt}\right| = \frac{6.0}{1.5\times 10^3} = 4.0\times 10^{-3}\text{H}.$$

4. The voltages are related as

$$\varepsilon - iR - L\frac{di}{dt} = 0 \Rightarrow \varepsilon = iR + L\frac{di}{dt} = 0$$

When the current is increasing:

$$9.0 = (2.0)R + L(0.50) \qquad \text{...(i)}$$

When the current is decreasing:

$$5.0 = (2.0)R + L(-0.50) \qquad \text{...(ii)}$$

(a) Subtracting [ii] from [i] gives

$$9.0 - 5.0 - L(1.0) \Rightarrow L = 4.0\text{H}$$

(b) Substituting the value for L in either equation gives

$$7.0 = (2.0)R \Rightarrow R = 3.5\ \Omega$$

5. Applying the loop theorem, $\varepsilon - L\left(\frac{di}{dt}\right) = iR$,

we solve for the (time-dependent) emf, with SI units understood:

$$\varepsilon = L\frac{di}{dt} + iR = L\frac{d}{dt}(3.0 + 5.0t) + (3.0 + 5.0t)R$$

$$= (6.0)(5.0) + (3.0 + 5.0t)(4.0)$$

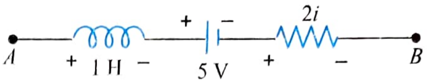
$$= (42 + 20t)\text{ V}$$

6. (a) Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\varepsilon - L\,di/dt = 0$.

So $i = \varepsilon t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0$ A. Thus,

$$t_0 = \frac{i_0 L}{\varepsilon} = \frac{(3.0\text{A})(5.0\text{H})}{10\text{V}} = 1.5\text{ s}$$

7. Applying KVL from A to B,



$$V_A - 1\frac{di}{dt} - 5 - 2i = V_B$$

(a) Put $i = 2$, $\frac{di}{dt} = 0$; $V_A - 5 - 4 = V_B$

$\therefore \quad V_A - V_B = 9$ V

(b) Put $i = 2$, $\frac{di}{dt} = 1$

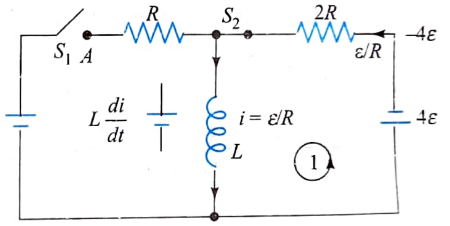
$$V_A - 1 - 5 - 4 = V_B$$

or $\quad V_A - V_B = 10$ V

(c) Put $i = 2$, $\frac{di}{dt} = -1$

$$V_A + 1 - 5 - 2\times 2 = V_B$$
$$V_A = 8\text{ V}$$

8. Before S_2 is closed and S_1 is opened, current in the left part of the circuit = ε/R. Now when S_2 is closed and S_1 is opened, current through the inductor cannot change suddenly, current ε/R will continue to flow through the inductor.

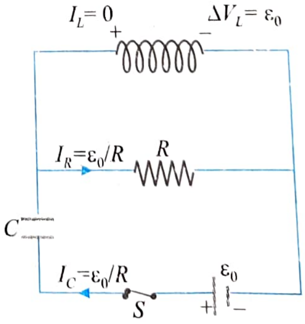


Applying KVL in loop 1,

$$L\frac{di}{dt} + \frac{\varepsilon}{R}(2R) + 4\varepsilon = 0 \Rightarrow \frac{di}{dt} = -\frac{6\varepsilon}{L}$$

Exercise 5.3

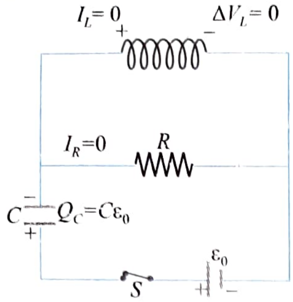
1. (a) The instant after the switch is closed, the capacitor acts as a conducting wire, and the inductor acts as infinite resistance to maintain zero current in itself. The situation is as shown in the circuit diagram



$$I_L = 0, I_c = \frac{\varepsilon_0}{R}, \text{I}_\text{R} = \frac{\varepsilon_0}{R}$$

$$\Delta V_L = \varepsilon_0, \Delta V_c = 0, \Delta V_R = \varepsilon_0$$

(b) After the switch has been closed a long time, the capacitor acts as an open switch. The steady state conditions will exist. The currents and voltages are:



$$I_L = 0, I_c = 0, I_R = 0$$

$$\Delta V_L = 0, \Delta V_C = \varepsilon_0, \Delta V_R = 0$$

2. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{(R_1 + R_2)} = \frac{110}{(10+20)} = \frac{11}{3}\text{A}.$$

(b) $i_2 = i_1 = \frac{11}{3}$A.

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1R_1 - i_2R_2 = 0 \quad \text{...(i)}$$

$$\varepsilon - i_1R_1 - (i_1 - i_2)R_3 = 0 \quad \text{...(ii)}$$

We solve these simultaneously for i_1 and i_2, and find

$$i_1 = \frac{\varepsilon(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$i_1 = \frac{(110)(20+30)}{(10)(20)+(10)(30)+(20)(30)} = 5.0\,\text{A},$$

(d) and $i_2 = \frac{\varepsilon R_3}{R_1R_2 + R_1R_3 + R_2R_3}$

$$i_2 = \frac{(110)(30)}{(10)(20)+(10)(30)+(20)(30)} = 3.0\text{A},$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$).

(f) The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is 5.0 A – 3.0 A = 2.0 A. The current in R_2 is the same but in the opposite direction as that in R_3, that is, $i_2 = -2.0$ A.

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g) $i_1 = 0$, and

(h) $i_2 = 0$.

3. For the increasing current $i = \frac{\varepsilon}{R}\left(1 - e^{-Lt/R}\right)$. The final value is $\frac{\varepsilon}{R}$, so the condition on Δt is

$$0.800\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-L\Delta t/R}\right)$$

$$e^{-L\Delta t/R} = 0.200 \Rightarrow e^{L\Delta t/R} = 5.00$$

$$\frac{L\Delta t}{R} = \ln 5.00 \Rightarrow \Delta t = \frac{R\ln 5.00}{L}$$

At the moment when the battery is removed, the current in the coil is quite precisely $\frac{\varepsilon}{R}$. During the decrease, $i = I_i e^{-Lt/R} = \frac{\varepsilon}{R}e^{-Lt/R}$.

(a) at $t = \Delta t = \frac{R\ln 5.00}{L}$,

$$\frac{i}{I_i} = e^{-L\Delta t/R} = 0.200 = 20.0\%$$

(b) at $t = 2\Delta t$,

$$\frac{i}{I_i} = e^{-L2\Delta t/R} = \left(e^{-L\Delta t/R}\right)^2$$

$$= (0.200)^2 = 0.0400 = 4.00\%$$

4. (a) When $i = 2.00$ A, the voltage across the resistor is

$$= \Delta V_R = iR = (2.0)(8.0) = 16.0 \text{ V}$$

Kirchhoff's loop rule tells us that the sum of the changes in potential around the loop must be zero:

$$\varepsilon - \Delta V_R - \varepsilon_L = 36.0 - 16.0 - \varepsilon_L = 0\ ,$$

so $\varepsilon_L = 20.0$ V and $\frac{\Delta V_R}{\varepsilon_L} = \frac{16.0}{20.0} = 0.80$

(b) Similarly, for $i = 4.50\text{ A}, \Delta V_R = iR = (4.50)(8.0) = 36.0$ V and $\varepsilon - \Delta V_R - \varepsilon_L = 36.0\text{ V} - 36.0\text{ V} - \varepsilon_L = 0$, so $\varepsilon_L = 0$.

5. (a) Applying the loop rule to the left loop gives $\varepsilon - i_1R_1 = 0$, so $i_1 = \varepsilon/R_1 = 10\text{ V}/5.0\,\Omega = 2.0$ A.

(b) Since now $\varepsilon_L = \varepsilon$, we have $i_2 = 0$.

(c) The junction rule gives $i_s = i_1 + i_2 = 2.0\text{ A} + 0 = 2.0$ A.

(d) Since $V_L = \varepsilon$, the potential difference across resistor 2 is $V_2 = \varepsilon - \varepsilon_L = 0$.

(e) The potential difference across the inductor is $V_L = \varepsilon = 10$ V.

(f) The rate of change of current is $\frac{di_2}{dt} = \frac{V_L}{L} = \frac{\varepsilon}{L} = \frac{10\text{V}}{5.0\text{H}} = 2.0\text{A/s}$.

(g) After a long time, we still have $V_1 = \varepsilon$, so $i_1 = 2.0$ A.

(h) Since now $V_L = 0$, $i_2 = \varepsilon/R_2 = 10\text{ V}/10\,\Omega = 1.0$ A.

(i) The current through the switch is now

$$i_s = i_1 + i_2 = 2.0\text{ A} + 1.0\text{ A} = 3.0\text{ A}.$$

(j) Since $V_L = 0$, $V_2 = \varepsilon - V_L = \varepsilon = 10$ V.

(k) With the inductor acting as an ordinary connecting wire, we have $V_L = 0$.

(l) The rate of change of current in resistor 2 is $\frac{di_2}{dt} \cdot \frac{V_L}{L} = 0$.

6. Because of the decay of current that occurs after the switches are closed on B, the flux will decay according to,

$$\Phi_1 = L_1i_1 = L_1i_{01}e^{-t/\tau_1} = \phi_{01}e^{-t/\tau_1}\text{, similarly } \Phi_2 = \phi_{02}e^{-t/\tau_2}$$

Setting the fluxes equal to each other and solving for time leads to

$$t = \frac{\ln(\Phi_{02}/\Phi_{01})}{(R_2/L_2) - (R_1/L_1)}$$

$$\Rightarrow \quad t = \frac{\ln(2.0)}{(30/0.006) - (20/0.005)} = \frac{\ln(2.0)}{1000}\text{ s}$$

7. Using $i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$, where $\tau_L = 2.0$ ns, we find

$$t = \tau_L \ln\left(\frac{1}{1 - iR/\varepsilon}\right) \approx 1.0\text{ ns}.$$

8. (a) As the switch closes at $t = 0$, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf (ε_{L1}) of the $L_1 = 0.30$ H inductor to cancel that of the battery.

$$\frac{di}{dt} = \frac{|\varepsilon_{L1}|}{L_1} = \frac{6.0}{0.30} = 20\text{A/s}.$$

(b) What is being asked for is essentially the current in the battery when the emfs of the inductors vanish (as $t \to \infty$). Applying the loop rule to the outer loop, with $R_1 = 8.0\ \Omega$, we have

$$\varepsilon - iR_1 - |\varepsilon_{L1}| - |\varepsilon_{L2}| = 0 \Rightarrow i = \frac{6.0\text{V}}{R_1} = 0.75\text{A}.$$

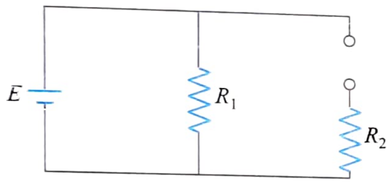
9. At $t = 0$, C will act as a simple wire, so R and L both will be short-circuited. So that there is no current in R and L.

$$I_1 = 0,\ I_2 = 0,\ I_3 = \frac{\varepsilon}{r}$$

At $t = \infty$, L will act as a simple wire so, R and C will be short-circuited.

$$I_1 = 0,\ I_2 = \frac{\varepsilon}{r},\ I_3 = 0$$

10. **(I)** At $t = 0$, inductor acts as open circuit:



(a) $i_1 = \dfrac{E}{R_1} = 2$ A

(b) $i_2 = 0$ A

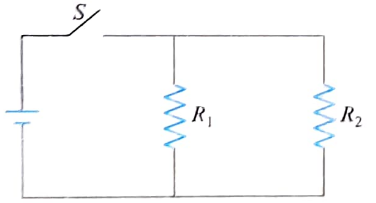
(c) $i = i_1 + i_2 = 2 + 0 = 2$ A

(d) V across resistor $R_2 = 0$

(e) V across $L = 10$ V

(f) $L\dfrac{di_2}{dt} = 10 \Rightarrow \dfrac{di_2}{dt} = \dfrac{10}{5} = 2$ A/s

(II) At $t = \infty$, inductor acts as a conducting wire:



(a) $i_1 = \dfrac{E}{R_1} = 2$ A

(b) $i_2 = \dfrac{E}{R_2} = \dfrac{10}{10} = 1$ A

(c) $i_{\text{switch}} = i_1 + i_2 = 3$ A

(d) $V_{R2} = 10$ V

(e) $V_L = 0$ V

(f) Current in R_2 is not changing with time, so

$$\frac{di_2}{dt} = 0$$

11. At $t = 0$, i_3 is zero since current cannot suddenly change due to the inductor.

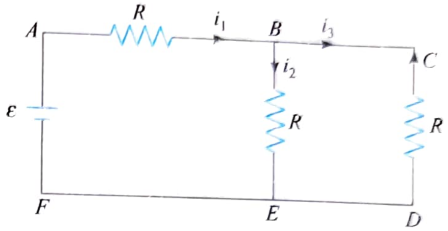
$\therefore \quad i_1 = i_2$ (from KCL)

Applying KVL in the part $ABEF$, we get $i_1 = i_2 = \dfrac{\varepsilon}{2R}$.

$$\varepsilon - i_1R - L\frac{di_3}{dt} = 0$$

$$\Rightarrow \quad \frac{di_3}{dt} = \frac{\varepsilon - i_1R}{L} = \frac{\varepsilon - (\varepsilon/2R)R}{L} = \frac{\varepsilon}{2L}$$

At $t = \infty$, i_3 will become constant and hence potential difference across the inductor will be zero. It is just like a simple wire and the circuit can be solved assuming it to be equivantent to the circuit diagram shown below.



$$i_2 = i_3 = \frac{\varepsilon}{3R},\ i_1 = \frac{2\varepsilon}{3R},\ \frac{di_3}{dt} = 0$$

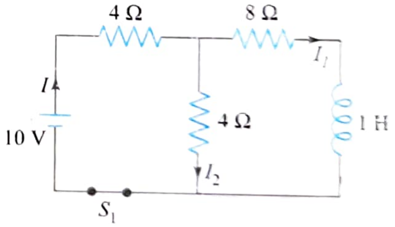
12. $10 - 4I - 8I_1 - \dfrac{dI_1}{dt} = 0 = 0$ ...(i)

$I_1 + I_2 = I$ ...(ii)

$10 - 4I - 4I_2 = 0$

$$\Rightarrow \quad I_2 = \left(\frac{10 - 4I}{4}\right)$$

$$I_1 + \frac{10}{4} = 2I \quad \Rightarrow \quad 4I = 2I_1 + 5$$



Put in equation (i), $10 - 2I_1 - 5 - 8I_1 = \dfrac{dI_1}{dt}$

$$dt = \frac{dI_1}{5 - 10I_1}$$

$$-t \times 10 = \ln\left(\frac{5 - 10I_1}{5}\right)$$

Current through inductor $I_1 = \dfrac{1}{2}(1 - e^{-10t})$

Current through switch $I = \dfrac{5}{4} + \dfrac{I_1}{2} = \dfrac{5}{4} + \dfrac{1}{4}(1 - e^{-10t})$

Exercise 5.4

1. At different times, $(U_C)_{\max} = (U_L)_{\max}$, so $\left[\dfrac{1}{2}C(\Delta V)^2\right]_{\max} = \dfrac{1}{2}LI_i^2$

Then, $I_i = \sqrt{\dfrac{C}{L}}(\Delta V)_{\max} = \sqrt{\dfrac{1.00 \times 10^{-6}\ \text{F}}{10.0 \times 10^{-3}\ \text{H}}}(40.0\ \text{V})$

$= 0.400\ \text{A} = 400\ \text{mA}$

2. When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_i = \frac{\varepsilon}{R}$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the LC loop. We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_i^2$.

$$\text{Then, } L = \frac{C(\Delta V)^2}{I_i^2} = \frac{C(\Delta V)^2 R^2}{\varepsilon^2}$$

$$= \frac{(1.0\times10^{-6})(100)^2(200)^2}{(50)^2}$$

$$= 0.16\,\text{H} = 160\text{ mH}$$

3. At different times, the maximum energy stored in the capacitor is equal to the maximum energy stored in the inductor.

$$\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \frac{1}{2}LI_i^2$$

$$\text{So } (\Delta V_C)_{\max} = \sqrt{\frac{L}{C}}I_i = \sqrt{\frac{20.0\times10^{-3}}{0.500\times10^{-6}}}(0.100) = 20.0\text{ V}$$

4. (a) The frequency of oscillation of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.0)(1.0\times10^{-6})}} = \frac{500}{\pi}\text{Hz}$$

(b) The maximum charge on the capacitor is

$$Q = C\varepsilon = (1.00\times10^{-6})(12.0) = 12.0\ \mu\text{C}$$

(c) To find the maximum current I_i, we equate

$$\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_i^2$$

Then solve for I_i to obtain

$$I_i = \varepsilon\sqrt{\frac{C}{L}} = 12.0\sqrt{\frac{1.00\times10^{-6}}{1.0}} = 12.0\text{ mA}.$$

(d) The total energy the circuit possesses at t = 3.00 s and at all times is

$$U = \frac{1}{2}C\varepsilon^2 = \frac{1}{2}(1.00\times10^{-6})(12.0)^2 = 72.0\ \mu\text{J}$$

5. (a) The voltage across the inductor is given by

$$\varepsilon_L = -L\frac{di}{dt} = -L\frac{d}{dt}(Kt) = -LK$$

(b) The current into the capacitor is $i = \frac{dq}{dt}$, so the charge that flows into the capacitor is

$$q = \int_0^t i\,dt = \int_0^t Kt\,dt = \frac{1}{2}kt^2$$

Going across the capacitor in the direction of the current, the potential drops from the positive to the negative side, so

$$\Delta V_C = \frac{-q}{C} = -\frac{kt^2}{2C}$$

(c) When $\frac{1}{2}C(\Delta V_C)^2 \geq \frac{1}{2}LI^2$,

$$\frac{1}{2}C\left(\frac{k^2t^4}{4C^2}\right) \geq \frac{1}{2}L(k^2t^2)$$

Thus, the energy in the capacitor begins to exceed the energy in the inductor after $t = 2\sqrt{LC}$.

6. The total energy equals the energy in the capacitor and inductor:

$$\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left(\frac{Q}{2}\right)^2 + \frac{1}{2}Li^2 \text{ so } i = \sqrt{\frac{3Q^2}{4CL}}$$

The flux through each turn of the coil is

$$\Phi_B = \frac{Li}{N} = \frac{q}{2N}\sqrt{\frac{3L}{C}}\text{, where } N \text{ is the number of turns.}$$

7. (a) The rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = Li\frac{di}{dt}$$

$$= L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right)$$

$$= \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}.$$

$$\text{Now, } \tau_L = \frac{L}{R} = \frac{2.0}{10} = 0.20\text{ s}$$

and ε = 100 V, so the above expression yields $dU_B/dt = 2.4 \times 10^2$ W when t = 0.10 s.

(b) The rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

At t = 0.10 s, this yields $P_{\text{thermal}} = 1.5 \times 10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9\times10^2\text{ W}.$$

8. The rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d(Li^2/2)}{dt} = Li\frac{di}{dt}$$

$$= L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right)$$

$$= \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}$$

where $\tau_L = L/R$ has been used. The rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

We equate this to dU_B/dt, and solve for the time:

$$\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2 = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}$$

$$\Rightarrow \quad t = \tau_L \ln 2 = 40.0 \ln 2\text{ ms.}$$

Exercises

Single Correct Answer Type

1. (1) $U = \frac{1}{2}Li^2$

$$\text{Rate} = \frac{dU}{dt} = (Li)\left(\frac{di}{dt}\right)$$

At $t = 0$, $i = 0$, $\therefore$ rate = 0 at $t = \infty$, $i = i_0$ but $\frac{di}{dt} = 0$,

Therefore, rate = 0.

2. (2) $e = E - iR$

Clearly, the graph is a straight line with negative slope.

3. (3) The mutual inductance M remains the same whether X or Y is used as the primary.

4. (2) At $t = 0$, charge on C is zero, so p.d. across C is zero, p.d. across R is also zero. Hence there is no current in R.

At $t = \infty$, current through L is maximum and constant, so p.d. across L is zero, therefore p.d. across R is zero. Hence no current in R.

5. (2) $I = \frac{I_0}{\alpha}$ at $t = t_0$

Since $I = I_0 e^{-t_0/\tau}$ $\Rightarrow$ $\frac{1}{\alpha} = e^{-t_0/\tau}$

$$\alpha = e^{t_0/\tau} \Rightarrow t_0 = \tau \log_e \alpha$$

$$\tau = \frac{t_0}{\log_e \alpha}$$

6. (3) Since, $\varepsilon = \frac{d\phi}{dt} = \frac{d}{dt}(NBA)$

$$\varepsilon = \frac{d}{dt}[NA(\mu_0 nI)];\ \varepsilon = NA\mu_0 n\left(\frac{dI}{dt}\right)$$

where N is total number of turns in the coil and n is the number of turns per unit length in the solenoid.

$$\varepsilon = (300)(1.2\times10^{-3})(4\pi\times10^{-7})\times\frac{2000}{0.3}\times\frac{4}{0.25}$$

$$\varepsilon = 4.8 \times 10^{-2}\ \text{V} = 48\ \text{mV}$$

7. (2) $\frac{L}{R} = 2 \times 10^{-3}$ ...(i)

$\frac{L}{R+90} = 0.5 \times 10^{-5}$ ...(ii)

From (i) and (ii), on solving, we get

$L = 60$ mH and $R = 30\ \Omega$

8. (4) Induced current in $B = 0.006\ \text{A} = 6 \times 10^{-3}$ A

Induced e.m.f. in $B = 6 \times 10^{-3} \times 4\ \text{V} = 24 \times 10^{-3}$ V

Now, $M\frac{dI}{dt} = 24 \times 10^{-3}$

or $dI = \frac{24\times10^{-3}\times0.02}{3\times10^{-3}}\ \text{A} = 0.16\ \text{A}$

9. (2) $I_{t=\infty} = I_0$

$$I_{t=1\text{s}} = I_0\left(1 - e^{-\frac{10}{5}\times1}\right)$$

$$= I_0(1-e^{-2}) = I_0\left(1-\frac{1}{e^2}\right)$$

$$\therefore \quad = \frac{I_{t=\infty}}{I_{t=1s}} = \frac{I_0 e^2}{I_0(e^2-1)} = \frac{e^2}{e^2-1}$$

10. (4) $V = RI + L\frac{dI}{dt}$, at $t = 0$, $I = 0$, thus we have

$$\frac{dI}{dt} = \frac{V}{L} = \frac{1.6}{0.2} = 8\ \text{A/s}$$

11. (3) $M = \frac{\mu_0 N_1 N_2 A}{l}a$, here $l = 1$ m

$= 503 \times 10^{-6}$ H

12. (1) Let I = current in one loop. The magnetic field at the centre of the other co-axial loop at a distance l from the centre of the first loop is

$$B = \frac{\mu_0}{4\pi}\frac{2I\pi a^2}{(a^2+l^2)^{3/2}}$$

where $p_m = I\pi a^2$

= magnetic moment of the loop

Flux through the other loop is

$$\phi_{12} = B\pi a^2$$

$$= \frac{\mu_0}{4\pi}\frac{2\pi a^2 I}{(a^2+l^2)^{3/2}}\pi a^2$$

or $$\text{M} = \frac{\phi_{12}}{I} = \frac{\mu_0}{4\pi}\frac{2\pi^2 a^4}{(a^2+l^2)^{3/2}}$$

$$= \frac{\mu_0 \pi a^4}{2(a^2+l^2)^{3/2}} = \frac{\mu_0\pi a^4}{2l^3} \quad (a << l)$$

13. (2) Let l_1 is the length of wire, then

$$l_1 = 2\pi rN \Rightarrow Nr = \frac{l_1}{2\pi} \Rightarrow L = \frac{\mu_0\pi}{l}\frac{l_1^2}{4\pi^2}$$

$$\Rightarrow \quad l_1 = \sqrt{\frac{Ll4\pi}{\mu_0}} = \sqrt{\frac{10^{-3}\times1\times4\pi}{4\pi\times10^{-7}}} = 10^2\ \text{m} = 0.10\ \text{km}$$

14. (4) Energy spent $= \frac{1}{2}LI^2$

15. (2) $B = \mu_0 nI = 4\pi \times 10^{-7} \times 200 \times 10^2 \times 1.5 = 3.8 \times 10^{-2}$ T

$\phi = BA = 3.8 \times 10^{-2} \times 3.14 \times 10^{-4} = 1.2 \times 10^{-5}$ Wb

When the current in the solenoid is reversed, the change in magnetic flux,

$d\phi = 2 \times (1.2 \times 10^{-5}) = 2.4 \times 10^{-5}$ Wb

$\therefore$ Induced e.m.f., $e = N\left(\frac{d\phi}{dt}\right)$

$$= 100 \times \left(\frac{2.4\times10^{-5}}{0.05}\right) = 0.048\ \text{V}$$

16. (4) Coefficient of mutual inductance M is given by

$$|M| = \frac{e_1}{(di_2/dt)} = \frac{\phi_2}{i_1}$$

$$\therefore \quad \phi_2 = \frac{e_1 i_1}{(di_2/dt)} = \frac{(25.0\times10^{-3})(3.6)}{(15)}$$

$= 6 \times 10^{-3} = 6$ mWb

17. (1) $i = i_0 e^{-t/t_0}$

$$\left(-\frac{di}{dt}\right) = \frac{i_0}{t_0}e^{-t/t_0}$$

$$\left(-\frac{di}{dt}\right) = \frac{i_0}{t_0} = r \qquad (\text{at } t = 0)$$

$\therefore$ The desired time is $\frac{i_0}{r}$ or t_0.

18. (2) Magnetic field at the centre of a large coil $E \propto \frac{\mu_0 I}{2R}$.

Magnetic flux linked with smaller coil $= \dfrac{\mu_0 I}{2R} \times \pi r^2$

$$M = \frac{\phi}{I} = \frac{\mu_0 \pi r^2}{2R}$$

$$\therefore \quad M \propto \frac{r^2}{R}$$

19. (3) $L = \dfrac{\mu_0 N^2 A}{l}$

If x is the length of the solenoid with r as radius, then

$x = 2\pi r N,\ A = \pi r^2$

$$\therefore \quad L = \mu_0 \left(\frac{x^2}{4\pi^2 r^2}\right) \frac{\pi r^2}{l} \qquad \left[\therefore \quad N = \frac{x}{2\pi r}\right]$$

$$\therefore \quad x = \sqrt{\frac{4\pi L l}{\mu_0}}$$

20. (2) The power dissipated in the resistor,

$$P = \frac{dW}{dt} = I^2 R$$

Since the current through resistor varies with time, we must integrate.

The total energy produced as heat in the resistor

$$W = \int_0^\infty I^2 R\, dt$$

The current in an RL circuit is $I = I_0 e^{-(R/L)t}$

$$W = \int_0^\infty I_0^2 e^{-(2R/L)t} R\, dt = \frac{I_0^2 R}{-2R/L}\left[e^{-\frac{2R}{L}t}\right]_0^\infty = \frac{1}{2} L I_0^2$$

We can integrate by substituting

Note that the total heat produced equals the energy (1/2) LI_0^2 originally stored in the conductor.

21. (2) The current is short circuited through inductor. In stready state, current will pass through inductor instead of 20 Ω.

Then current through 30 Ω is:

$$I = \frac{3}{30} = 0.1 \text{ A}$$

22. (1) For a solenoid, $L = \mu_0 N^2 \dfrac{A}{l}$. If x is the length of the wire and a is the area of cross-section, then

$$R = \frac{\rho x}{a} \text{ and } m = axD$$

$$Rm = \frac{\rho x}{a} axD, \quad x = \sqrt{\frac{Rm}{\rho D}}$$

Also, $x = 2\,\pi r N,\ N = \dfrac{x}{2\pi r}$ $\qquad \left(\therefore \quad L = \dfrac{\mu_0 N^2 A}{l}\right)$

$$\therefore \quad L = \mu_0 \left(\frac{x}{2\pi r}\right)^2 \frac{\pi r^2}{l} = \frac{\mu_0}{4\pi l} \frac{Rm}{\rho D}$$

23. (3) ω_r = constant

$\Rightarrow \quad LC$ = constant

$$\Rightarrow \quad L\,dC + C\,dL = 0 \quad \Rightarrow \quad \frac{dL}{L} = -\frac{dC}{C} = -1\%$$

24. (4) $L = \dfrac{\phi}{I},\ \phi = NAB$

$$B = \mu_0 n I$$

where $\quad n = \dfrac{N}{2\pi R}$

$$\therefore \quad \phi = N\pi r^2 \left(\mu_0 \frac{N}{2\pi R} I\right)$$

$$\phi = \frac{\mu_0 N^2 r^2 I}{2R}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 r^2}{2R}$$

25. (4) $E = \dfrac{Mdi}{dt} = Ma, \quad i = \dfrac{Ma}{R}\left(1 - e^{-tR/L}\right)$

26. (3) Flux through a closed circuit containing an inductor does not change instantaneously.

$$\therefore \quad L\left(\frac{E}{R}\right) = \frac{L}{4}(i) \quad \Rightarrow \quad i = \frac{4E}{R}$$

27. (1) The current in L for steady state $= \dfrac{E}{R_1}$

$$\therefore \quad \text{Energy stored in } L = \frac{1}{2} L I_0^2 = \frac{1}{2} L \frac{E^2}{R_1^2}$$

28. (2) $|e| = B \dfrac{dA}{dt}$

$$|e| = B \frac{(A_2 - A_1)}{(t_2 - t_1)}$$

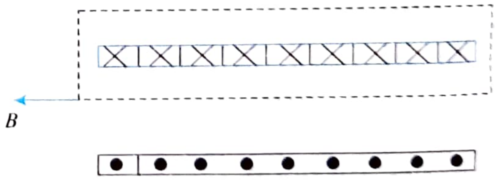
$$|e| = \frac{1\left(\frac{\pi}{4} - 0\right)}{(1 - 0)}$$

$$e = \frac{\pi}{4} \text{ V}$$

29. (1) $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I \quad \Rightarrow \quad B2b = \mu_0 I$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2b} \rightarrow \text{due to one tape}$$

Net field = 2 B



Magnetic flux passing through this double tape

$$\phi = 2BA = 2B(\ell h) \quad \Rightarrow \quad \phi = \frac{\mu_0 I}{b} \ell h$$

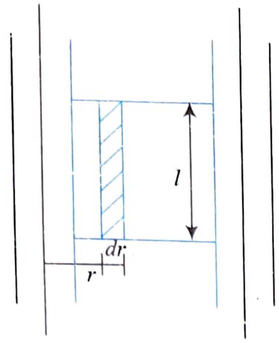
$$L = \frac{\phi}{I} = \frac{\mu_0 \ell h}{b} \quad \Rightarrow \quad \frac{L}{\ell} = \frac{\mu_0 h}{b}$$

30. (2) Since the wires are infinite, so the system of these two wires can be considered as a closed rectangle of infinite length and breadth equal to d. Flux through the strip:

$$\phi = \int_a^{d-a} \frac{\mu_0 I}{2\pi r}(l\,dr) = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{d-a}{a}\right)$$

The other wire produces the same result, so the total flux through the dotted rectangle is

$$\phi_{\text{total}} = \frac{\mu_0 I l}{\pi} \ln\left(\frac{d-a}{a}\right)$$



The total inductance of length l

$$L = \frac{\phi_{\text{total}}}{I} = \frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right)$$

Inductance per unit length $= \frac{L}{l} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$

31. (2) Let current i_1 in the straight wire be upward. Then the magnetic field due to the straight wire has magnitude $B_1 = \mu_0 i_1/2\pi r$ at a distance r. In accordance with right hand rule, B_1 points inward to the plane of page. We consider a differential strip of thickness dr, area $dA_2 = a\,dr$. Magnetic flux through area dA,

$$d\phi_B = B_1\,(a\,dr)$$

Total flux through the loop,

$$\phi_B = \int B_1 a dr = \int_c^{c+b} \frac{\mu_0 i_1}{2\pi r}\,a\,dr$$

$$= \frac{\mu_0 i_1 a}{2\pi}\int_c^{c+b}\frac{dr}{r} = \frac{\mu_0 i_1 a}{2\pi}\ln\left(\frac{c+b}{c}\right)$$

Therefore mutual inductance,

$$M = \frac{\phi}{i_1} = \frac{\mu_0 a}{2\pi}\ln\left(1+\frac{b}{c}\right)$$

32. (2) Inductors 5 mH and 10 mH are connected in parallel, hence equivalent inductance $L_{eq} = \frac{5\times 10}{5+10} = \frac{50}{15} = \frac{10}{3}$ mH

Current at steady state, $I = \frac{20}{5} = 4$ A

As L_1 and L_2 are in parallel.

$$I_1 = I\left(\frac{L_2}{L_1+L_2}\right) = 4\left[\frac{10}{10+5}\right] = \frac{8}{3}\text{ A}$$

33. (1) When energy on both is same, means energy on capacitor is half of its maximum energy.

$$\frac{q^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \Rightarrow q = \frac{Q}{\sqrt{2}}$$

$$\Rightarrow \quad Q\cos\omega t = \frac{Q}{\sqrt{2}} \Rightarrow \cos\omega t = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}$$

34. (4) $E = L\frac{dI}{dt}$ or $dI = \frac{E}{L}dt$

or $\quad I = \frac{2}{4}t = 0.5\,t \Rightarrow t = 2I = 2\times 5 = 10$ s

35. (2) In steady state: $P = I^2R$

Energy in inductor $= \frac{1}{2}LI^2$

After connecting x and z, the whole energy of inductor will go into heat so heat produced: $H = \frac{1}{2}LI^2 = \frac{1}{2}L\frac{P}{R} = \frac{1}{2}P\tau$

36. (1) $\Delta q = \frac{\Delta\phi}{R}$

$$\phi_i = 0$$

$$\phi_f = \left(\frac{\mu_0}{2}\frac{i}{b}\right)(\pi a^2) = \frac{\mu_0 i a^2\pi}{2b}$$

$$\therefore \quad \Delta\phi = \phi_f - \phi_i = \frac{\mu_0 i a^2\pi}{2b}$$

So, $\quad \Delta q = \frac{\mu_0 i a^2\pi}{2Rb}$

37. (3) The current at time t is given by

$$i = i_0(1-e^{-t/\tau})$$

Here $i_0 = \frac{E}{R}$ and $\tau = \frac{L}{R}$

$$\therefore \quad q = \int_0^\tau i dt = \int_0^t i_0(1-e^{-t/\tau})\,dt$$

$$= \frac{i_0\tau}{e} = \frac{\left(\frac{E}{R}\right)\left(\frac{L}{R}\right)}{e} = \frac{EL}{eR^2}$$

38. (1) At $t = 0$, the branch containing L will offer infinite resistance while the branch containing the capacitor will be effectively a short circuit. Hence, $(i)_{t=0} = \frac{\varepsilon}{R}$. Similarly at $t = \infty$, L will offer zero resistance whereas C will be an open circuit. Hence effective resistance $= R + \frac{6\times 3R}{6+3} = 3R$ $(i)_{t=\infty} = \frac{\varepsilon}{3R}$.

The required ratio $= \frac{\varepsilon}{R}\times\frac{3R}{\varepsilon} = 3:1$

39. (3) Induced e.m.f. $e = L\frac{dI}{dt} = A\frac{dB}{dt}$

$$\Rightarrow \quad \int_0^I dI = \int_0^B \frac{A}{L}dB \Rightarrow I = \frac{A}{L}B$$

$$\Rightarrow \quad I_{\max} = \frac{A}{L}B_{\max} = \frac{10^{-2}}{10\times 10^{-3}}\times 0.1 = 0.1\text{ A} = 100\text{ mA}$$

40. (2) $E_i - ir_1 = 0 \Rightarrow i = \frac{E_1}{r_1} = \frac{3}{1} = 3\,\Omega$

$$i = \frac{10}{3R+2}, \frac{10}{3R+2} = 3, R = \frac{4}{9}\Omega$$

41. (2) Use $i = \frac{E}{R}[1-e^{-t/\tau}]$ and $U = \frac{1}{2}Li^2$

42. (3) Constancy of flux implies that $\frac{E}{R}L_1 = i(L_1+L_2)$

i.e., $\quad i = \frac{EL_1}{R(L_1+L_2)}$

43. (3) Let i_1 be the current in the circuit before shifting

$$i_1 = \frac{E}{R} \qquad \text{...(i)}$$

Since the flux associated with the inductors will be same just before and just after shifting, therefore

$$i_1\,5L = i_2\,9L$$

$$i_2 = \frac{5}{9}\frac{E}{R}$$

4. **(4)** At $t=0$, for the purpose of current calculation in circuit, inductor can be assumed as open and capacitor as short circuited.

45. **(1)** $L=\dfrac{\mu_0 N^2 \pi r^2}{\ell}$

Length of wire $= N\,2\pi r =$ Constant ($= C$, suppose)

$$\therefore \quad L=\mu_0\left(\frac{C}{2\pi r}\right)^2\frac{\pi r^2}{\ell}$$

$$\therefore \quad L\propto\frac{1}{\ell}$$

$\therefore$ Self-inductance will become $2L$.

46. **(2)** Equivalent inductance

$$L_{eq}=L+2L=3L,\ C_{eq}=C+2C=3C$$

$\therefore$ Frequency of oscillation

$$f=\frac{1}{2\pi\sqrt{L_{eq}C_{eq}}}=\frac{1}{6\pi\sqrt{LC}}$$

47. **(3)** Just before opening the switch, the current in the inductor is $\dfrac{E}{R}$.

Energy stored in it $=\dfrac{1}{2}L\left(\dfrac{\varepsilon}{R}\right)^2$

This energy will dissipate in resistors R_1 and R_2 in the ratio $\dfrac{1}{R_1}$ and $\dfrac{1}{R_2}$.

48. **(1)** Given, $L_1=1$ mH, $L_2=2$ mH, $R_1=1\Omega$, $R_2=2\Omega$

In the first circuit,

$$L=L_1+L_2 \text{ and } R=R_1+R_2$$

$$\tau_1=\frac{L}{R}=\frac{3\text{ mH}}{3\Omega}=1\text{ ms}$$

In the second circuit,

$$L=\frac{L_1L_2}{L_1+L_2}=\frac{2\times10^{-6}}{3\times10^{-3}}=\frac{2}{3}\times10^{-3}$$

$$R=\frac{R_1R_2}{R_1+R_2}=\frac{2}{3}\Omega$$

$$\tau_2=\frac{\frac{2}{3}\times10^{-3}}{2/3}=1\text{ ms}$$

In the third circuit,

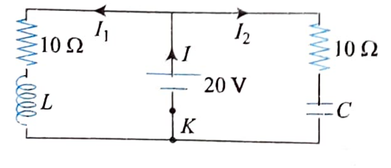
$$L=L_1+L_2=3\text{ mH}$$

$$R=\frac{R_1R_2}{R_1+R_2}=\frac{2}{3}\Omega$$

$$\tau_3=\frac{L}{R}=\frac{3\times10^{-3}}{2/3}=\frac{9}{2}\text{ ms}$$

49. **(3)** Current in branches containing L and R will flow independently

$$I_1=\frac{20}{10}\left(1-e^{-\frac{t}{5\times10^{-4}}}\right)=\frac{3}{2}=1.5\text{ A}$$



$$I_2=\frac{20}{10}e^{-\frac{t}{10^{-3}}}=1.0\text{ A}$$

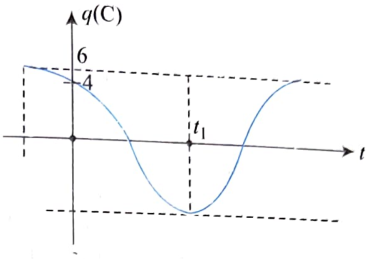
$$I=I_1+I_2=2.5\text{ A}$$

50. **(1)** For $t<t_0$, $E=L\dfrac{di}{dt}\ \Rightarrow\ i=\dfrac{E}{L}t$

For $t>t_0$, $L\dfrac{di}{dt}=0\ \Rightarrow\ i=$ constant

51. **(1)** $i=\sqrt{5}$ A

$$\frac{q_m^2}{2C}=\frac{q^2}{2C}+\frac{1}{2}Li^2\ \Rightarrow\ q_{\max}=6\text{ C}$$



$q=Q\cos(\omega t+\delta)$, at $t=0$, $q=4$

$$\Rightarrow\quad 4=6\cos\delta\ \Rightarrow\ \cos\delta=\frac{2}{3}\ \Rightarrow\ \delta=\cos^{-1}\left(\frac{2}{3}\right)$$

Charge on the capacitor is maximum at $t=t_1$

for this $\omega t_1+\delta=\pi$

$$\Rightarrow\quad t_1=\frac{1}{\omega}(\pi-\delta)=\sqrt{LC}\,(\pi-\delta)=2\left[\pi-\cos^{-1}\left(\frac{2}{3}\right)\right]$$

52. **(1)** $R_{eq}=\dfrac{5R}{6}\ \Rightarrow\ I=\dfrac{6E}{5R}=1$ A

53. **(1)** $\omega=\dfrac{1}{\sqrt{L_{eq}C_{eq}}}=\dfrac{1}{\sqrt{2LC/2}}=\dfrac{1}{\sqrt{LC}}$

54. **(1)** At $t=0$, i.e., when the key is just pressed, no current exists inside the inductor. So 10 Ω and 20 Ω resistors are in series and a net resistance of (10 + 20) = 30 Ω exists across the circuit.

Hence, $I_1=\dfrac{2}{30}=\dfrac{1}{15}$ A

As $t\to\infty$, the current in the inductor grows to attain a maximum value, i.e., the entire current passes through the inductor and no current passes through 10 Ω resistor.

Hence, $I_2=\dfrac{2}{20}=\dfrac{1}{10}$ A

55. **(3)** $U_{\max}=\dfrac{1}{2}LI_0^2$, $U=\dfrac{U_{\max}}{2}$

$$\Rightarrow\quad \frac{1}{2}LI^2=\frac{1}{2}\left[\frac{1}{2}LI_0^2\right]\ \Rightarrow\ I_0^2\left[1-e^{-t/\tau}\right]^2=\frac{I_0^2}{2}$$

$$\Rightarrow\quad e^{-t/\tau}=1-\frac{1}{\sqrt{2}}=\frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\Rightarrow\quad -\frac{t}{\tau}=\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\ \Rightarrow\ t=\tau\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

56. **(3)** The magnetic field produced by the square loop is parallel to the plane of the circular loop. Hence the mutual inductance is zero.

57. **(4)** Let current i flows in the bigger ring, then the magnetic field on its axis

$$B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$

Flux linked with the smaller ring: $\phi = B\pi r^2$

$$\phi = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}} \pi r^2 = M i$$

$$\therefore \quad M = \frac{\mu_0 \pi R^2 r^2}{2(R^2 + x^2)^{3/2}}$$

58. (1) Maximum mutual inductance:

$$M = \sqrt{L_1 L_2} = 200 \text{ mH}$$

59. (2) Current through the inductor before closing the switch $= 10/(5 + 5) = 1$ A

Current through the inductor after closing the switch (in steady state) $I = \frac{20}{5} = 4$ A

$$\therefore \quad \Delta\phi = L\Delta I = 1.5 \text{ Wb}$$

60. (1) Initially no current in the inductor, so initial current

$$i_1 = \frac{15}{R_1 + R_2 + R_3} = 0.25 \text{ A}$$

In steady state, no current in R_2 and R_3. So final current through battery: $i_2 = \frac{15}{R_1} = 0.5$ A

61. (2) Since resistance of the circuit is increasing and hence current in the circuit is decreasing, so di/dt is negative.

Current in the circuit is given by $i = \dfrac{6 - L\dfrac{di}{dt}}{12}$

Since (di/dt) is negative so the value of numerator will be more than 6V and hence current in the circuit at that instant will be more than 0.5 A.

62. (4) Induced emf in the coil is given by $E = -\frac{L\,di}{dt}$

From the graph it is clear that for time t_1, current does not change with time, i.e., $\frac{di}{dt} = 0$. So, induced emf is zero.

Hence, graph (1) and (2) are wrong.

From time interval t_1 to t_2, (di/dt) is negative and constant.

$$\text{So, } E = -\frac{L\,di}{dt} = -L\left(-\frac{di}{dt}\right) \Rightarrow E = \frac{L\,di}{dt}$$

Hence, induced emf is positive and constant.

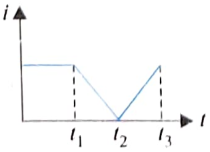
From t_2 to t_3, (di/dt) is positive and constant.

$$\text{So, induced emf } E = -L\left(+\frac{di}{dt}\right) = -\frac{L\,di}{dt}$$

Hence, induced emf is negative and constant.

Hence, choice (4) is correct.

Choice (3) is wrong because in the graph the induced emf is never negative and is not constant.



63. (2) Maximum current $I_m = \frac{100}{50} = 2$ A

Potential drop across the resistor is 2 × 50 = 100 V

Initial potential difference across capacitor is zero

$$L_{eq} = (200 + 50) = 250 \text{ mH}$$

$$\text{Energy stored} = \frac{1}{2} L_{\text{eff}} I_M^2 = \frac{1}{2} \times 250 \times 10^{-3} \times 2^2 = 0.5 \text{ J}$$

When S_1 is opened $\frac{1}{2} L_{\text{eff}} I_M^2 = \frac{1}{2} C V_{\text{max}}^2$

$$V_{\text{max}} = \sqrt{\frac{2 \times 0.5}{100 \times 10^{-6}}} = 100 \text{ V}$$

64. (1) See the diagram in the problem

The $(L - R)$ and $(C - R)$ sections having the same time constant are in parallel.

The current through the $R - L$ branch will be unaffected by the parallel $R - C$ branch

$$i_L(t) = \frac{E}{R}\left[1 - e^{\frac{-Rt}{L}}\right] \quad \text{...(i)}$$

Similarly the current in the RC branch is given by

$$i_C(t) = \frac{E}{R} e^{\frac{-t}{RC}} \quad \text{...(ii)}$$

Hence the current through the battery is

$$i(t) = i_L(t) + i_C(t) = \frac{E}{R}\left[1 - e^{-\frac{R}{C}t} + e^{-\frac{t}{RC}}\right] \quad \text{...(iii)}$$

Time constant of RL branch is $\tau_L = \frac{L}{R}$ and the time constant of RC branch is $\tau_C = RC$.

If these are equal, $\tau_L = \tau_C = T$, equation (iii) becomes

$i(t) = \dfrac{E}{R}\left[1 - e^{-\frac{t}{T}} + e^{-\frac{t}{T}}\right] = \dfrac{E}{R}$ for all values of $t > 0$. Hence the result.

65. (4) The inductance acts as a short-circuit. The whole of the current. The whole of the current flows through the inductance. Since no current flows through the bulb does not glow.

66. (3) Steady state current $I_0 = \dfrac{E}{R}$

$$= \frac{12}{12/4} = 4 \text{ A}$$

67. (2) $\phi = MI = MI_0 t$

Hence, induced emf $e = \dfrac{d\phi}{dt} = MI_0$

Current through ring $= \dfrac{e}{R} = \dfrac{MI_0}{R}$

68. (1) Notice that magnetic field due to left coil is towards left. Since magnetic field due to right coil is also towards left, so it increases flux associated with left coil. To oppose it, induced current due to mutual induction opposes the original current in left coil.

69. (4) Angular frequency of oscillation

$$\omega = \frac{1}{\sqrt{LC_1}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 8 \times 10^{-6}}}$$

$$= 2.5 \times 10^4 \text{ rad/s} = 25 \times 10^3 \text{ rad/s}$$

$$\frac{Q}{C_1} - L\frac{dI}{dt} = 0 \Rightarrow \frac{Q}{C_1} + L\frac{d^2Q}{dt^2} = 0$$

The solution of this equation is $Q = Q_0 \sin(\omega t + \phi)$ where at $t = 0$, $Q = Q_0$

$$Q_0 = C_{eq}V = \frac{C_1C_2}{C_1+C_2}V$$

$$= \frac{8\times2\times10^{-12}\times20}{10\times10^{-6}} = 32\times10^{-6}\text{ C}$$

$$\phi = \frac{\pi}{2}$$

$$Q = (32\ \mu\text{C})\cos(2.5\times10^4\times125\times10^{-6}) \simeq -32\ \mu\text{C}$$

70. **(1)** Magnetic field energy stored $= U = \frac{1}{2}Li^2$

$$32 = \frac{1}{2}L(4)^2 \quad\text{or}\quad L = 4\text{ H}$$

Power dissipated as heat is $P = i^2R$

$$320 = 4^2 R \text{ or } R = 20\ \Omega$$

Time constant of circuit $= \tau = \frac{L}{R} = \frac{4}{20} = 0.2$ s

71. **(1)** Time constant of circuit $= \frac{L}{R} = \frac{50\text{ mH}}{10} = 5$ ms

Current at time T is given by

$$i = i_0(1 - e^{-T/\tau})$$

For $i = \frac{i_0}{2}, \frac{i_0}{2} = i_0(1 - e^{-T/\tau})$

$$e^{-T/\tau} = \frac{1}{2} \quad\text{or}\quad \frac{T}{\tau} = \log_e 2$$

$$T = \tau\log_2 2 = 5\text{ ms}\times0.693 = 3.2\text{ ms}$$

72. **(1)** Current at time t is given by

$$i = i_0(1 - e^{-t/\tau}) \text{ where } i_0 = E/R$$

The charge passed through the battery during the period t to $t + dt$ is idt. Thus, the total charge passed during time 0 to τ is

$$Q = \int_0^\tau idt = i_0\int_0^\tau (1 - e^{-t/\tau})\,dt$$

$$= i_0\left[t + \frac{e^{-t/\tau}}{\tau}\right]_0^\tau = i_0[\tau + \tau(e^{-1} - 1)]$$

$$= \frac{i_0\tau}{e} = \frac{E}{R}\frac{\tau}{e}$$

when $t = \infty$.

Multiple Correct Answers Type

1. **(3), (4)**

When switch is just closed in the circuit shown, at that moment current through the circuit is zero. Hence, e.m.f. induced across inductance L will be equal to e.m.f. E of the battery.

But as the current through the circuit increases, the induced e.m.f. in the solenoid decreases. But induced e.m.f. in the solenoid is equal to $|e| = L\frac{di}{dt}$.

Since di/dt decreases as time passes, therefore the graph for induced e.m.f. e and time t will be as shown in option (3).

Hence (1) is wrong and (3) is correct.

The graph for current should be such that at initial moment current is zero and current increases with time in such a way that the rate of increase of current gradually decreases. Hence, slope of the current–time curve should decrease with time. Therefore, the graph between current and time will be as shown in option (4).

Hence (2) is wrong and (4) is correct.

2. **(1), (3)**

In the circuit shown in the figure in problem, 6 Ω and 12 Ω resistances are in parallel with each other and their parallel combination is in series with 4 Ω and the inductance of 2H. Hence, equivalent resistance of these three resistances is equal to 8 Ω. Therefore,the time constant for the circuit is

$$\lambda = \frac{L}{R} = \frac{2}{8} = 0.25\text{ s}$$

Hence (1) is correct.

In steady state, no e.m.f. will be induced in the inductance. Hence current through the circuit will be equal to E/R where R is the equivalent resistance. Hence, the steady state current will be equal to $6/8 = 0.75$ A.

Hence (3) is correct and (2) is wrong.

3. **(1), (2)**

The coils are in parallel, so

$$L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} \Rightarrow \int L_1\,dI_1 = \int L_2dI_2$$

$\Rightarrow$ Initially $I_1 = 0, I_2 = 0 \Rightarrow C = 0$

So $L_1I_1 = L_2I_2$

4. **(1), (3)**

$$V = L\frac{dI}{dt} \Rightarrow \int_0^I dI = \frac{1}{L}\int_0^t Vdt$$

$\Rightarrow$ $I = \frac{1}{L}$ [Area under v–t graph from $t = 0$ to $t = t$]

At $t = 2$ s, $I = \frac{1}{2}\left[\frac{1}{2}\times2\times10\right] = 5$ A

For $t = 0$ to 2 s, $V = 5t$

$$I = \frac{1}{L}\int_0^t 5t\,dt = \frac{1}{2}\left[\frac{5t^2}{2}\right]_0^t = \frac{5t^2}{4}$$

For $t = 2$ to 4 s, $V = -5t + 20$

$$I = \frac{1}{2}\int_0^t(-5t + 20)\,dt \Rightarrow I = \frac{-5t^2}{4} + 10t$$

Hence the correct graph is (3).

5. **(1), (2)**

At $t < 0, I_L = \frac{6}{6} = 1$ A

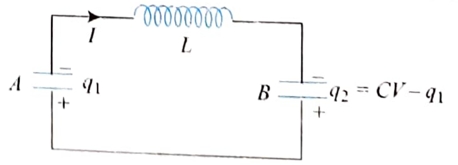
At $t >> 0, I_L = \frac{12}{3} = 4$ A

$\therefore$ $|\phi| = L[i_f - i_i] = 500\times10^{-3}\times3 = 1.5$ Wb

$\therefore$ (1) and (2) are the correct choices.

6. **(1), (3)**

Let at any time, charge and current in the circuit wire are as shown.



$$I = \frac{dq_1}{dt}$$

Applying Kirchhoff's law:

$$\frac{CV - q_1}{C} - \frac{q_1}{C} - L\frac{dI}{dt} = 0 \Rightarrow \frac{CV - 2q_1}{LC} = L\frac{d^2q_1}{dt^2}$$

$$\Rightarrow \quad \frac{d^2q_1}{dt^2} = -\frac{2}{LC}\left(q_1 - \frac{CV}{2}\right)$$

$$\Rightarrow \quad q_1 - \frac{CV}{2} = A\sin(\omega t + \delta) \qquad \text{...(i)}$$

where $\omega = \sqrt{\frac{2}{LC}} = 100\,\pi$ rad/s

At $t = 0, q_1 = 0 \quad \Rightarrow \quad 0 - \frac{CV}{2} = A\sin\delta$ ...(ii)

Differentiating (i), $\frac{dq_1}{dt} - 0 = A\omega\cos(\omega t + \delta)$

$$\Rightarrow \quad I = A\omega\cos(\omega t + \delta) \qquad \text{...(iii)}$$

At $t = 0, I = 0 \quad \Rightarrow \quad 0 = A\omega\cos\delta \quad \Rightarrow \quad \delta = \frac{\pi}{2}$

From (ii), $A = -\frac{CV}{2}$

From (i), $q_1 = \frac{CV}{2} - \frac{CV}{2}\sin\left(\omega t + \frac{\pi}{2}\right)$

$$\Rightarrow \quad q_1 = \frac{CV}{2}(1 - \cos\omega t) \qquad \text{...(iv)}$$

$$\Rightarrow \quad q_1 = 50(1 - \cos 100\,\pi t)\text{ mC}$$

Now $q_2 = CV - q_1 = \frac{CV}{2}(1 + \cos\omega t)$ ...(v)

$$= 50\,(1 + \cos 100\,\pi t)$$

From (ii), $I = -A\omega\sin\omega t$

Current in the circuit is maximum for $\omega t = \frac{\pi}{2}$.

And for $\omega t = \frac{\pi}{2}$, we see from (iv) and (v) that $q_1 = q_2 = \frac{CV}{2}$

7. (1), (2)

$$L\frac{di}{dt} = Bv\ell \quad \Rightarrow \quad \int di = \frac{B\ell}{L}\int v\,dt \quad \Rightarrow \quad i = \frac{B\ell}{L}x \qquad \text{...(i)}$$

$$F = ma \quad \Rightarrow \quad -iB\ell = mv\frac{dv}{dx}$$

$$\Rightarrow \quad -\frac{B^2\ell^2 x}{L} = mv\frac{dv}{dx} \quad \Rightarrow \quad -\frac{B^2\ell^2}{mL}\int_0^d x\,dx = \int_{v_0}^{v_0/2} v\,dv$$

$$\Rightarrow \quad -\frac{B^2\ell^2 d^2}{2mL} = \frac{-3v_0^2}{8},\ v_0 = \frac{J}{m}$$

$$\Rightarrow \quad d = \sqrt{\frac{3J^2L}{4B^2\ell^2 m}}$$

Put $x = d$ in (i), $i = \frac{B\ell}{L}\sqrt{\frac{3J^2L}{4B^2\ell^2 m}} = \sqrt{\frac{3J^2}{4Lm}}$

8. (1), (2), (3), (4)

At any time: $L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt}$

$$\Rightarrow \quad \int_0^{i_1} L_1 di_1 = \int_0^{i_2} L_2 di_2 \quad \Rightarrow \quad L_1 i_1 = L_2 i_2 \quad \Rightarrow \quad \frac{i_1}{i_2} = \frac{L_2}{L_1} \qquad \text{...(i)}$$

Total steady state current

$$i = E/R = i_1 + i_2 \qquad \text{...(ii)}$$

from equation (i) and (ii)

$$i_1 = \frac{EL_2}{R(L_1 + L_2)},\ i_2 = \frac{EL_1}{R(L_1 + L_2)}$$

$$U = \frac{1}{2}L_{eq}i^2 = \frac{1}{2}\left(\frac{L_1L_2}{L_1 + L_2}\right)\frac{E^2}{R^2}$$

9. (1), (3), (4)

When the current becomes constant, the p.d. across inductor will become zero. Hence the entire p.d. will be across R.

Also energy stored in the inductor becomes constant if current through it becomes constant.

10. (1), (2), (3)

$V = 2t \quad \Rightarrow \quad L(di/dt) = 2t$

$2(di/dt) = 2t \quad \Rightarrow \quad di/dt = t$

$i = t^2/2 \quad \Rightarrow \quad i - t$ graph is parabola

at $t = 2$s, $i = 2$ A

$U = (1/2)Li^2 = 4$ J

$dU/dt = Li(di/dt) = t^3 = 1$ J/s

11. (1), (2), (3)

Initially current $I = 0$, so $U = \frac{1}{2}Li^2 = 0$

$P = ei = 0$

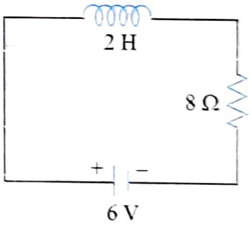
But initially $\frac{di}{dt} \neq 0$, so $e = L\frac{di}{dt} \neq 0$

12. (2), (3)

Based on theory

13. (1), (3)

In the circuit shown in figure of the question, 6 Ω and 12 Ω resistances are in parallel with each other and their parallel combination is in series with 4 Ω and the inductance of 2H. Hence, equivalent resistance of these three resistances is equal to 8 Ω. Therefore, this circuit may be reduced to the circuit as shown in figure.



The time constant for the circuit is

$$\tau = \frac{L}{R} = \frac{2}{8} = 0.25\text{ sec}$$

Hence, option (1) is correct.

In steady state, no emf will be induced in the inductance. Hence, current through the circuit will be equal to E/R where R is equivalent resistance. Hence, the steady state current will be equal to $\frac{6}{8} = 0.75$ amp.

Hence, option (3) is correct and (2) is wrong.

14. (2), (4)

Steady state current for both the circuits is same. Therefore,

$$\frac{V}{R_1} = \frac{V}{R_2} \text{ or } R_1 = R_2$$

Further $\tau_{L_1} < \tau_{L_2}$ (τ_L = time constant)

$$\frac{L_1}{R_1} < \frac{L_2}{R_2} \text{ or } L_1 < L_2 \qquad (R_1 = R_2)$$

Linked Comprehension Type

For Problems 1–2

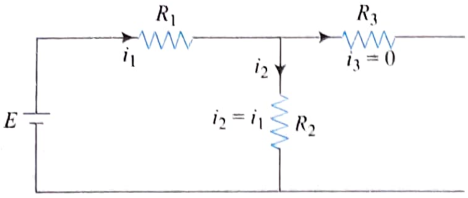
1. **(2)** 2. **(3)**

At $t = 0$, capacitor will behave like a short circuit and the inductor as open circuit but as $t \to \infty$, the nature is just opposite.

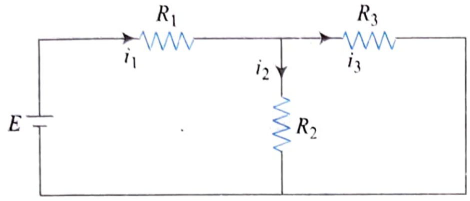
For Problems 3–5

3. (1) **4. (1)** **5. (2)**

At $t = 0$, circuit can be considered as follows:



After a long time, circuit can be considered as follows:



Immediately after closing S, $\frac{di_3}{dt} \neq 0$. Because induced e.m.f.

$$\frac{Ldi_3}{dt} \neq 0.$$

For Problems 6–11

6. (2) **7. (1)** **8. (3)** **9. (1)**

10. (3) **11. (2)**

Instantaneous current in the capacitor,

$$q = CV_C = (2)(3e^{-2t}) = 6e^{-2t}\text{ A}$$

$$\text{Current, } i_C = \frac{dq}{dt} = -12e^{-2t}\text{ A}$$

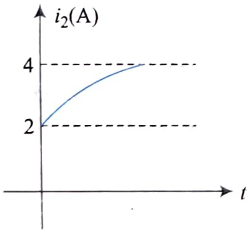
This current flows from B to O.

From KVL, we have

$$i_L = i_1 + i_2 + i_C = 10e^{-2t} + 4 - 12e^{-2t}$$
$$= (4 - 2e^{-2t})\text{ A} = [2 + 2(1 - e^{-2t})]\text{ A}$$

i_L vs. time graph is as shown in figure.

i_L increases from 2 A to 4 A exponentially.

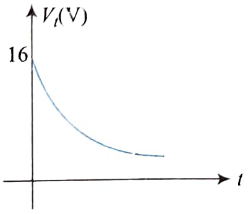


$$V_L = L\frac{di_L}{dt}$$

$$= (4)\frac{d}{dt}(4 - 2e^{-2t}) = 16e^{-2t}\text{ V}$$

V_L decreases exponentially from 16 A to 0 as shown in figure.

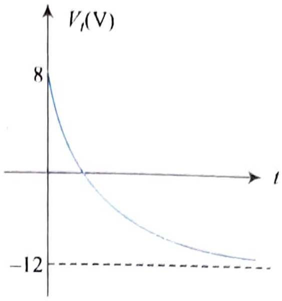
To determine V_{AC}, we begin from A and end at C. From KVL, we have



$$V_A - i_1R_1 + i_2R_2 = V_C$$
$$V_A - V_C = i_1R_1 - i_2R_2$$

Substituting the values, we have

$$V_{AC} = (10e^{-2t})(2) - (4)(3)$$



$$V_{AC} = (20e^{-2t} - 12)\text{ V}$$

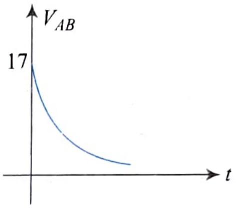
At $t = 0$, $V_{AC} = 8$ V

At $t = \infty$, $V_{AC} = -12$ V

Therefore, V_{AC} decreases exponentially from 8 V to – 12 V.

Similarly, we have from A to B

$$V_A - i_1R_1 + V_C = V_B$$
$$V_{AB} = V_A - V_B = i_1R_1 - V_C$$



Substituting the values, we have

$$V_{AB} = (10e^{-2t})(2) - 3e^{-2t}$$
$$V_{AB} = 17e^{-2t}\text{ V}$$

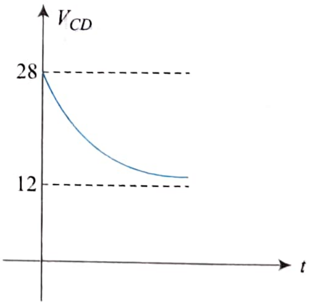
Thus, V_{AB} decreases exponentially from 17 V to 0.

As we move from C to D,

$$V_C - i_2R_2 - V_L = V_D$$
$$V_{CD} = V_C - V_D = i_2R_2 + V_L$$

Substituting the values we have ,

$$V_{CD} = (4)(3) + 16e^{-2t}$$



$$V_{CD} = (12 + 16e^{-2t})\text{ V}$$

At $t = 0$, $V_{CD} = 28$ V

and at $t = \infty$, $V_{CD} = 12$ V

i.e., V_{CD} decreases exponentially from 28 V to 12 V.

For Problems 12–13

12. (4) **13. (1)**

12. $E = \frac{r}{2}\frac{dB}{dt} = \frac{r}{2}\mu_0 n\frac{dI}{dt} \Rightarrow E = -\frac{\mu_0 nr}{2}I_{\max}\omega\sin\omega t$

13. $E = \frac{R^2}{2r}\frac{dB}{dt} = -\frac{R^2}{2r}\mu_0 n\, I_{\max}\omega \sin \omega t$

$$= -\frac{\mu_0 n I_{\max}\omega R^2}{2r}\sin \omega t$$

For Problems 14–16

14. **(2)** **15. (4)** **16. (1)**

S_1 and S_2 are closed for 1 s.

Charge on capacitor,

$$q = CE\,(1 - e^{-t/RC}) = 1 - \frac{1}{e} \qquad \text{...(i)}$$

The current in inductor,

$$I = \frac{E}{R}(1 - e^{-tR/L}) = 1 - \frac{1}{e} \qquad \text{...(ii)}$$

Now S_1 and S_3 are opened and S_2 is closed,

It is LC circuit,

$$q = q_{\max}\sin(\omega t + \phi) \qquad \text{...(iii)}$$

$$I = (q_{\max})\,\omega\cos(\omega t + \phi) \qquad \text{...(iv)}$$

As total energy (Magnetic + electrical) is constant

$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}\frac{q_{\max}^2}{C} = \frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C} \qquad \text{...(v)}$$

$$q_{\max} = \sqrt{2}\left(1 - \frac{1}{e}\right) \qquad \text{...(vi)}$$

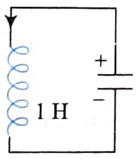
$$I_{\max} = \sqrt{2}\left(1 - \frac{1}{e}\right) \qquad \text{...(vii)}$$

From (iii) at $t = 0$, we get

$$\left(1 - \frac{1}{e}\right) = \sqrt{2}\left(1 - \frac{1}{e}\right)\sin\phi \;\Rightarrow\; \phi = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

After closing S_2, circuit will be as shown. Direction of current shows that charge on capacitor will be decreasing. Hence $\phi = 3\pi/4$.

$$q = \sqrt{2}\left(1 - \frac{1}{e}\right)\sin\left(\omega t + \frac{3\pi}{4}\right)$$



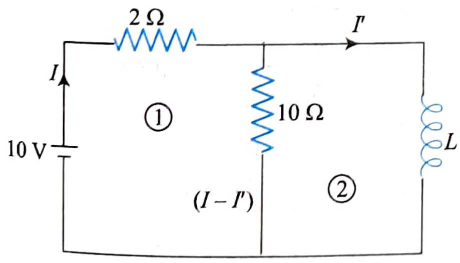
where, $\omega = \frac{1}{\sqrt{LC}} = 1$

$$q = \sqrt{2}\left(1 - \frac{1}{e}\right)\sin\left(t + \frac{3\pi}{4}\right)$$

For Problems 17–19

17. **(2)** **18. (2)** **19. (1)**

From loop by applying Kirchhoff's law,



$$12\,I - 10\,I' = 10 \qquad \text{...(i)}$$

From loop (ii)

$$-L\frac{dI'}{dt} + 10\,I - 10\,I' = 0$$

$$I - I' = \frac{L}{10}\frac{dI'}{dt} \qquad \text{...(ii)}$$

Solving simultaneously (i) and (ii), we have

$$I' = 5 - 5e^{-\frac{5t}{3L}} \qquad \text{...(iii)}$$

and $$I = 5 - \frac{25}{6}e^{-\frac{5t}{3L}} \qquad \text{...(iv)}$$

$$I - I' = \frac{5}{6}e^{-\frac{1000t}{3}}$$

$$E_L = \frac{1}{2}L(I')^2;\; E_L = \frac{125}{2}\left(1 - e^{-\frac{1000t}{3}}\right)^2 \text{ mJ}$$

Current in the inductor at $t = \infty$, $I' = 5$ A

$$E_L(t \to \infty) = \frac{1}{2}\times 5\times 10^{-3}(5)^2$$

$$E_L = 62.5 \text{ mJ}$$

$$E_C(t \to \infty) = \frac{1}{2}CV^2 = \frac{1}{2}\times 20\times 10^{-6}\times 100 = 1 \text{ mJ}$$

For Problems 20–21

20. **(2)** **21. (2)**

The equivalent inductance $L = 500$ mH. Just before opening the switch, current through the inductors is

$$I_M = \frac{200}{100} = 2 \text{ A}$$

Potential drop across the resistor is 2 × 100 = 200 V

Hence, across capacitor potential difference is zero.

So energy stored

$$U = \frac{1}{2}L(2)^2 = \frac{1}{2}\times 500\times 10^{-3}\times 2^2 = 1 \text{ J}$$

$$\frac{1}{2}LI_M^2 = \frac{1}{2}CV_{\max}^2 = 1 \text{ J}$$

$$V_{\max} = I_M\sqrt{\frac{L}{C}} = 2\times\sqrt{\frac{500\times 10^{-3}}{50\times 10^{-6}}} = 200 \text{ V}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{500\times 10^{-3}\times 50\times 10^{-6}}} = 200 \text{ rad s}^{-1}$$

For Problems 22–24

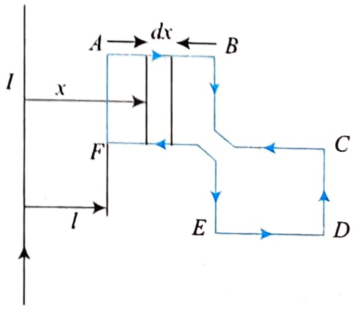
22. **(1)** **23. (1)** **24. (2)**

Consider a strip at a distance x from the wire of thickness dx.

Magnetic flux associated with this strip $\phi = B(x)\,a\,dx = \frac{\mu_0 Ia}{2\pi x}dx$

$$\phi = \frac{\mu_0 Ia}{2\pi}\left[\int_l^{a+l}\frac{dx}{x} + \int_{a+l}^{2a+l}\frac{dx}{x}\right] = \frac{\mu_0 Ia}{2\pi}\ln\left(\frac{2a+l}{l}\right)$$

$$M = \frac{\phi}{I} \;\Rightarrow\; M = \frac{\mu_0 a}{2\pi}\ln\left(\frac{2a+l}{l}\right)$$



$$e = -M\frac{dI}{dt}$$

$$e = -MI_0 = -\frac{\mu_0 I_0 a}{2\pi}\ln\left(\frac{2a+l}{l}\right)$$

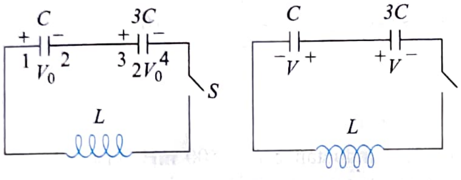
$$\text{Heat produced} = \frac{\varepsilon^2}{R}t = \frac{\left[\frac{\mu_0 I_0}{2\pi}\ln\left(\frac{2a+l}{l}\right)\right]^2 at}{8\lambda}$$

For Problems 25–27

25. (1) **26. (3)** **27. (3)**

When current is maximum $\frac{di}{dt} = 0$

$\therefore$ e.m.f. across $L = 0$, so potential difference across the capacitor will be same.



From the law of conservation of charge on plates 2 and 3,

$$3CV + CV = 6\,CV_0 - CV_0$$

$$\Rightarrow \quad V = \frac{5V_0}{4}$$

Loss in energy of capacitor = energy stored in inductor

$$\Rightarrow \quad \frac{1}{2}CV_0^2 + \frac{1}{2}3C(2V_0)^2 - \frac{1}{2}\times 4CV^2 = \frac{1}{2}LI^2$$

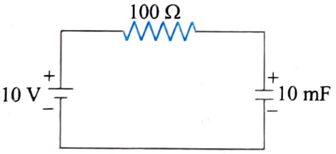
$$\Rightarrow \quad I = \frac{3}{2}V_0\sqrt{\frac{3C}{L}}$$

For Problems 28–30

28. (2) **29. (1)** **30. (4)**

Initial charge $= CV_0 = Q_0$

$= 10 \times 10^{-3} \times 5 = 50$ mC



When capacitor is connected at position 1.

$$E - IR - \frac{q}{C} = 0$$

$$\int_0^t \frac{1}{RC}dt = \int_{Q_0}^{q}\frac{dq}{EC-q} \quad \text{or} \quad q = 50\,[2 - e^{-t}]\text{ mC}$$

At $t = 1$ s, $q = 50\,[2 - e^{-1}]$

Voltage across the capacitor at that time

$$V = \frac{q}{C} = \frac{50[(2-1/e)]}{10\times 10^{-3}} = 5\times 10^3[2-(1/e)]\text{ V}$$

$$\frac{1}{2}Li^2 = \frac{1}{2}CV^2 \Rightarrow i = \left(2 - \frac{1}{e}\right)\times 10^4\text{ A}$$

$$\text{Frequency} = \frac{1}{2\pi\sqrt{LC}} = \frac{100}{\pi}\text{ Hz}$$

For Problems 31–33

31. (2) **32. (4)** **33. (3)**

Initially there is no current in the inductor.

So initially, $V_A - V_B = \left(\frac{6}{1+5}\right)\times 1 = 1$ V ...(i)

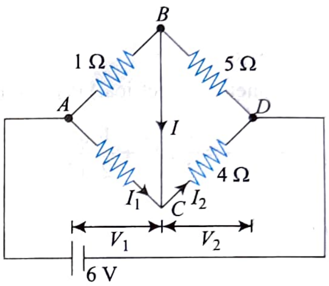
$$V_A - V_C = \left(\frac{6}{2+4}\right)\times 2 = 2\text{ V} \quad \text{...(ii)}$$

From (i) and (ii), $V_B - V_C = 2 - 1 = 1$ V

$$\Rightarrow \quad V_B - V_C = L\frac{di}{dt}$$

$$\Rightarrow \quad 1 = 0.1\frac{di}{dt} \Rightarrow \frac{di}{dt} = 10\text{ As}^{-1}$$

Current through 6 Ω resistor will remain constant because it is independently connected to 6 V. After a long time, inductor will behave like a simple wire.



1 Ω and 2 Ω are in parallel, their equivalent is $\frac{2}{3}$ Ω.

5 Ω and 4 Ω are in parallel, their equivalent is $\frac{20}{9}$ Ω.

$$V_1 = \frac{\frac{2}{3}\times 6}{\frac{2}{3}+\frac{20}{9}} = \frac{18}{13}\text{ V}, V_2 = V - V_1 = 6 - \frac{18}{13} = \frac{60}{13}\text{ V}$$

$$I_1 = \frac{V_1}{2} = \frac{18}{13\times 2} = \frac{9}{13}\text{ A}, I_2 = \frac{V_2}{4} = \frac{60}{13\times 4} = \frac{15}{13}\text{ A}$$

Current through inductor: $I = I_2 - I_1 = \frac{15}{13} - \frac{9}{13} = \frac{6}{13}$ A

For Problems 34–36

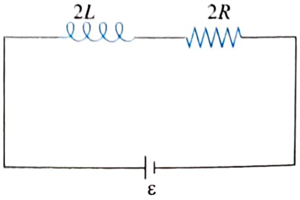
34. (2) **35. (3)** **36. (2)**

Since the field is increasing in inside direction in bigger loop, so the current will be induced to oppose this increasing flux. Hence in anticlockwise direction.

$$\phi = BA = B(l^2 + b^2)$$

$$|\varepsilon| = \left|\frac{d\phi}{dt}\right| = \frac{dB}{dt}(\ell^2 + b^2) = 0.5\text{ V}$$

$$i = \frac{\varepsilon}{2R}\left[1 - e^{-\frac{t2R}{2L}}\right] = \frac{0.5}{20}\left[1 - e^{-t}\right]$$



$$i = \frac{1}{40}\left[1 - e^{-t}\right]$$

For Problems 37–39

37. (3) **38. (1)** **39. (3)**

When inductance of coil is changed abruptly, flux in it remains same. i.e. $\phi = LI$ = constant. Since here inductance will increase on insertion of iron rod, so current will decrease. Now current will increase to attain the steady state value (E/R). During this time, inductance (nL) will remain same and p.d. across A and B will also remain same. But as current will change with time so rate of production of heat will vary.

For Problems 40–42

40. (4) $L_1 = L,\ L_2 = \eta L,\ \phi = L_1 I_1 = L_2 I_2,\ I_1 = E/R$

$$\Rightarrow \quad I_2 = \frac{L_1}{L_2} I_1 = I_1/\eta$$

$$W = U_f - U_i = \frac{1}{2} L_2 I_2^2 - \frac{1}{2} L_1 I_1^2$$

$$= \frac{1}{2}\eta L\left(\frac{I_1}{\eta}\right)^2 - \frac{1}{2} L I_1^2 = \frac{LE^2}{2\eta R^2}[1-\eta]$$

41. (1) As current increases on pulling out the iron piece, the current will decrease to achieve its steady state value E/R again. At any time:

$$E = IR + \eta L \frac{dI}{dt} \quad \Rightarrow \quad \int_{I_2}^{i} \frac{dI}{E - IR} = \int_0^t \frac{dt}{\eta L}$$

Solve to get: $i = \dfrac{E}{R}\left[1 - \left(1 - \dfrac{1}{\eta}\right) e^{-t/\lambda}\right]$

42. (4) $P = Ei = \dfrac{E^2}{R}\left[1 - \left(1 - \dfrac{1}{\eta}\right) e^{-t/\lambda}\right]$

For Problems 43–45

43. (4) Since $V_L = \dfrac{Ldi}{dt}$

$$Ldi = V_L dt$$

Integrating $L\int di = \int V_L dt$

$$L(i_f - i_i) = \int_0^t V_L dt = \text{area under } V_L\text{–}t \text{ graph.}$$

44. (2) Form explanation on previous page

$$L(i_f - i_i) = \text{area under } V_L\text{–}t \text{ graph}$$

$$2(i_f - 0) = \frac{1}{2} \times 1 \times 8 = 4$$

$$i_f = 2\text{A}$$

45. (2) For $t \le 1$ s

$$V_L = 8t$$

$$Ldi = V_L dt$$

$$\int Ldi = \int 8t\, dt$$

$$Li = \frac{8t^2}{2} = 4t^2$$

$$i = 2t^2$$

So current versus time graph is a parabola.

For $1\text{s} \le t \le 2\text{s}$

$$V_L = 16 - 8t$$

$$\int Ldi = \int (16 - 8t)\, dt$$

$$Li = 16t - \frac{8t^2}{2}$$

$$i = 8t - 2t^2$$

So i–t graph in a parabola.

So for entire time shown, i–t graph in a parabola.

Matrix Match Type

1. i. → a.; ii. → b.; iii. → c.; iv. → a.

i. Just after switch S is closed, flux in M starts increasing in left direction, so in N also the flux starts increasing in left direction. This will induce current in N in a direction so that the flux is in right direction. This is possible if induced current in N is from A to B.

ii. In this case just reverse of (i) will happen, because after closing the switch, the flux in M starts decreasing in left direction.

iii. After a long time of closing the switch, flux becomes constant. Hence, no current is induced.

iv. Just after closing S, flux starts increasing, but because M moves away, so due to this flux through N will decrease. But there will be a net increase in flux in N in left direction. This is the case similar to (i).

2. i. → c.; ii. → c.; iii. → a., c.; iv. → a., c.

i. Current in inductor when switch is open:

$$I_0 = \frac{E}{R}$$

Initially induced e.m.f. will be equal to E and finally it is zero. So, energy stored will be zero.

ii. Same as (i).

iv. Here current becomes zero suddenly.

So, $\dfrac{dI}{dt}$ is large.

Hence, induced e.m.f. $L\dfrac{dI}{dt}$ will be large. Finally, energy stored in inductor will be zero.

3. i. → c.; ii. → a.; iii. → d.; iv. → b.

When the switch is connected with a for a long time, current in the circuit would be E/R and E/R is greater for (i) and (iii) than for (ii) and (iv).

Next, comparison is made on the basis of time constant. Shorter time constant means faster decay of the current.

4. i. → a., c.; ii. → a., c., d.; iii. → a., b.; iv. → a., c., d.

i. Let $I = I_0 \sin(\omega t + \phi)$, so current leads emf by ϕ

$$\tan\phi = \frac{-X_L}{R}, \text{ so } \phi \ne 0 \quad \Rightarrow \quad \cos\phi \ne 1$$

ϕ is negative, so current lags emf

$$P_{av} = E_v I_v \cos\phi \ne 0$$

ii. $R = 0,\ \omega > \dfrac{1}{\sqrt{LC}}$,

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{\frac{1}{\omega C} - \omega L}{R} = -\infty \quad \Rightarrow \quad \phi = -\pi/2$$

$\cos\phi = 0$, current lags emf. $P_{av} = E_v I_v \cos\phi = 0$

iii. $\tan\phi = \dfrac{X_C - X_L}{R} > 0 \quad \Rightarrow \quad \phi > 0$

$\cos\phi \ne 1$ also $P_{av} \ne 0$ because $\cos\phi \ne 0$

Current leads emf

iv. $\tan\phi = \frac{-X_L}{0} = -\infty \quad \Rightarrow \quad \phi = -\pi/2$

$$P_{av} = E_v I_v \cos\phi = 0$$

Numerical Value Type

1. **(8)** $V_A - IR + E - L\frac{dI}{dt} = V_B$

$\Rightarrow \quad V_A - 5\times 1 + 8 - 5\times 10^{-3}(-10^{-3}) = V_B$

$\Rightarrow \quad V_B - B_A = 8\text{ V}$

2. **(8)** Potential difference across the coil is $V = L\frac{di}{dt}$

or $\quad V = (1)(4) = 4\text{ V}$

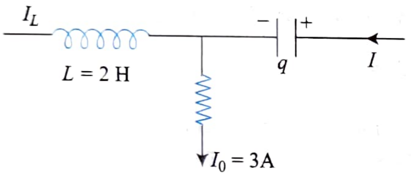
Now energy stored per unit time = power

$= Vi = (4)(2) = 8\text{ J/s}$

3. **(4)** $e = M\frac{dI}{dt} = M4t = 20t$

$$i = \frac{e}{R} = \frac{20t}{10} = 2t, \quad q = \int Idt = \int_0^2 2tdt = 4\text{ C}$$

4. **(4)** $q = CV,\ I = \frac{dq}{dt} = C\frac{dv}{dt} = V\frac{dC}{dt},\ I_L = I_0 - I$



$$\frac{dI_L}{dt} = \frac{dI_0}{dt} - \left[C\frac{d^2V}{dt^2} + \frac{dV}{dt}\frac{dC}{dt} + \frac{dV}{dt}\frac{dC}{dt} + V\frac{d^2C}{dt^2}\right]$$

$$= -1\times 10^{-3} - \left[6\times 10^{-3}\times\frac{1}{2} + 2(-0.5\times 10^{-3}) + 2(-0.5\times 10^{-3}) + 0\right]$$

$$= -2\times 10^{-3}\text{ A/s}$$

$$V_L = L\frac{dI_L}{dt} = 2\times 2\times 10^{-3} = 4\times 10^{-3}\text{ V} = 4\text{ mV}$$

5. **(1)** After charging, charge on capacitor = $C\varepsilon$

Now, at $t = 0$ two circuits are formed.

6. **(8)** The mutual inductance of solenoid coil system

$$M = \mu_0 n_1 N_2 A_2 = \mu_0 n_1 N_2 \pi r_2^2$$
$$= 4\pi\times 10^{-7}\times 2\times 10^4\times 100\times\pi\times(0.01)^2$$
$$= 8\pi^2\times 10^{-5}\text{ H}$$

EMF induced in the coil: $e_2 = -M\frac{\Delta i}{\Delta t}$

$$= -8\pi^2\times 10^{-5}\times\left(\frac{-2-2}{0.05}\right) = 640\,\pi^2\times 10^{-5}\text{ V}$$

Required charge:

$$q = i\Delta t = \frac{e}{R}\Delta t = \frac{640\times\pi^2\times 10^{-5}}{40\pi^2}\times 0.05 = 8\,\mu\text{C}$$

7. **(6)** From energy conservation: $\frac{1}{2}CV_0^2 = \frac{1}{2}CV^2 + \frac{1}{2}LI^2$

$$\Rightarrow \quad \frac{1}{2}\times 2\times 10^{-6}\times 12^2 = \frac{1}{2}\times 2\times 10^{-6}\times 6^2 + \frac{1}{2}\times 6\times 10^{-6}I^2$$

$\Rightarrow \quad I = 6\text{ A}$

8. **(5)** Induced EMF $= \frac{1}{2}B\omega\ell^2$

Maximum current: $i_0 = \frac{B\omega\ell^2}{2R}$

Torque about the hinge P is

$$\tau = \int_0^\ell i(dx)Bx \quad\Rightarrow\quad \tau = \frac{1}{2}iB\ell^2$$

Putting $i = i_0/2$, we get: $\tau = \frac{B^2\omega\ell^4}{8R} = 5\text{ mNm}$

9. **(1)** At $t = 0$, inductor behave as open circuit so $i = \frac{10}{10} = 1\text{ A}$

10. **(4)** $U = \frac{1}{2}Li^2$

Energy stored per unit time

$$\frac{dU}{dt} = \frac{1}{2}L2i\frac{di}{dt} = Li\frac{di}{dt} = 2\times 2\times 1 = 4\text{ J/s}$$

11. **(5)** Induced emf $= \frac{1}{2}B\omega l^2$

Maximum current: $i_0 = \frac{B\omega l^2}{2R}$

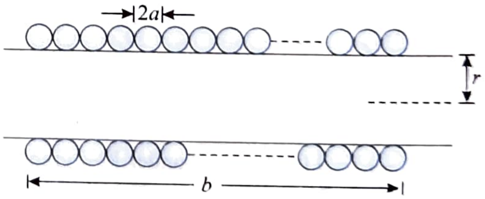
Torque about the hinge P is

$$\tau = \int_0^l i(dx)Bxd \quad\Rightarrow\quad \tau = \frac{1}{2}iBl^2$$

Put $i = i_0/2$, we get: $\tau = \frac{B^2\omega l^4}{8R} = 5\text{ mNm}$

12. **(8)** Length of wire $l = \frac{\text{volume }(V)}{\pi a^2}$

Winding is as shown in the figure.



Number of turns, $N = \frac{l}{2\pi r}$

Length of the helix, $b = 2a\cdot N = \frac{al}{\pi r}$

Number of turns per meter length $n = \frac{1}{2a}$

$\therefore$ Self-inductance, $L = \pi\mu_0 n^2 r^2 b$

$$= \pi\mu_0\left(\frac{1}{2a}\right)^2\cdot r^2\cdot\frac{al}{\pi r}$$

$$= \frac{1}{4}\mu_0 r\frac{l}{a} = \frac{1}{4}\mu_0 r\frac{V}{a(\pi a^2)}$$

$$L = \frac{\mu_0 r}{4}\frac{V}{a^3} \qquad \therefore\ L \propto \frac{1}{a^3}$$

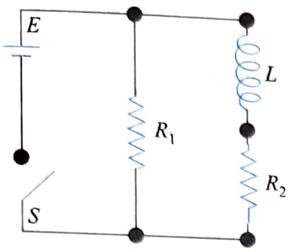
$$\therefore \quad \frac{L_1}{L_2} = \left(\frac{2a}{a}\right)^3 = \frac{8}{1}$$

Archives

JEE Main

Single Correct Answer Type

1. (4)



$$I_1 = \frac{12}{2} = 6\text{ A}$$

$$E = L\frac{dI_2}{dt} + R_2 \times I_2$$

$$I_2 = I_0\,(1 - e^{-t/tc})$$

$$\Rightarrow \quad I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\text{ A}$$

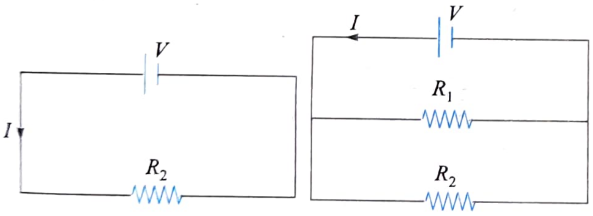
$$t_C = \frac{L}{R} = \frac{400 \times 10^{-3}}{2} = 0.2$$

$$I_2 = 6(1 - e^{-t/0.2})$$

Potential drop across L is

$$E - R_2 I_2 = 12 - 2 \times 6\,(1 - e^{-bt}) = 12e^{-5t}$$

2. **(2)** Just after closing the key (say at time t = 0) the inductor coil offers infinite resistance, hence it acts as an open circuit. The corresponding equivalent circuit diagram is as shown in Fig. (i).



(i) **(ii)**

The current through battery is $I = \dfrac{V}{R_2}$

At time $t = \infty$, the inductor acts as a conducting wire. The corresponding equivalent circuit diagram is as shown in Fig. (ii).

∴ The current through the battery is

$$I = \frac{V}{R_{eq}} = \frac{V}{\dfrac{R_1R_2}{R_1+R_2}} \quad (R_1 \text{ and } R_2 \text{ are in parallel})$$

$$= \frac{V(R_1+R_2)}{R_1R_2}$$

3. **(2)** Charge on the capacitor at any time 't',

$$q = q_0 \cos \omega t \qquad \text{...(i)}$$

At time when energy stored equally in electric and magnetic field, at this time

Energy of a capacitor $= \dfrac{1}{2}$ Total energy

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right) \Rightarrow q = \frac{q_0}{\sqrt{2}}$$

From equation (i)

$$\frac{q_0}{\sqrt{2}} = q_0 \cos \omega t$$

$$\cos \omega t = \frac{1}{\sqrt{2}} \Rightarrow \omega t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} \quad \left(\because \; \omega = \frac{1}{\sqrt{LC}}\right)$$

4. **(4)** Applying Kirchhoff's law in closed loop,

$$-V_R - V_C = 0$$

$$\Rightarrow \quad V_R/V_C = -1$$

[**Note:** The sense of voltage drop has not been defined.]
The answer could have been 1.

5. **(4)** $I = I_0 e^{-\frac{t}{\tau}}, \tau = \dfrac{L}{R} = \dfrac{15}{150} e^{-\frac{1 \times 10^{-3}}{1/5 \times 10^3}} = 0.67\text{ mA}$

6. **(1)** For a damped pendulum, $A = A_0 e^{-bt/2m}$

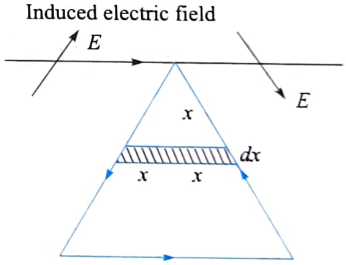
$$\Rightarrow \quad A = A_0 e^{-\left(\frac{R}{2L}\right)t}$$

(Since L plays the same role as m)

JEE Advanced

Multiple Correct Answers Type

1. **(2), (4)**

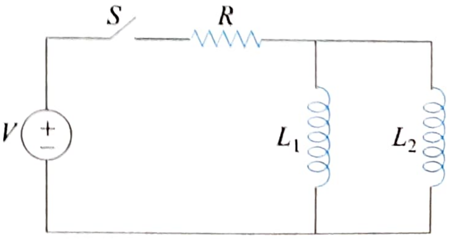


By direction of induced electric field, current in wire is in same direction of current along the hypotenuse. Flux through triangle if wire have current $i = \displaystyle\int_0^{0.1}\left(\frac{\mu_0 i}{2\pi x}\right)(2x\,dx) = \frac{\mu_0 i}{10\pi}$

$$\Rightarrow \text{Mutual inductance} = \frac{\mu_0}{10\pi}$$

$$\text{Induced emf in wire} = \frac{\mu_0}{10\pi}\frac{di}{dt} = \frac{\mu_0}{10\pi} \times 10 = \frac{\mu_0}{\pi}$$

2. **(1), (2), (3)**



Since inductors are connected in parallel

$$V_{L_1} = V_{L_2}$$

$$L_1\frac{dI_1}{dt}=L_2\frac{dI_2}{dt}$$

$$L_1I_1=L_2I_2$$

$$\frac{I_1}{I_2}=\frac{L_2}{L_1}$$

∴ Option (1) is correct.

Current through resistor at any time t is given by

$$I=\frac{V}{R}(1-e^{-\frac{RT}{L}})\text{ where } L=\frac{L_1L_2}{L_1+L_2}$$

After long time $I=\frac{V}{R}$

$$I_1+I_2=I \qquad \text{...(i)}$$

$$L_1I_1=L_2I_2 \qquad \text{...(ii)}$$

From (i) and (ii), we get

$$I_1=\frac{V}{R}\frac{L_2}{L_1+L_2},\ I_2=\frac{V}{R}\frac{L_1}{L_1+L_2}$$

∴ Options (2) and (3) are correct.

Value of current is zero at $t=0$

Value of current is V/R at $t=\infty$

Hence option (4) is incorrect.

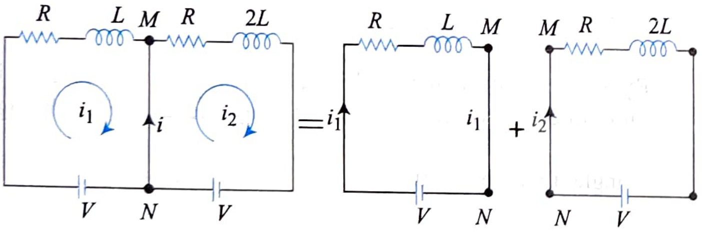
3. (2), (4)

Hence current in wire MN; $i=i_1-i_2$

or $i_{\max}=(i_1-i_2)_{\max}$

$$i=(i_1-i_2)=\frac{V}{R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right]-\frac{V}{R}\left[1-e^{-\left(\frac{R}{2L}\right)t}\right]$$

$$=\frac{V}{R}\left[e^{\left(-\frac{R}{2L}\right)t}-e^{\left(-\frac{R}{L}\right)t}\right]$$



For current to be maximum, $\frac{di}{dt}=0$

$$\frac{V}{R}\left[-\frac{R}{2L}e^{-\left(\frac{R}{L}\right)t}-\left(-\frac{R}{L}\right)e^{-\left(\frac{R}{L}\right)t}\right]=0$$

or $$e^{-\left(\frac{R}{L}\right)t}=\frac{1}{2}e^{-\left(\frac{R}{2L}\right)t}$$

$$e^{-\left(\frac{R}{2L}\right)t}=\frac{1}{2}\Rightarrow\quad\left(\frac{R}{2L}\right)t=\ln 2$$

or $$t=\frac{2L}{R}\ln 2\rightarrow\text{It is the time when } i \text{ is maximum}$$

$$i_{\max}=\frac{V}{R}\left[e^{-\frac{R}{2L}\left(\frac{2L}{R}\ln 2\right)}-e^{-\left(\frac{R}{L}\right)\left(\frac{2L}{R}\ln 2\right)}\right]=\frac{V}{R}\left[e^{-\ln 2}-e^{-\ln 4}\right]$$

$$|i_{\max}|=\frac{V}{R}\left[\frac{1}{2}-\frac{1}{4}\right]=\frac{V}{4R}$$

Numerical Value Type

1. (6) Flux through circular ring

$$\phi=\left(\frac{\mu_0 I}{L}\right)\pi r^2 \quad\Rightarrow\quad \phi=\frac{\mu_0}{L}\pi r^2 I_0\cos 300t$$

Current in the ring $i=\frac{1}{R}\frac{d\phi}{dt}$

$$i=\frac{\mu_0\pi r^2 I_0}{RL}\cdot\sin 300t\times 300$$

$$=\mu_0 I_0\sin 300t\left[\frac{\pi r^2\cdot 300}{RL}\right]$$

$$M=I\cdot\pi r^2=\mu_0 I_0\sin 300t\left[\frac{\pi^2 r^4\cdot 300}{RL}\right]$$

Taking $\pi^2=10$, we get

$$M=6\mu_0 I_0\sin 300t$$

thus $N=6$

2. (7) $B=\frac{\mu_0 iR^2}{2(R^2\times X^2)^{3/2}}$

$$B=\frac{\mu_0 iR^2}{2(R^2\times 3R^2)^{3/2}}=\frac{\mu_0 iR^2}{2(4R^2)^{3/2}}=\frac{\mu_0 iR^2}{2\cdot 2^3 R}=\frac{\mu_0 i}{16R}$$

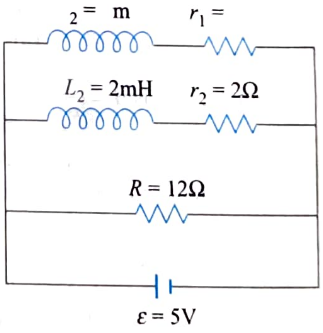
$$\phi=2\frac{\mu_0 i}{16R}a^2\frac{1}{\sqrt{2}}=\frac{\mu_0 ia^2}{8\sqrt{2}R}$$

$$M=\frac{\phi}{i}$$

$$M=\frac{\mu_0 a^2}{2^{7/2}R}=\frac{\mu_0 a^2}{2^{p/2}R}$$

$$p=7$$

3. (8)



$$I_{\max}=\frac{\varepsilon}{R}=\frac{5}{12}A\text{ (Initially at } t=0)$$

$$I_{\min}=\frac{\varepsilon}{R_{eq}}=\varepsilon\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{R}\right)\text{ (finally in steady state)}$$

$$=5\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{12}\right)=\frac{10}{3}A$$

$$\frac{I_{\max}}{I_{\min}}=8$$

Chapter 6

Concept Application Exercises

Exercise 6.1

1. Yes, because here also (as when 50 Hz), fluctuation in current will be so rapid [30 times/s] that the bulb will appear glowing continuously due to persistence (lesser than 30/s) of vision.
2. The brightness will decrease, because L increases on inserting the iron rod and impedence $Z = \sqrt{R^2 + (\omega L)^2}$ increases. This decreases the current, hence brightness decreases.
3. The brightness will decrease, because Z increases on increasing w.
4. First decreases, then becomes same. Because as the rod is inserted, L increases. From $f = LI$, I decreases. But soon the current will become same as original.
5. $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$ decreases on increasing w. This increases current, hence the brightness will increase.
6. $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$ increases on decreasing C. This decreases current, hence the brightness will decrease.
7. True, $Z = \sqrt{R^2 + (\omega L)^2} > R$
8. If resistance by dc is R, then by ac impedance:

   $$Z = \sqrt{R^2 + (\omega L)^2} > R$$

   Current will be lesser in case of ac. Hence, heat produced in case of ac will be less.
9. False, True

   In series current at any time is same in entire circuit. Hence, current is in phase in both L and C. In resistor, voltage is in same phase as current but in inductor, voltage leads current by $\pi/2$. So inductor voltage leads resistance voltage by $\pi/2$.
10. False, if frequency of the source is more than resonance frequency then $X_L > X_C$. The circuit becomes inductive. In inductive circuit current lags voltage.
11. True, voltage across inductor is given by

    $$V_{Lv} = I_v X_L = \frac{E_v X_L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

    For certain values of R, X_L, and X_C, X_L can be greater than $Z = \sqrt{R^2 + (X_L - X_C)^2}$. In that case V_{Lv} can be greater than E_v
12. $P = E_v I_v \cos\phi$. If $\cos\phi$ is small, I_v is large for same P.

    So loss $= I_v^2 R$ is large.
13. True
14. 1
15. No

Exercises

Single Correct Answer Type

1. (2) $E_v = \sqrt{120^2 + 160^2} = 200$ V
2. (1) The equivalent primary load is

   $$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2 = \left(\frac{20}{1}\right)^2 (6.0) = 2400\ \Omega$$

   Current in the primary coil

   $$= \frac{240}{R_1} = \frac{240}{2400} = 0.1\ \text{A}$$
3. **(1)** Here $V_L = V_C$. They are in opposite phase. Hence, they will cancel each other. Now the resultant potential difference is equal to the applied potential difference = 100 V

   $$Z = R \qquad (\because\ X_L = X_C)$$

   $$\therefore\quad I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{100}{50} = 2\ \text{A}$$
4. **(3)** According to the given question,

   $$\tan 60° = \frac{\omega L}{R} \text{ and } \tan 60° = \frac{1/\omega C}{R}$$

   $$\therefore\quad \omega L = (1/\omega C)(\text{case of resonance})$$

   $$\text{Now } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 100\ \Omega$$

   $$\therefore\quad I_{rms} = \frac{E_{rms}}{Z} = \frac{200\ \text{V}}{100\ \Omega} = 2\ \text{A}$$
5. **(2)** Given that $E_0 = 10$ V, $t = \frac{1}{600}$ s

   $$\therefore\quad E = E_0 \cos 2\pi f t$$

   $$= 10 \cos\left[2\pi \times 50 \times \frac{1}{600}\right]$$

   $$= 10 \cos(\pi/6) = 10(\sqrt{3}/2) = 5\sqrt{3}\ \text{V}$$
6. **(2)** Wattless component of ac

   $$= I_v \sin\phi = \frac{E_v}{Z}\frac{X_L}{Z} = \frac{E_v X_L}{Z^2} = \frac{220 \times \omega L}{(R^2 + \omega^2 L^2)}$$

   As $\omega L = 0.7 \times 2\pi \times 50 = 220\ \Omega$

   Hence, wattless component of ac

   $$= \frac{200 \times (220)}{(220^2 + 220^2)} = 0.5\ \text{A}$$
7. **(3)** Clearly, $X_L = R$

   or $L \times 2 \times 3.14 \times 1000 = 100$

   or $L = \frac{100}{2 \times 3.14 \times 1000}$ H

   $$= 15.9 \times 10^{-3}\ \text{H} = 15.9\ \text{mH} \approx 16\ \text{mH}$$
8. **(1)** Current remains unchanged in R. However, it becomes half in L, because reactance is doubled on doubling the frequency.
9. **(3)** The current of 1.6 A lags emf in phase by $\pi/2$. The current of 0.4 A leads emf in phase by $\pi/2$. So, these two currents are 180° out of phase with each other.

   $\therefore$ Net current, $I_1 = (1.6 - 0.4)$ A $= 1.2$ A

   $$1.6 = \frac{E_v}{X_L} \text{ and } 0.4 = \frac{E_v}{X_C}$$

   $$\Rightarrow \frac{X_C}{X_L} = 4 \Rightarrow \frac{1}{\omega C \omega L} = 4$$

   $$\Rightarrow \omega = \frac{1}{2\sqrt{LC}}$$

   $$\Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{4\pi\sqrt{LC}}$$

10. **(2)** $X_C = \dfrac{1}{2\pi fC} = \dfrac{10^6}{2\pi 50 \times 500} = \dfrac{20}{11}\,\Omega$

$$X_L = 2\pi fL = 2\pi \times 50 \times 10 \times 10^{-3} = \pi\Omega$$

Since $X_L < X_C$, so inductive branch has less impedance, and so more current. Hence B_2 will be brighter.

11. **(1)** Since the capacitor is connected in series to the resistor, current I_C from the supply and I_R through the resistor is in phase as represented by choice (1).

12. **(2)** Root mean square current of the sinusoidal waveform, $I = \dfrac{I_0}{\sqrt{2}}$.

Power output of the heater,

$$P = I^2R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{I_0^2R}{2}$$

13. **(4)** Both V and I are in the same phase. So, let us calculate the time taken by the voltage to change from peak value to rms value.

Now $220 = 220 \sin 100\pi t_1$

or $100\pi t_1 = \dfrac{\pi}{2}$ or $t_1 = \dfrac{1}{200}$ s

Again, $\dfrac{220}{\sqrt{2}} = 220 \sin 100\pi t_2$

or $\dfrac{1}{\sqrt{2}} = \sin 100\pi t_2$ or $100\pi t_2 = \dfrac{3\pi}{4}$

or $t_2 = \dfrac{3}{400}$ s

Required time $= t_2 - t_1 = \dfrac{1}{400}$ s $= 2.5 \times 10^{-3}$ s

14. **(3)** Quality factor

$$= \frac{f_0}{f_2 - f_1} = \frac{600}{650 - 550} = \frac{600}{100} = 6$$

15. **(1)** $E = E_0 \cos \omega t$

$\therefore \quad \omega = 50\,\pi$

$2\,\pi f = 50\,\pi \Rightarrow f = 25$ Hz

In one cycle ac current becomes zero twice.

Therefore, 50 times the current becomes zero in 1 s.

16. **(3)** $\lambda = 300$ m, $c = 3 \times 10^8$ m s^{-1}

Frequency $f = \dfrac{c}{\lambda} = \dfrac{3 \times 10^8}{300} = 10^6$ Hz

Resonance frequency

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad L = \frac{1}{C4\pi^2 f^2}$$

$$\therefore \quad L = \frac{1}{4\pi^2(10^6)^2 \times 2.4 \times 10^{-6}} \equiv 10^{-8}\,\text{H}$$

17. **(1)** Change on the capacitor,

$$q_0 = CV = 1 \times 10^{-6} \times 1 = 10^{-6}\,\text{C}$$

or $I_0 = \omega q_0 =$ Maximum current

Now $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{10^{-9}}} = (10^9)^{1/2}$

$\therefore \quad I_0 = (10^9)^{1/2} \times (1 \times 10^{-6}) = \sqrt{10^{-3}}$ A $= \sqrt{1000}$ mA

18. **(2)** $E = E_0 \sin \omega t$

Voltage read is rms value.

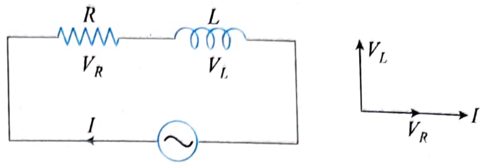
$\therefore \quad E_0 = \sqrt{2} \times 234\text{ V} = 331\text{ V}$

and $\quad \omega t = 2\pi ft = 2\pi \times 50 \times t = 100\pi t$

Thus, the equation of the line voltage is given by

$$E = 331 \sin(100\pi t)$$

19. **(2)** Here inductance and resistance are connected in series. We know that in case of resistance, both current and potential difference are in the same phase. In inductor, voltage leads current by $\pi/2$.



20. **(2)** Here $R = X_L = X_C$ $\quad$ ($\because$ voltage across them is same)

When capacitor is short circuited,

$$I = \frac{10}{(R^2 + X_L^2)^{1/2}} = \frac{10}{\sqrt{2}R}$$

$\therefore$ Potential drop across inductance $= IX_L = IR = 10/\sqrt{2}$ V

21. **(1)** $R = \dfrac{125}{12.5} = 10\,\Omega$

$$X_L = \omega L = 2\pi fL = \frac{V}{I} = \frac{125}{10} = 12.5$$

$\therefore \quad X_L' = 2\pi L \times f' = 0.25 \times 40 = 10\,\Omega$

Impedance of the circuit

$$Z = \sqrt{R^2 + X_L^2} = 10\sqrt{2}\,\Omega$$

$\therefore \quad$ Current $= \dfrac{100}{10\sqrt{2}} = 10/\sqrt{2}$ A

22. **(2)** Given $I = 5 + 10 \sin \omega t$

$$I_{\text{eff}} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (5 + 10 \sin \omega t)^2\, dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T (25 + 100 \sin \omega t + 100 \sin^2 \omega t)\right]^{1/2}$$

But as $\dfrac{1}{T}\displaystyle\int_0^T \sin \omega t\, dt = 0$

and $\dfrac{1}{T}\displaystyle\int_0^T \sin^2 \omega t\, dt = \dfrac{1}{2}$

so $\quad I_{\text{eff}} = \left[25 + \dfrac{1}{2} \times 100\right]^{1/2} = 5\sqrt{3}$ A

23. **(1)** $X_L = L\omega = 5 \times 10^{-3} \times 2000 = 10\,\Omega$

$$X_C = \frac{1}{C\omega} = \frac{1}{50 \times 10^{-6} \times 2000} = \frac{100}{10}\,\Omega = 10\,\Omega$$

Since $X_L = X_C$, therefore,

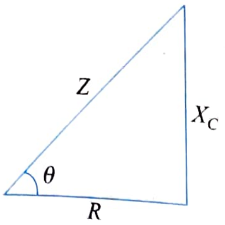
$Z = R = 5.9 + 0.10 + 4 = 10\,\Omega$

$$I_v = \frac{E_v}{Z} = \frac{20}{10}\,\text{A} = 2\,\text{A}$$

24. **(1)** $P = E_v I_v \cos \phi$; $P = E_v \dfrac{E_v}{Z}\dfrac{R}{Z}$

or $\quad P = \dfrac{E_v^2 R}{Z^2} = \dfrac{110 \times 110 \times 11}{22 \times 22} = 275$ W

25. (2) $V_C^2 + V_R^2 = V^2$



$$50^2 + V_R^2 = 110^2 \Rightarrow V_R = \sqrt{160 \times 60} = 98 \text{ V}$$

Then, $I_V = \dfrac{\sqrt{160 \times 60}}{50}$ $(\because \ R = 50\ \Omega)$

Also, $I_V = \dfrac{110}{\sqrt{R^2 + X_C^2}}$ $\therefore$ $\dfrac{98}{50} = \dfrac{110}{\sqrt{50^2 + X_C^2}}$

Flux X_C and now using $X_C = \dfrac{1}{\omega C}$, we get $C = 104\ \mu\text{F}$

26. (1) $q_0 = CE_0 = 8 \times 10^{-6} \times 220\sqrt{2} = 2.5 \times 10^{-3}$ C

27. (1) Resultant current is superposition of two currents, i.e., I (instantaneous total current) $= 6 + I_0 \sin \omega t$

dc ammeter will read average value

$$= \overline{6 + I_0 \sin\omega t} = 6 \qquad (\because \ \overline{I_0 \sin\omega t} = 0)$$

ac ammeter will read

$$= \sqrt{\overline{(6 + I_0 \sin\omega t)^2}}$$

$$= \sqrt{36 + 12 I_0 \overline{\sin\omega t} + I_0^2 \overline{\sin^2\omega t}} \qquad (\because \ \overline{I_0 \sin\omega t} = 0)$$

Since $\overline{\sin^2\omega t} = \dfrac{1}{2}$ and $I_{\text{rms}} = 8 = \dfrac{I_0}{\sqrt{2}} \Rightarrow I_0 = 8\sqrt{2}$ A

Therefore, ac reading

$$= \sqrt{36 + \frac{I_0^2}{2}} = \sqrt{36 + 64} = 10 \text{ A}$$

28. (3) $I = I_1 \cos\omega t + I_2 \sin\omega t$

$$(I^2)_{\text{mean}} = I_1^2 \overline{\cos^2\omega t} + I_2^2 \overline{\sin^2\omega t} + 2I_1 I_2 \overline{\cos\omega t \sin\omega t}$$

$$= I_1^2 \times \frac{1}{2} + I_2^2 \times \frac{1}{2} + 2I_1 I_2 \times 0$$

$$I_{\text{rms}} = \sqrt{(I^2)_{\text{mean}}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

29. (2) Resistance of bulb: $R = \dfrac{V_0^2}{P_0} = \dfrac{(220)^2}{1100} = 44\ \Omega$

when L is maximum, power consumed will be minimum which is $P = \dfrac{1100}{5} = 220$ W

Now $P = I_v^2 R \Rightarrow 220 = I_v^2 \times 44 \Rightarrow I_v = \sqrt{5}$ A

$$I_v = \frac{E_v}{Z} \Rightarrow \sqrt{5} = \frac{220}{\sqrt{44^2 + X_L^2}} \Rightarrow X_L = 88\ \Omega$$

$$\Rightarrow \ 2\pi f L_{\text{max}} = 88 \Rightarrow 2 \times \frac{22}{7} \times 50\, L_{\text{max}} = 88$$

$$\Rightarrow \ L_{\text{max}} = 0.28 \text{ H}$$

30. (3) $E_1 = E_0 \sin\omega t$; $E_2 = E_0 \sin[\omega t + (\pi/3)]$

$$E = E_2 + E_1$$

$$= E_0 \sin[\omega t + (\pi/3)] + E_0 \sin\omega t$$

$$= E_0\, [2 \sin\{\omega t + (\pi/6)\} \cos(\pi/6)]$$

$$= \sqrt{3}\, E_0 \sin[\omega t + (\pi/6)]$$

31. (4) If an ac source $E = E_0 \sin\omega t$ is applied across an inductance and capacitance in parallel, the current in inductance will lag the applied voltage while that across the capacitor will lead, and so,

$$I_L = \frac{E_0}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) = -0.8\sqrt{2}\cos\omega t$$

$$I_C = \frac{V}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) = 0.6\sqrt{2}\cos\omega t$$

So the current drawn from the source

$$I = I_L + I_C = -0.2\sqrt{2}\ \cos\omega t$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{0.2\sqrt{2}}{\sqrt{2}} = 0.2 \text{ A}$$

32. (4) As current at any instant in the circuit will be

$$I = I_{dc} + I_{ac} = a + b \sin\omega t$$

so, $$I_{\text{eff}} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (a + b\sin\omega t)^2 dt\right]^{1/2}$$

i.e., $$I_{\text{eff}} = \left[\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2 \sin^2\omega t)\, dt\right]^{1/2}$$

but as $\dfrac{1}{T}\displaystyle\int_0^T \sin\omega t\, dt = 0$ and $\dfrac{1}{T}\displaystyle\int_0^T \sin^2\omega t\, dt = \dfrac{1}{2}$

So, $$I_{\text{eff}} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$$

33. (3) The equation of a semi-circular wave is

$$x^2 + y^2 = a^2 \quad \text{or} \quad y^2 = a^2 - x^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{2a}\int_{-a}^{+a} y^2 dx}$$

$$I^2_{\text{rms}} = \frac{1}{2a}\int_{-a}^{+a} (a^2 - x^2)\, dx$$

$$= \frac{1}{2a}\int_{-a}^{+a} (a^2 - x^2)\, dx = \frac{1}{2a}\left|a^2 x - \frac{x^3}{3}\right|_{-a}^{+a}$$

$$= \frac{1}{2a}\left(a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3}\right) = \frac{2a^2}{3}$$

$$I_{\text{rms}} = \sqrt{\frac{2a^2}{3}} = \sqrt{\frac{2}{3}}\, a$$

34. (2) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{100 \times 10^{-6}} = 10^4\ \Omega$

So, $I_{\text{rms}} = \dfrac{E_{\text{rms}}}{Z} = \dfrac{50\sqrt{2}}{\sqrt{2} \times 10^4} = 5$ mA

35. (2) At resonance, the series combination of L and C gives zero impedance.

At resonance, the voltages across L and C are equal but opposite in phase.

36. (2) As resistance of the lamp

$$R = \frac{V_S^2}{P_0} = \frac{100^2}{50} = 200\ \Omega$$

The rms current $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}$ A.

So when the lamp is put in series with a capacitance and run at 200 V ac, from $V = IZ$, we have

$$Z = \frac{V}{I} = \frac{200}{(1/2)} = 400\ \Omega$$

Now as in case of C–R circuit,

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

i.e., $R^2 + \left(\frac{1}{\omega C}\right)^2 = 160000$

or, $\left(\frac{1}{\omega C}\right)^2 = 16\times 10^4 - (200)^2 = 12\times 10^4$

$$\frac{1}{\omega C} = \sqrt{12}\times 10^2$$

$$C = \frac{1}{100\pi\times\sqrt{12}\times 10^2}\ \text{F}$$

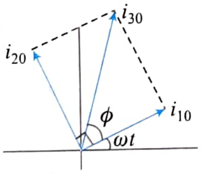
$$= \frac{100}{\pi\sqrt{12}}\ \mu\text{F} = \frac{50}{\pi\sqrt{3}} = 9.2\ \mu\text{F}$$

37. (2) Here, $X_L = \omega L = 2\pi f L = 2\pi\times 50\times 1 = 100\,\pi\,\Omega$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi\times 50\times 10\times 10^{-6}} = \frac{10^3}{\pi}\ \Omega$$

So, $X = |X_L - X_C| = \left|100\pi - \frac{10^3}{\pi}\right| = \left|10^2\left[\frac{\pi^2-10}{\pi}\right]\right|\ \Omega$

38. (1) $i_3 = i_1 + i_2 = 3\sin\omega t + 4\cos\omega t$
$= 3\sin\omega t + 4\sin(\omega t + 90°)$



$$i_{30} = \sqrt{i_{10}^2 + i_{20}^2} = \sqrt{3^2+4^2} = 5$$

$$\tan\phi = \frac{4}{3} \Rightarrow \phi = 53°$$

So $\quad i_3 = i_{30}\sin(\omega t + \phi) = 5\sin(\omega t + 53°)$

39. (2) $V_{rms} = \sqrt{16^2 + 20^2} = 25.6$ V

40. (3) $i = 3\sin\omega t + 4\cos\omega t$

$$= 5\left[\frac{3}{5}\sin\omega t + \frac{4}{5}\cos\omega t\right] = 5[\sin(\omega t + \delta)] \qquad \text{...(i)}$$

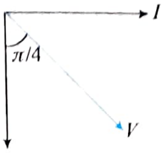
$$\text{rms value} = \frac{5}{\sqrt{2}}\ \text{A}$$

If voltage applied is $V = V_m \sin\omega t$ then I, given by Eq. (i), indicates that it is ahead of V by δ where $0 < \delta < 90$ which indicates that the circuit contains R and C.

If we find average value for the first half period (0 to $T/2$), then it will be $6/\pi$ A. But for different time interval, it will be different.

41. (1) $v = v_0\sin[\omega t + (\pi/4)] = v_0\cos[\omega t - (\pi/4)]$

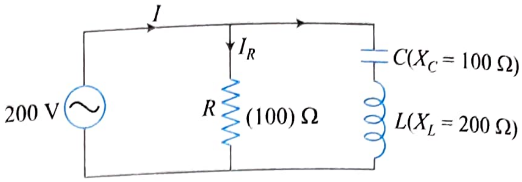
Since V lags current, an inductor can bring it in phase with current.

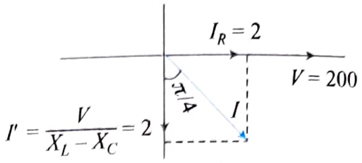


42. (2) $I_R = \frac{V}{R} = \frac{200}{100} = 2$ A

$$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2\ \text{A}$$

$$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2}\ \text{A}$$





43. (3) The circuit will have inductive nature if

$$\omega > \frac{1}{\sqrt{LC}}\left(\omega L > \frac{1}{\sqrt{LC}}\right)$$

Hence (1) is false. Also, if circuit has inductive nature, the current will lag behind voltage. Hence, (4) is also false.

If $\omega = \frac{1}{\sqrt{LC}}\left(\omega L = \frac{1}{\omega C}\right)$, the circuit will have resistance nature. Hence, (2) is false.

Power factor, $\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = 1$

If $\omega L = \frac{1}{\omega C}$. Hence, (3) is true.

44. (3) $X_L = X_C$ at resonance

$\frac{X_L}{X_C} = 1$ for both circuits.

Impedance may be different if applied voltage is different.

45. (4) Since $\cos\phi = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$

(Also cos ϕ can never be greater than 1)

Hence, (c) is wrong.

Also $IX_C > IX_L \Rightarrow X_C > X_L$

$\therefore$ Current will be leading.

In an LCR circuit,

$$V = \sqrt{(v_L - v_C)^2 + v_R^2} = \sqrt{(6-12)^2 + 8^2}$$

$V = 10$; which is less than voltage drop across capacitor.

46. (3) Since current is leading emf, capacitor must be present in the circuit. Along with capacitor, there can be either (i) only resistor or (ii) resistor and inductor if $X_C > X_L$.

In both of the above conditions, we can obtain a phase difference of $\pi/4$ with current leading emf.

47. (1) $V = \frac{V_0}{T/4}t;\ V = \frac{4V_0}{T}t$

$$V_{rms} = \sqrt{<V^2>} = \frac{4V_0}{T}\sqrt{<t>}$$

$$= \frac{4V_0}{T}\left\{\frac{\int_0^{T/4} t^2\,dt}{\int_0^{T/4} dt}\right\}^{1/2} = \frac{V_0}{\sqrt{3}}$$

(4) $$\langle V \rangle = \frac{\int_0^T V dt}{\int_0^T dt} = 0$$

(2) $$(0.5)^2 R\left(\frac{T}{2}\right) + (0.5)^2 R\left(\frac{T}{2}\right) = I^2_{\text{rms}} RT$$

or $$I_{\text{rms}} = \frac{1}{2}\text{ A} = 0.5\text{ A}$$

(3) $$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

Putting the values, $C = 500\ \mu\text{F}$

(1) Given:

Voltage in primary coil, $V_p = 200$ V
Current in primary coil, $i_p = 2$ A
Voltage in secondary coil, $V_s = 2000$ V
The relation for the current in the secondary coil is

$$\frac{V_s}{V_p} = \frac{i_p}{i_s} \Rightarrow \frac{2000}{200} = \frac{2}{i_s} = \frac{2 \times 200}{2000} = 0.2\text{ A}$$

(3) $P_{av} = E_v I_v \cos\phi = 100 \times 10 \cos\phi = 1000 \cos\phi$ Watt
So depending upon value of ϕ, P_{av} can be equal to or less than 1000 W.

(1) $E = 120 \sin 100\,\pi t \cos 100\,\pi t = 60 \sin(200\,\pi t)$
So peak voltage $E_0 = 60$ V

(4) Both B and I are is same phase. So, let us calculate the time taken by the voltage to change from peak value of rms value.
Now, $220 = 220 \sin 100\,\pi t_1$

or $$100\pi t_1 = \frac{\pi}{2} \text{ or } t_1 = \frac{1}{200}\ s$$

Again, $$\frac{200}{\sqrt{2}} = 200 \sin 100\pi t_2$$

or $$\frac{1}{\sqrt{2}} = \sin 100\,\pi t_2 \text{ or } 100\,\pi t_2 = \frac{\pi}{4}$$

or $$t_2 = \frac{1}{400}\text{ s}$$

Required time $= t_1 - t_2$.

$$= \frac{1}{200} - \frac{1}{400} = \frac{2-1}{400} = \frac{1}{400}\text{ s} = 2.5 \times 10^{-3}\text{ s}$$

55. (1) $$I^2_{\text{rms}} T = \int_0^T (3 + 4\sin\omega t)^2\, dt$$

56. (2) $$\tan\theta = \frac{X_L}{R} = 1 \quad \therefore \quad \theta = 45^\circ$$

$$t = \theta \times \frac{\text{Time period}}{360}$$

57. (3) For LR circuit, $XL = \omega L$.

$$\tan\phi = \frac{\omega L}{R} = \frac{X_L}{R}$$

For $\phi = 45^\circ$, $\tan 45^\circ = \frac{X_L}{R}$ or $X_L = R$

58. **(3)** $H(DC) = I^2Rt = 2^2Rt$ joule

$$H(AC) = I^2Rt = \left(\frac{I_0}{\sqrt{2}}\right)^2 Rt = \frac{2^2}{2} Rt$$

$$\therefore \quad = \frac{H(DC)}{H(AC)} = \frac{2}{1}$$

59. **(3)** Voltage across L and C cancel out.
So, voltage across k is 220 V.

Again, $$I_0 = \frac{220}{100}\text{ A} = 2.2\text{ A}$$

60. **(2)** Let I_1 and I_2 be currents through B_1 and B_2 then

$$I_1 \times \sqrt{R^2 \times X_L^2} = 220$$

$$\frac{I_2}{I_1} = \frac{\sqrt{R^2 \times X_C^2}}{\sqrt{R^2 + X_L^2}} = \frac{\sqrt{R^2 + \left(\frac{1}{c\omega}\right)^2}}{\sqrt{R^2 + L^2\omega^2}}$$

$$= \frac{\sqrt{R^2 + \left(\frac{1}{500 \times 10^{-6} \times 2\pi \times 50}\right)^2}}{\sqrt{R^2 + (10 \times 10^{-3} \times 2\pi \times 50)^2}}$$

$$= \sqrt{R^2 + 40}/\sqrt{R^2 + 9.87},\ I_2 > I_1$$

Bulb B_2 will be brighter. As frequency and increases, X_C decreases X_L increases. I_2 becomes less and I_1 increases.
$\therefore$ brightness of B_1 will increase and that of B_2 decreases.

Multiple Correct Answers Type

1. **(1), (3)**

$$f_{\text{res}} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi} \times \frac{1}{\pi} \times 10^{-6}}} = 500\text{ Hz}$$

At resonance, $Z = R$,

So current $$I = \frac{V}{Z} = \frac{V}{R}$$

When L and C are in series, voltage across capacitor and inductor is 180° out of phase.

2. **(1), (2)**

$(f = 900\text{ Hz}) < (f_{\text{res}} = 1000\text{ Hz})$, so $X_C > X_L$, circuit becomes capacitive and current leads emf.

At resistance, impedance is minimum.

Not only at resonance, but at all conditions voltage across L and C differs in phase by 180° when connected in series.

3. **(1), (2), (3)**

$H = I^2Rt/4.2$ (Heat produced in a conductor is independent of the nature of current. If the current flowing in a conductor changes its direction, then also heat being produced in it will not change). Hence, (1), (2), and (3) are true.

4. **(1), (2), (3)**

The frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.6 \times 10 \times 10^{-6}}} = 65\text{ Hz}$$

The current at resonance is

$$I_0 = \frac{200\text{ V}}{5\ \Omega} = 40\text{ A}$$

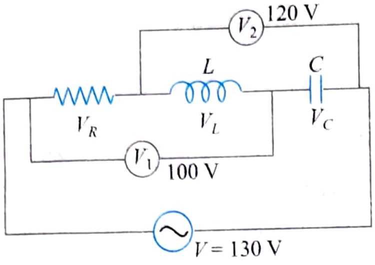
The quantity $V_0 = I_0\sqrt{R^2 + (\omega L)^2}$

$$= 40\sqrt{25^2 + (0.6\times 2\pi\times 65)^2} = 9.8\text{ kV}$$

$$V_1 = \frac{I_0}{\omega C} = \frac{40\times 10^5}{2\pi\times 65} = 9.8\text{ kV}$$

5. (1), (3), (4)

$V_R^2 + V_L^2 = 100^2, |V_C - V_L| = 120$



$$130^2 = V_R^2 + (V_C - V_L)^2 \Rightarrow V_R = 50\text{ V}$$

$$\Rightarrow \quad V_L = \sqrt{100^2 - 50^2} = 50\sqrt{3}\text{ V}$$

$$V_C = 120 + 50\sqrt{3}\text{ V}$$

$$\cos\phi = \frac{V_R}{V} = \frac{50}{130} = \frac{5}{13}$$

Since $V_C < V_L$, so circuit is capacitive.

6. (1), (3)

$$\text{average current} = \frac{\text{total charge}}{\text{time}}$$

Total charge will be equal to area of I–t graph, which is zero for $T/2$ to $3T/2$, hence average current will be zero for this time interval.

for $T/2$ to $3T/2$:

$$I = \frac{2I_0 t}{T} - 2I_0$$

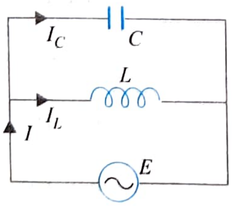
$$I_{\text{rms}} = \sqrt{\frac{\int_{T/2}^{3T/2} I^2 dt}{3T/2 - T/2}} = I_0/\sqrt{3}$$

7. (2), (3)

$I = I_C + I_L$

$$\Rightarrow \quad I \;= I = \frac{E_0}{X_C}\sin(\omega t + \pi/2) + \frac{E_0}{X_L}\sin(\omega t - \pi/2)$$

$$= \left[\frac{E_0}{X_C} - \frac{E_0}{X_L}\right]\cos\omega t$$



To find: $I_v = \left|\frac{1}{\sqrt{2}}\left[\frac{E_0}{X_C} - \frac{E_0}{X_L}\right]\right|$ (i)

Given: $I_{Cv} = \frac{E_0}{\sqrt{2}X_C} = 0.4\text{ A}$ (ii)

$I_{Lv} = \frac{E_0}{\sqrt{2}X_L} = 1.6\text{ A}$ (iii)

From equations (i), (ii) and (iii) $I_v = 1.2$ A

Dividing (ii) by (iii)

$$\frac{X_L}{X_C} = \frac{1}{4} \Rightarrow \frac{\omega L}{1/\omega C} = \frac{1}{4} \Rightarrow \omega^2 LC = \frac{1}{4} \Rightarrow \omega = \frac{1}{2\sqrt{LC}}$$

8. (1), (3), (4)

$V_1 = V_2 \Rightarrow XL = XC$

$$\Rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}} = 125\text{ Hz}$$

$$I_{\text{rms}} = \frac{V}{R} = \frac{200}{100} = 2\text{ A} \qquad (\because\; Z = R)$$

$$V_1 = V_2 = IXL = I.(\omega L) = 1000\text{ volt}$$

Linked Comprehension Type

For Problems 1–3

1. (1) **2. (2)** **3. (1)**

1. (1) The peak value, $V_0 = 220\sqrt{2} = 311$ V

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{220\sqrt{2}}{\sqrt{2}} = 200\text{ V}$$

2. (2) $V_{\text{av}} = \frac{2}{\pi}V_0 = \frac{2}{\pi}\times 311 = \frac{622}{\pi}$ V

3. (1) As $\omega = 2\pi f$, $2\pi f = 314$, i.e., $f = \frac{314}{2\times\pi} = 50$ Hz

For Problems 4–5

4. (4) **5. (1)**

4. (4) Equation of line AB is given by $y = 10 + \left(\frac{10}{T}\right)t$

$$y_{\text{av}} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T\left(10 + \frac{10}{T}t\right)dt = \frac{1}{T}\left|10t + \frac{5t^2}{T}\right|_0^T = 15$$

5. (1) Mean square value

$$= \frac{1}{T}\int_0^T y^2 dt = \int_0^T\left(10 + \frac{10}{T}t\right)^2 dt$$

$$= \frac{1}{T}\int_0^T\left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right)$$

$$= \frac{1}{T}\left|100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right|_0^T = \frac{700}{3}$$

rms value $= 10\sqrt{7/3}$

For Problems 6–7

6. (3) **7. (1)**

6. (3) When dc is applied, $I = \frac{V}{R}$, i.e., $R = \frac{12}{4} = 3\,\Omega$

and when ac is applied, $Z = \left(\frac{V}{I}\right) = \left(\frac{12}{2.4}\right) = 5\,\Omega$

$$R^2 + X_L^2 = 5^2 \quad (\text{as } Z = \sqrt{R^2 + X_L^2})$$

So, $X_L^2 = 5^2 - R^2 = 5^2 - 3^2 = 4^2$, i.e., $X_L = 4\,\Omega$

but as, $X_L = \omega L$, $L = \frac{X_L}{\omega} = \frac{4}{50} = 0.08$ H

7. (1) Now when the capacitor is connected to the above circuit in series,

$$X_C = \frac{1}{\omega C} = \frac{1}{50\times 2500\times 10^{-6}} = \frac{10^3}{125} = 8\,\Omega$$

so, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (4-8)^2} = 5\,\Omega$

and hence, $I_{\text{rms}} = \frac{V}{Z} = \frac{12}{5} = 2.4$ A

So, $P_{av} = V_{rms} I_{rms} \cos\phi = (I_{rms} \times Z) \times I_{rms} \times \left(\frac{R}{Z}\right)$

i.e., $P_{av} = I_{rms}^2 R = (2.4)^2 \times 3 = 17.28$ W

For Problems 8–9

8. (1) **9. (1)**

$Z^2 = (X_C - X_L)^2 + R^2 = (31.85 - 6.28)^2 + (50)^2 = 3154$

$$P = \left(\frac{E_{rms}^2}{Z^2}\right) R = \frac{(10/\sqrt{2})^2}{3154} \times 50 = 0.8 \text{ W}$$

Heat produced in 20 min = (0.8)(20 × 60) = 960 J

$X_C - X_L = 31.85 - 2(6.28) = 19.29$

$$I_m = \frac{10}{19.29} = 0.52$$

Hence, $I = 0.52 \sin(314\,t + \pi/2) = 0.52 \cos 314t$

For Problems 10–13

10. (1) **11. (4)** **12. (2)** **13. (4)**

10. (1) As this circuit is a series LCR circuit, current will be maximum at resonance, i.e.,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(4.9 \times 10^{-3})(10^{-6})}} = \frac{10^5}{7} \text{ rad s}^{-1}$$

with $I = \frac{V}{R} = \frac{10}{(32 + 68)} = \frac{1}{10}$ A

So the impedance, $Z_P = [R_1^2 + (1/\omega C)^2]^{1/2} = \sqrt{5924} = 77\ \Omega$

11. (4) $Z_Q = [R_2^2 + (\omega L)^2]^{1/2}$

$= [(68)^2 + (4.9 \times 10^{-3} \times 10^5/7)^2]^{1/2} = \sqrt{9524} = 97.6\ \Omega$

12. (2) $V_P = IZ_P = \frac{1}{10} \times (77) = 7.7$ V

13. (4) $V_Q = IZ_Q = \frac{1}{10} \times (97.6) = 9.76$ V

For Problems 14–16

14. (2) **15. (1)** **16. (3)**

At resonance as $X = 0$, $I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2}$ A

as $V_L = IX_L = I\omega L$, $L = \frac{V_L}{I\omega}$

So, $L = \frac{40}{(1/2) \times 4 \times 10^5} = 0.2$ mH

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega^2}$$

i.e., $C = \frac{1}{0.2 \times 10^{-3} \times (4 \times 10^5)^2} = \frac{1}{32}$ μF

Now in case of series LCR circuit,

$$\tan\phi = \frac{X_L - X_C}{R}$$

So current will lag the applied voltage by 45° if,

$$\tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$1 \times 120 = \omega \times 2 \times 10^{-4} - \frac{1}{\omega(1/32) \times 10^{-6}}$$

$$\omega^2 - 6 \times 10^5\,\omega - 16 \times 10^{10} = 0$$

$$\text{i.e., } \omega = \frac{6 \times 10^5 \pm \sqrt{(6 \times 10^5)^2 + 64 \times 10^{10}}}{2}$$

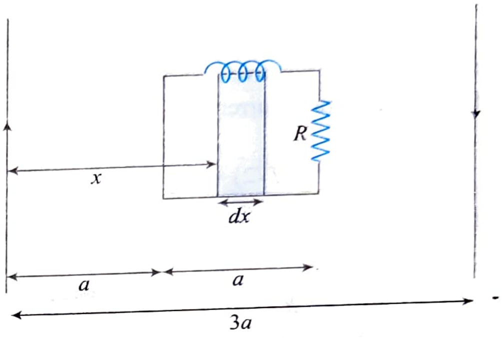
$$\text{i.e., } \omega = \frac{6 \times 10^5 + 10 \times 10^5}{2} = 8 \times 10^5 \text{ rad s}^{-1}$$

For Problems 17–19

17. (1) **18. (1)** **19. (4)**

$d\phi = BdA$

$$d\phi = \left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3a - x)}\right] a dx$$



$$\phi = \frac{\mu_0 I}{2\pi}\left[\int_a^{2a} \frac{dx}{x} + \int_a^{2a} \frac{dx}{(3a - x)}\right] a; \quad \phi = \frac{\mu_0 I a}{\pi} \ln 2$$

Magnitude of emf in this circuit:

$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0 a (\ln 2)}{\pi} \left|\frac{dI}{dt}\right|$$

$$= \frac{\mu_0 a \ln 2}{\pi} I_0 \omega \sin\omega t$$

ac current, $I = \frac{\mu_0 a \ln 2 I_0 \omega}{\pi\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \phi)$

Matrix Match Type

1. **i. → b., c.; ii. → b., c.; iii. → a., b., c., d.; iv. → b., c., d.**

i. $L = \frac{\mu N^2 A}{l}$

So, L depends upon shape, size (l, A) and medium (μ) inserted.

ii. $C = \frac{\varepsilon A}{d}$

So, C depends upon shape, size (A, d) and medium (ε) inserted.

iii. $Z = \sqrt{R^2 + x^2}$

When $x_L = \omega L$, ω depends upon the external voltage source. So, Z depends upon all the factors in column II.

iv. $X_C = \frac{1}{\omega C}$ (independent of resistivity)

2. **i. → d.; ii. → a., d.; iii. → b., d.; iv. → b.**

i. For sinusoidal curve $i_{rms} = i_0/\sqrt{2}$ and average current for positive half cycle is $2i_0/\pi$.

ii. Let us find the average current (or rms current) for $t = 0$ to $T/4$. Then it will also be average current (or rms current) for half cycle, i.e., from $t = 0$ to $T/2$.

For $t = 0$ to $T/4$: $i = \dfrac{4i_0}{T}t$

$$\text{then} \quad i_{\text{av}} = \frac{\int_0^{T/4} i\,dt}{T/4} = \frac{\int_0^{T/4} \frac{4i_0}{T}t\,dt}{T/4} = \frac{i_0}{2}$$

$$\text{and} \quad i_{\text{rms}} = \sqrt{\frac{\int_0^{T/4} i^2\,dt}{T/4}} = \sqrt{\frac{64i_0^2}{T^3}\int_0^{T/4} t^2\,dt} = \frac{i_0}{\sqrt{3}}$$

Full cycle average current is zero.

iii. For positive half cycle, average current

$$= \frac{\int i\,dt}{\int dt} = \frac{i_0(T/2)}{T/2} = i_0$$

Full cycle average current is zero.

iv. Average current for positive half cycle is i_0.
For full cycle, average current

$$= \frac{\int i\,dt}{\int dt} = \frac{i_0(T/2)+0}{T} = \frac{i_0}{2}$$

$$\text{and} \quad i_{\text{rms}} = \sqrt{\frac{\int_0^{T/2} i_0^2\,dt}{T}} = \frac{i_0}{\sqrt{2}}$$

3. i. → b., c.; ii. → a., d.; iii. → a., c.; iv. → b., d.

i. $\tan\phi = \dfrac{1/\omega C}{R} = 1 \Rightarrow \phi = \dfrac{\pi}{4}$, current leads source voltage because reactance is capacitive.

ii. Pure inductive circuit $\phi = \dfrac{\pi}{2}$, current lags behind source voltage because reactance is inductive.

iii. as $R = 0$, $\tan\phi = \infty$ $(X_C > X_L)$
$\phi = \pi/2$, current leads source voltage because reactance is capacitive.

iv. $\tan\phi = \dfrac{\omega L}{R} = 1 \Rightarrow \phi = \dfrac{\pi}{4}$, current lags behind source voltage because reactance is inductive.

4. i. → b., d.; ii. → a..; iii. → a.; iv. → b., c.

$$X_L = \omega L = 2\pi \times 50 \times \frac{100}{\pi} \times 10^{-3} = 10\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times \frac{100}{\pi} \times 10^{-6}}\ \Omega = 100\ \Omega$$

Impedence of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 134.5\ \Omega$$

rms value of the current through the circuit is

$$I_{\text{rms}} = \frac{220}{Z} = 1.63\ \text{A}$$

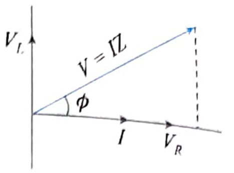
rms value of voltage drop across the capacitor is
$V_{Cv} = I_{\text{rms}} X_C = 163$ V
Average power dissipated in the resistor is

$$P_{\text{av}} = I_{\text{rms}}^2 R = 1.63^2 \times 100 = 265.7\ \text{W}$$

Average power dissipated in inductor and capacitor would be zero.

5. i. → a., b.; ii. → c.; iii. → a., b., c., d.; iv. → d.

i. For LR series circuit, the phasor diagram is as shown below
I lags voltage by an angle ϕ $(< \pi/2)$.
I lags V_L by an angle $\pi/2$.



ii. For RC series ac circuit, I leads V by an angle less than $\pi/2$.

iii. Depending on the value of L, C and R, circuit would be either capacitive, inductive or purely resistive. All possibilities can be there.

iv. In purely resistive circuit, current and voltage are in same phase.

Numerical Value Type

1. a. (4), b. (1) In such cases it is difficult to develop a single equation. Hence, it is usual to consider two equations, one applicable from 0 to 1 ms and another from 1 to 2 ms.

For t lying between 0 and 1 ms, $V_1 = 4$. For t lying between 1 and 2 ms, $V_2 = -4t + 4$.

$$\text{(a)} \quad V_{\text{rms}} = \sqrt{\frac{1}{2}\left(\int_0^1 V_1^2\,dt + \int_1^2 V_2^2\,dt\right)}$$

$$V_{\text{rms}}^2 = \frac{1}{2}\left[\int_0^1 4^2\,dt + \int_1^2 (-4t+4)^2\,dt\right]$$

$$= \frac{1}{2}\left[\left|16t\right|_0^1 + \left|\frac{16t^3}{3}\right|_1^2 + \left|16t\right|_1^2 - \left|\frac{32t^2}{2}\right|_1^2\right] = \frac{32}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{32}{3}} = 4\sqrt{\frac{2}{3}}\ \text{V, Hence } N = 4$$

$$\text{(b)} \quad V_{\text{av}} = \frac{1}{2}\left[\int_0^1 V_1\,dt + \int_1^2 V_2\,dt\right] = \frac{1}{2}\left[\int_0^1 4\,dt + \int_1^2 (-4t+4)\,dt\right]$$

$$= \frac{1}{2}\left[\left|4t\right|_0^1 + \left|-\frac{4t^2}{2} + 4t\right|_1^2\right] = 1\ \text{V}$$

2. (5) From the $i - t$ graph (figure), area from $t = 0$ to $t = 2$ s

$$\frac{1}{2} \times 2 \times 10 = 10\ \text{A s}$$

$$\therefore \quad \text{Average current} = \frac{10}{2} = 5\ \text{A}$$

3. (0) $$\langle i \rangle = \frac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin\omega t\,dt}{\frac{3\pi}{2\omega} - \frac{\pi}{2\omega}} = \frac{I_m\left(-\frac{\cos\omega t}{\omega}\right)_{\pi/2\omega}^{3\pi/2\omega}}{\frac{\pi}{\omega}} = 0$$

4. (5) Net current: $I = 3 + 4\sqrt{2}\sin\omega t$

$$\text{Now find } I_v = \sqrt{\frac{\int_0^T (3 + 4\sqrt{2}\sin\omega t)^2\,dt}{T}} = 5\ \text{A}$$

5. (8) $P_{\text{av}} = E_v I_v \cos\phi = E_v \dfrac{E_v}{Z}(R/Z)$

Z has to be same in both cases.

$$Z_1 = Z_2 \Rightarrow \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\Rightarrow \quad X_L = X_C - X_L \quad \Rightarrow \quad 2X_L = X_C \quad \Rightarrow \quad 2\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \quad C = \frac{1}{2\omega^2 L}$$

$$\Rightarrow \quad C = \frac{1}{2\times(2\pi f)^2 L} = \frac{1}{2\times 4\pi^2 \times (50)^2 \times 5/8} = 8\times 10^{-6}\text{ F}$$

6. **(4)** $V_C - V_L = 120$

$$V_R^2 + (V_C - V_L)^2 = E_v^{\,2}$$

$$V_R^2 + (120)^2 = (200)^2 \quad \Rightarrow \quad V_R = 160$$

$$I = \frac{V_R}{R} = \frac{160}{40} = 4\text{ A}$$

Archives

JEE Main

Single Correct Answer Type

1. **(4)** When only the capacitance is removed, the phase difference between the current and voltage is

$$\tan\phi = \frac{X_L}{R}$$

$$\tan 30^\circ = \frac{X_L}{R} \quad \text{or} \quad X_L = \frac{1}{\sqrt{3}}R$$

When only the inductance is remove, the phase difference between current and voltage is

$$\tan\phi' = \frac{X_C}{R} \Rightarrow \tan 30^\circ = \frac{X_C}{R} \ \textit{or}\ X_C = \frac{1}{\sqrt{3}}R$$

As $X_L = X_C$, therefore the given series LCR is in resonance.

$\therefore$ Impedance of the circuit is $Z = R = 200\ \Omega$

The power dissipated in the circuit is

$$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

$$= \frac{V_{rms}^2}{Z}\cos\phi \qquad \left(\because I_{rms} = \frac{V_{rms}}{Z}\right)$$

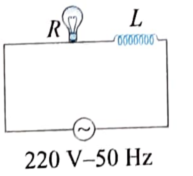
At resonance, power factor $\cos\phi = 1$

$$\therefore \quad P = \frac{V_{rms}^2}{Z} = \frac{(220V)^2}{(200\Omega)} = 242\text{ W}$$

2. **(4)** Resistance of lamp $R = \frac{V}{i} = \frac{80}{10} = 8\Omega$

$$V_R = iR = 80$$

$$\left(\frac{220}{\sqrt{R^2 + X_L^2}}\right)R = 80 \Rightarrow \sqrt{R^2 + X_L^2} = \frac{11}{4}R$$



$$X_L^2 = \frac{105}{16}R^2 = \frac{105}{16}\times 8^2$$

$$X_L^2 = 420 \Rightarrow X_L = 20.5$$

$$2\pi fL = 20.5 \Rightarrow 2\times 3.14\times 50L = 20.5$$

On solving we get, $L = 0.065$ H

3. **(2)** Quality factor $Q = \frac{\omega_0}{\Delta\omega}$

Band width $\Delta\omega = \frac{R}{L}$

Hence $Q = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R}$

4. **(3)** $P_{avg} = V_{rms} I_{rms}\cos\phi$

$$= \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\phi = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45^\circ$$

$$= \frac{1000}{\sqrt{2}}\text{ watt}$$

Wattless current $= I_{rms}\sin\phi$

$$= \frac{I_0}{\sqrt{2}}\sin\phi = \frac{20}{\sqrt{2}}\sin 45^\circ = 10\text{ A}.$$

JEE Advanced

Single Correct Answer Type

1. **(2)** Impedance $Z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

As ω increases, Z decreases. Hence, bulb will glow brighter.

Multiple Correct Answers Type

1. **(2), (3)**

Case (A): $Z_A = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Case (B): $Z_B = \sqrt{R^2 + \left(\frac{1}{\omega KC}\right)^2}$

So $Z_B < Z_A$

$$I_R^A = \frac{V}{Z_A} \quad \text{and} \quad I_R^B = \frac{V}{Z_B} \quad \text{clearly} \quad I_R^A < I_R^B$$

Since current in case (B) is greater, so p.d. across R will increase in case (B) and thus across capacitor will decrease.

Hence $V_C^A > V_C^B$ $\qquad (\because\ V_R^2 + V_C^2 = V_0^2)$

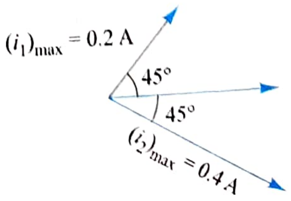
2. **(1), (3)**

$$C = 100\ \mu\text{F},\ \frac{1}{\omega C} = \frac{1}{(100)(100\times 10^{-6})}$$

$$X_L = 100\ \Omega,\ X_L = \omega L = (100)(0.5) = 50\ \Omega$$

$$Z_1 = \sqrt{X_L^2 + 100^2} = 100\sqrt{2}\ \Omega$$

$$Z_2 = \sqrt{X_L^2 + 50^2} = \sqrt{50^2 + 50^2} = 50\sqrt{2}$$



$$\varepsilon = 20\sqrt{2}\sin\omega t$$

$$i_1 = \frac{1}{5}\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$i_2 = \frac{20\sqrt{2}}{50\sqrt{2}}\sin\left(\omega t - \frac{\pi}{4}\right) = \frac{2}{5}\sin\left(\omega t - \frac{\pi}{4}\right)$$

$$i = \sqrt{(0.2)^2 + (0.4)^2} = (0.2)\sqrt{1+4} = \frac{1}{5}\sqrt{5} = \frac{1}{\sqrt{5}}$$

$$(I)_{\text{rms}} = \frac{1}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \approx 0.3 \text{ A}$$

$$(V_{100\Omega}) = (I_1)_{\text{rms}} \times 100 = \left(\frac{0.2}{\sqrt{2}}\right) \times 100 = 10\sqrt{2} \text{ V}$$

$$(V_{50\Omega})_{\text{rms}} = \left(\frac{0.4}{\sqrt{2}}\right) \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2} \text{ V}$$

3. (3), (4)

If q represents the charge on capacitor's upper plate:

$$I(t) = I_0\cos(\omega t) = \frac{dq}{dt} \Rightarrow q(t) = \frac{I_0}{\omega}\sin(\omega t)$$

$$\text{Max charge} = \frac{1 \text{ A}}{500 \text{ rad s}^{-1}} = 2 \times 10^{-3} \text{ C}$$

$$\text{Charge on upper plate at } t = \frac{7\pi}{6\omega} = \frac{I_0}{\omega}\sin\left(\frac{7\pi}{6}\right) = -\frac{I_0}{2\omega}$$

When capacitor is fully charged, charge on upper plate $= 50 \text{ V} \times 20\ \mu\text{F} = 1 \times 10^{-3}\text{C}$

$$\therefore\ Q = 1 \times 10^{-2} \text{ C} - \left(-\frac{I_0}{2\omega}\right) = 2 \times 10^{-3} \text{ C}$$

Voltage across capacitor when A and D are connected

$$= \frac{1 \times 10^{-3} \text{ C}}{20 \times 10^{-6}\ \mu\text{F}} = 50 \text{ V}$$

Total voltage across resistor = 100 V

$$\Rightarrow \text{current} = \frac{100 \text{ V}}{10\ \Omega} = 10 \text{ A}$$

Current is anticlockwise.

4. (1), (3)

We know $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}$

At $\omega \sim 0$,

$X_L \sim 0$ and $X_C \to \infty$

$\therefore$ Current will be nearly zero.

For $\omega = 10^6$, $X_L = \omega L = 10^6 \times 10^{-6} = 1\,\Omega$

and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{10^6 \times 10^{-6}} = 1\,\Omega$

For $\omega >> 10^6$, $X_L >> 1\,\Omega$ and $X_C << 1\,\Omega$ or $X_c \to 0$

Hence circuit does not behave like a capacitor.

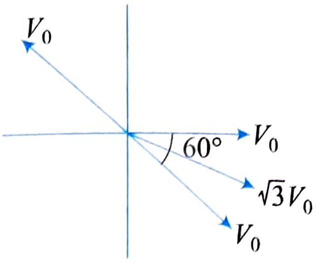
At resonance frequency (ω_R) i.e. when $X_C = X_L$ current will be in phase with voltage and frequency is independent of R.

Resonant frequency

$$\omega_R = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6}10^{-6}}} = 10^6 \text{ rad s}^{-1}$$

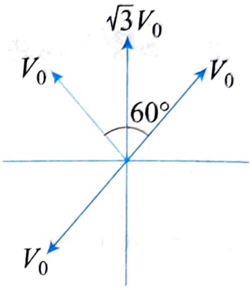
5. (3), (4)

$$V_{XY} = V_0\sin\omega t - V_0\sin\left(\omega t + \frac{2\pi}{3}\right)$$



$$V_{XY}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}}$$

$$V_{YZ} = V_0\sin\left(\omega t + \frac{2\pi}{3}\right) - V_0\sin\left(\omega t + \frac{4\pi}{3}\right)$$



$$V_{YZ}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}}$$

$\therefore$ Options (3) and (4) are correct.

Matrix Match Type

1. i. → c., d., e.; ii. → b., c., d., e.; iii. → a., b.; iv. → b., c., d., e.

a. In this case, steady state current is zero ($I = 0$) because of the presence of capacitor. Entire potential will be across capacitor in steady state. Hence $V_1 = 0$ and $V_2 = V$.

b. In steady state $V_1 = 0$, so $V_2 = 0$. Also V_2 is proportional to I.

c. $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 6 \times 10^{-3} = 1.885\ \Omega$

$R = 2\ \Omega$

Since $R > X_L$, so $V_2 > V_1$

Here $I \neq 0$

V_1 and V_2 both are proportional to I.

d. $X_L = 1.885\ \Omega$,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 3 \times 10^{-6}} = 1061\ \Omega$$

Hence $X_C > X_L$, so $V_2 > V_1$

e. $X_C = 1.061$ kΩ, $R = 1$ kΩ

$X_C > R$, so $V_2 > V_1$

Numerical Value Type

1. (4) $\omega = 500$ rad s^{-1}

$$Z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2} = R\sqrt{1.25}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = R^2(1.25) \Rightarrow \left(\frac{1}{\omega C}\right)^2 = \frac{R^2}{4}$$

$$\Rightarrow\quad CR = \frac{2}{\omega} = \frac{2}{500} \text{ s} = \frac{2}{500} \times 10^3 \text{ ms} = 4 \text{ ms}$$

Chapter 7

Exercises

Single Correct Answer Type

1. **(2)** Intensity is given by, $\frac{P}{A}=\frac{3.9\times10^{26}}{4\pi r^2}=5.62\times10^7\ \text{W/m}^2$

2. **(1)** Electric or magnetic field has zero average value in e.m.f. waves

3. **(3)** $Z=\sqrt{\frac{\mu_r}{\varepsilon_r}}\times\sqrt{\frac{\mu_0}{\varepsilon_0}}=\sqrt{\frac{50}{2}}\times376.6\Omega=1883\ \Omega$

4. **(1)** $v=1057$ MHz

$$\lambda=\frac{c}{v}=\frac{3\times10^8}{1057\times10^6}\text{m}=0.28\text{ m}=28\text{ cm}\quad\text{(radio waves)}$$

5. **(2)** $\lambda_m T=\text{constant}$

$$\Rightarrow\quad\lambda_m\times(2.7\text{ K})=0.2896\text{ cm K}$$

$$\therefore\quad\lambda_m=\frac{0.2896}{2.7}\text{cm}=0.10\text{ cm}=1\text{ mm}$$

6. **(3)** $E=hv\ \therefore\ E\propto v$

7. **(2)** $E=\frac{hc}{\lambda},\ E\propto\frac{1}{\lambda}$ or $eV\propto\frac{1}{\lambda}$

8. **(1)** X-rays are not reflected from the target, so cannot be used for radar.

9. **(3)** $E=hv$

$V=E/h=5\times10^{17}$ Hz

10. **(4)** $eV=hv=\frac{hc}{\lambda}\quad\therefore\quad\lambda=\frac{hc}{eV}$

11. **(2)** Area surrounding the TV tower is

$$A=\pi r^2\qquad(r=d)$$

$$=\pi d^2=\pi\left(\sqrt{2hR}\right)^2=\pi.2hR$$

$$a\propto h\qquad\therefore\qquad a'=2a.\ h'=2h$$

12. **(1)** Here, $\lambda=\frac{c}{v}=\frac{3\times10^8}{8.2\times10^6}=36.6$ m

13. **(2)** Comparing the given equation with equation of plane electromagnetic wave, $E_z=E_0\cos(\omega t+kx)$, we have

$\omega=6\times10^{-8}$ and $k=4$.

Velocity of light in medium

$$v=\frac{\omega}{k}=\frac{6\times10^8}{4}=\frac{3}{2}\times10^8\text{ m/s}$$

Refractive index, $\mu=\frac{c}{v}=\frac{3\times10^8}{(3/2)\times10^8}=2$

14. **(3)** $\mu=\frac{c}{v}$

Now $c=\frac{1}{\sqrt{\mu_0K_0}}$ and $v=\frac{1}{\sqrt{\mu_0K_0\mu_rK_r}}$

$$\therefore\quad\mu=\sqrt{\mu_rK_r}$$

But $\mu_r=\mu_0$ and $K_r=K_0$

$$\therefore\quad\mu=\sqrt{\mu_0K_0}$$

15. **(2)** $\mu=\frac{c}{v}=\frac{1}{\sqrt{\mu_0\varepsilon_0}}\times\frac{\sqrt{\mu\varepsilon}}{1}=\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

16. **(3)** $c=\frac{E_0}{B_0}$

$$\therefore\quad B_0=\frac{E_0}{c}=1.6\times10^{-6}\ \text{Wb/m}^2$$

17. **(1)** As $B\propto r$, since the point is on the axis, where $r=0$, so $B=0$.

18. **(3)** $I_{av}=\frac{1}{2}\varepsilon_0E_0^2\times c=1.719\ \text{W/m}^2$

19. **(1)** Average energy density of electromagnetic wave

$$U_{av}=\frac{1}{2}\varepsilon_0E_0^2=\frac{1}{2}\times(8.85\times10^{-12})\times(50)^2$$

$$\approx10^{-8}\text{ J/m}^3$$

20. **(3)** $C=\frac{\varepsilon_0KA}{d}=\frac{(8.85\times10^{-12})\times10\times1}{10^{-3}}=8.85\times10^{-8}\text{ F}$

$$i=\frac{d}{dt}(CV)=C\frac{dV}{dt}=8.85\times10^{-8}\times25=2.2\times10^{-6}\text{ A}$$

$$=2.2\ \mu\text{A}$$

21. **(2)** Charge on capacitor plates at time t is, $q=it$

Electric field between the plates at this instant.

$$E=\frac{q}{A\varepsilon_0}=\frac{it}{A\varepsilon_0}$$

Electric flux through the given area

$$\phi_E=\left(\frac{A}{2}\right)E=\frac{it}{2\varepsilon_0}$$

Therefore, displacement current

$$i_d=\varepsilon_0\frac{d\phi_E}{dt}=\varepsilon_0\frac{d}{dt}\left(\frac{it}{2\varepsilon_0}\right)=\frac{i}{2}$$

22. **(3)** Total power = Solar constant × Area

$$=10^4\times(10\times10)\times10^6\text{ W}$$

23. **(4)** Electric energy density

$$u_e=\frac{1}{2}\varepsilon_0E_{rms}^2$$

$$E_{rms}=\frac{E_0}{\sqrt{2}}$$

$$\therefore\quad u_e=\frac{1}{4}\varepsilon_0E_0^2$$

24. **(4)** Power = Area × Solar constant = 1.62×10^5 W

25. **(4)** Force of radiation $=\frac{\text{Power}}{c}=\frac{1.6\times10^5}{3\times10^8}=5.32\times10^{-4}\text{ N}$

26. **(1)** Transferred momentum/second to mirror is

$$P=\frac{2AS}{C}=1.21\times10^{-10}\text{ kg m s}^{-1}$$

27. **(2)** Number of oscillation in coherence length

$$\frac{\ell}{\lambda}=\frac{0.024}{5900\times10^{-10}}=4.068\times10^4$$

28. **(4)** $B = \dfrac{E}{c} = \dfrac{10^{-4}}{3\times10^{8}} = 3.3\times10^{-13}\,\mathrm{T}$

29. **(2)** $B_0 = \dfrac{E_0}{c} = \dfrac{10^{-3}}{3\times10^{8}} = 3.3\times10^{-12}\,\mathrm{T}$

30. **(3)** $\oint \vec{E}.d\vec{l} = -\dfrac{d\phi_B}{dt}$

or $E2\pi r = \dfrac{d}{dt}(Kt\times\pi r^2)$

or $E = \dfrac{Kr}{2}$

31. **(3)** Radiation force = Momentum transferred per second by electromagnetic wave to the mirror

$$= \frac{2I_{\text{av}}A}{c} = \frac{2\times(10)\times(20\times10^{-4})}{(3\times10^{8})} = 1.33\times10^{-10}\ \mathrm{N}$$

32. **(4)** The pointing vector $\vec{S} = \dfrac{1}{\mu_0}(\vec{E}\times\vec{B})$

$$= \frac{1}{4\pi\times10^{-7}}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 80 & 32 & -64\\ 0.2 & 0.08 & 0.29\end{vmatrix} = 11.52\hat{i} - 28.8\hat{j}$$

33. **(2)** $\omega = 2\pi v \dfrac{2\pi c}{\lambda} = \dfrac{2\pi\times3\times10^{8}}{6\times10^{-3}}$

$= \pi\times10^{11}$ rad/s

The equation for the electric field, along y-axis in the electromagnetic wave is

$$E_y = E_0\sin\omega\left(t - \frac{x}{c}\right) = 33\sin\pi\times10^{11}(t - x/c)$$

34. **(4)** $B_0 = \dfrac{E_0}{c} = \dfrac{48}{3\times10^{8}} = 16\times10^{-8}\,\mathrm{T}$

$$B_{\text{rms}} = \frac{B_0}{\sqrt{2}} = \frac{16\times10^{-8}}{\sqrt{2}} = 8\sqrt{2}\times10^{-8}\,\mathrm{T}$$

$= 11.3\times10^{-8}$ T

35. **(4)** Wave is propagating along positive X-axis; magnetic field is directed along Z-axis so electric field must be directed along $(\hat{k}\times\hat{i}) = \hat{j}$, along Y-axis.

Amplitude of electric field

$$E_0 = B_0c = 2\times10^{-7}\times3\times10^{8} = 60\ \mathrm{V/m}$$

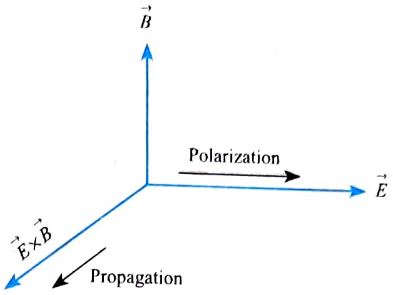
So choice (4) is correct.

Archives

JEE Main

Single Correct Answer Type

1. **(2)**



2. **(1)** $E_0 = CB_0$

$= 3\times10^{8}\times20\times10^{-9}$

$= 6$ V/m

3. **(2)** Infrared waves → To treat muscular strain

Radio waves → for broadcasting

X-rays → To detect fracture of bones

Ultraviolet rays → Absorbed by the ozone layer of the atmosphere

4. **(1)** Energy is equally divided between electric and magnetic field.

5. **(2)** $I = \dfrac{P}{4\pi r^2} = U_{\text{av}}\times c$ (i)

$U_{av} = \dfrac{1}{2}\varepsilon_0E_0^2$ (ii)

$$\Rightarrow \quad \frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0E_0^2\times c$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2P}{4\pi r^2\varepsilon_0c}} = 2.45\ \mathrm{V/m}$$

6. **(4)** In air $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\dfrac{z}{c} - t\right)\right]$

In medium $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z - ct)]$

During refraction, frequency remains unchanged, whereas wavelength gets changed.

Velocity of wave in medium $v = \dfrac{\omega}{k} = \dfrac{kc}{2k} = \dfrac{c}{2}$

$$\therefore \quad \frac{1}{\sqrt{\mu_0\varepsilon_{r_2}}} = \frac{1}{2}\frac{1}{\sqrt{\mu_0\varepsilon_{r_1}}} \Rightarrow \sqrt{\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

APPENDIX

1

Chapterwise Solved January 2019 JEE Main Questions (All Sets)

Chapter 1
Magnetic Field and Magnetic Forces

1. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron $=1.6 \times 10^{-19}$C)

(1) 2.0×10^{-24} kg
(2) 1.6×10^{-19} kg
(3) 1.6×10^{-27} kg
(4) 9.1×10^{-31} kg

2. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter up to 15 volt, is

(1) 80 Ω
(2) 120 Ω
(3) 125 Ω
(4) 100 Ω

3. The region between $y = 0$ and $y = d$ contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. If $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is

(1) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$
(2) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{\sqrt{2}}\hat{j}\right)$
(3) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$
(4) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$

4. A particle of mass m and charge q is in an electric and magnetic field given by

$\vec{E} = 2\hat{i}+3\hat{j}$; $\vec{B} = 4\hat{j}+6\hat{k}$

The charged particle is shifted from the origin to the point $P(x = 1; y = 1)$ along a straight path. The magnitude of the total work done is

(1) $(0.35)q$
(2) $(0.15)q$
(3) $(2.5)q$
(4) $5q$

5. A proton and an α-particle (with their masses in the ratio of 1:4 and charges in the ratio of 1:2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_a$ of the circular paths described by them will be

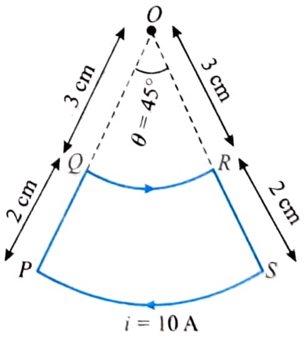
(1) $1:\sqrt{2}$
(2) 1 : 2
(3) 1 : 3
(4) $1:\sqrt{3}$

Chapter 2
Sources of Magnetic Field

6. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be:

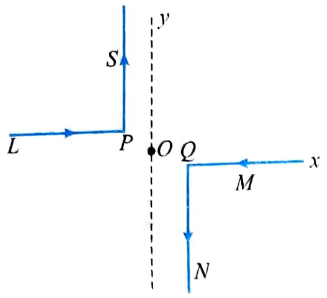
(1) $\frac{1}{N}$
(2) N^2
(3) $\frac{1}{N^2}$
(4) N

7. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to:



(1) 1.0×10^{-5} T
(2) 1.5×10^{-5} T
(3) 1.0×10^{-7} T
(4) 2.0×10^{-7} T

8. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If $OP = OQ = 4$ cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}):



(1) 40 A, perpendicular into the page

(2) 40 A, perpendicular out of the page

(3) 20 A, perpendicular out of the page

(4) 20 A, perpendicular into the page

Chapter 3
Permanent Magnets and Magnetic Properties of Matter

9. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is:

(1) 1200 A/m (2) 2600 A/m

(3) 520 A/m (4) 285 A/m

10. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic neeedle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is

(1) 3.6×10^{-5} N (2) 6.5×10^{-5} N

(3) 1.3×10^{-5} N (4) 1.8×10^{-5} N

11. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then:

(1) $T_h = 0.5\ T_c$ (2) $T_h = 2\ T_c$

(3) $T_h = 1.5\ T_c$ (4) $T_h = T_c$

12. An insulating thin rod of length l has a x linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin ($x = 0$) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is:

(1) $\frac{\pi}{4} n\rho l^3$ (2) $n\rho l^3$

(3) $\pi n\rho l^3$ (4) $\frac{\pi}{3} n\rho l^3$

13. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is

(1) 2.3×10^{-2} (2) 3.3×10^{-2}

(3) 3.3×10^{-4} (4) 4.3×10^{-2}

14. A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is

(1) 3.672×10^{-4} (2) 3.726×10^{-4}

(3) 3.267×10^{-4} (4) 2.672×10^{-4}

15. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, of 0.3×10^{-4} Wb/m^2. The value of the induced emf in wire is

(1) 2.5×10^{-3} V (2) 1.1×10^{-3} V

(3) 0.3×10^{-3} V (4) 1.5×10^{-3} V

Chapter 5
Inductance

16. The self-induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is:

(1) 437.5 J (2) 637.5 J

(3) 740 J (4) 540 J

17. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self-inductance of the coil:

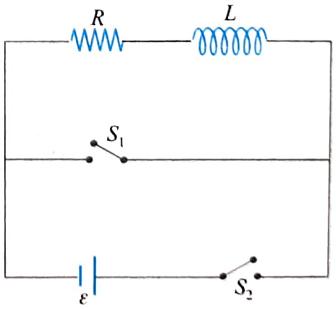
(1) Decreases by a factor of $9\sqrt{3}$

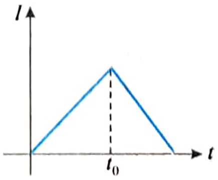
(2) Increases by a factor of 3

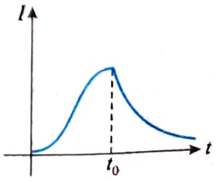
(3) Decreases by a factor of 9

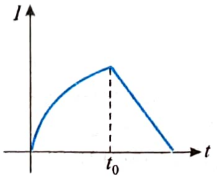
(4) Increases by a factor of 27

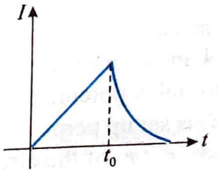
18. In the circuit shown, the switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviors of the current I as a function of time 't' is given by



(1)


(2)


(3)


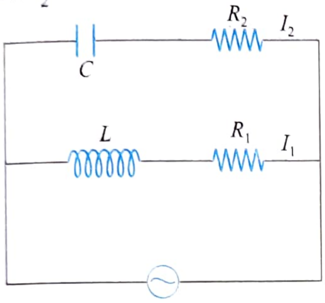
(4)


19. There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The rate of mutual inductance to the self-inductance of the inner-coil is

(1) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$ (2) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$

(3) $\frac{n_1}{n_2}$ (4) $\frac{n_2}{n_1}$

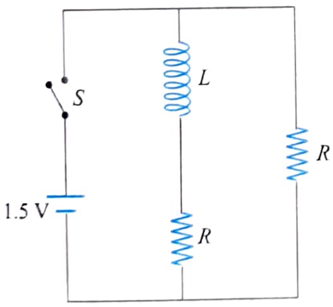
20. In the above circuit, $C = \frac{\sqrt{3}}{2}\,\mu\text{F}$, $R_2 = 20\,\Omega$, $L = \frac{\sqrt{3}}{10}\,\text{H}$ and $R_1 = 10\,\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of A.C source is given by $V = 200\sqrt{2}\sin(100t)$ between I_1 and I_2 is:



(1) 30° (2) 0°

(3) 90° (4) 60°

21. In the figure shown, a circuit contains two identical resistors with resistance $R = 5\Omega$ and an inductance with $L = 2$ mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed?



(1) 6 A (2) 7.5 A

(3) 5.5 A (4) 3 A

Chapter 6
Alternating Current

22. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is:

(1) 2.26×10^3 J (2) 3.39×10^3 J

(3) 5.65×10^2 J (4) 5.17×10^2 J

Chapter 7
Electromagnetic Waves

23. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then:

(1) $U_E = \frac{U_B}{2}$ (2) $U_E < U_B$

(3) $U_E = E_B$ (4) $U_E > U_B$

24. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\,\hat{j}$ V/m The corresponding magnetic field $\vec{B}$, at that point will be:

(1) $18.9 \times 10^{-8}\,\hat{k}$T (2) $6.3 \times 10^{-8}\,\hat{k}$T

(3) $2.1 \times 10^{-8}\,\hat{k}$T (4) $18.9 \times 10^{8}\,\hat{k}$T

25. The electric field of a plane polarized electromagnetic wave in free space at time $t = 0$ is given by an expression

$\vec{E}(x, y) = 10\hat{j}\cos[(6x + 8z)]$

The magnetic field B (x, z, t) is given by : (c is the velocity of light)

(1) $\frac{1}{c}(6\hat{k} + 8\hat{i})\cos[(6x - 8z + 10ct)]$

(2) $\frac{1}{c}(6\hat{k} - 8\hat{i})\cos[(6x + 8z - 10ct)]$

(3) $\frac{1}{c}(6\hat{k} + 8\hat{i})\cos[(6x + 8z - 10ct)]$

(4) $\frac{1}{c}(6\hat{k} - 8\hat{i})\cos[(6x + 8z + 10ct)]$

26. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8 m/s)

$B = 100 \times 10^{-6}\sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$ then the maximum electric field associated with it is:

(1) 4×10^4 N/C (2) 4.5×10^4 N/C

(3) 6×10^4 N/C (4) 3×10^4 N/C

27. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by [Given permittivity of space $\varepsilon_0 = 9 \times 10^{-12}$ SI units, Speed of light $c = 3 \times 10^8$ m/s]

(1) 1 kV/m (2) 2 kV/m

(3) 1.4 kV/m (4) 0.7 kV/m

28. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by:

(1) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (2) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

(3) $(\sqrt{n}, \sqrt{n})$ (4) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

29. The mean intensity of radiation on the surface of the Sun is about 10^8 W/m^2. The rms value of the corresponding magnetic field is closest to:

(1) 10^2 T (2) 10^{-4} T

(3) 1 T (4) 10^{-2} T

Answers Key

1. (1) **2.** (1) **3.** (*) **4.** (4) **5.** (1)
6. (3) **7.** (1) **8.** (4) **9.** (1) **10.** (2)
11. (4) **12.** (1) **13.** (3) **14.** (3) **15.** (4)

16. (1) **17.** (2) **18.** (2) **19.** (4) **20.** (*)
21. (1) **22.** (4) **23.** (3) **24.** (3) **25.** (2)
26. (4) **27.** (3) **28.** (2) **29.** (2)